The Electron Density Distribution in the Lower Ionosphere Produced through Impact Ionization by Precipitating Electrons and through Photoionization by the Associated Bremsstrahlung X-Rays

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Abstract

In this paper characteristic features of the synthetic effect of a flux of precipitating mono-energy electrons are studied for the representative incident energies. The maximum rate of impact ionization produced by an electron flux of $2\pi \times 10^7 \text{cm}^{-2}\text{sec}^{-1}$ is found to be about $1.7 \times 10^3 \text{cm}^{-3}\text{sec}^{-1}$ at about the 116 km-level for the incident electron energy $E_0=10 \text{ keV}$, and about $2.3 \times 10^4 \text{cm}^{-3}\text{sec}^{-1}$ at about the 88 km-level for $E_0=100 \text{ keV}$. On the other hand, the maximum rate of photoionization produced through the absorption of bremsstrahlung x-rays caused by the same flux is about $3.6 \times 10^{-2} \text{cm}^{-3}\text{sec}^{-1}$ at about 97 km for $E_0=10 \text{ keV}$, and about $1.9 \times 10^4 \text{cm}^{-3}\text{sec}^{-1}$ at about 45 km for $E_0=100 \text{ keV}$. The consequent maximum electron density is estimated to be about $8.4 \times 10^4 \text{cm}^{-3}$ at about 117 km for $E_0=10 \text{ keV}$, and about $3.0 \times 10^5 \text{cm}^{-3}$ at about 90 km for $E_0=100 \text{ keV}$. In the latter case where $E_0=100 \text{ keV}$, the electron density of the order of $10^3 \text{cm}^{-3}$ extends to a level as low as 65 km. The associated auroral luminosity is estimated to be 0.21 kR (kilo-rayleighs) at 3914 Å, 0.14 kR at 15577 Å, and 0.17 kR at 26300 Å for $E_0=10 \text{ keV}$; and 1.4 kR at 3914 Å, 0.15 kR at 5577 Å, and 0.046 kR at 6300 Å for $E_0=100 \text{ keV}$. Estimates are also made of the total effects of a flux of precipitating electrons having an energy spectrum expressed as $2\pi i_0 \exp (-E_0/\beta)$ at the top of the atmosphere. With $i_0=10^6 \text{cm}^{-2}\text{sec}^{-1}\text{kev}^{-1}\text{ster}^{-1}$ and $\beta=5 \text{ keV}$, the maximum electron density is found to be about $3.2 \times 10^5 \text{cm}^{-3}$ at about 114 km. It is suggested that the electron flux of $2\pi \times 4.5 \times 10^4 \text{cm}^{-2}\text{sec}^{-1}$ having such an energy spectrum would be required in order to explain the observed electron density in the nighttime E layer at middle latitudes, as due to the influx of energetic electrons.

1. Introduction

Attempts have been repeatedly made to estimate the ionization and the excitation produced by the impact of energetic electrons precipitating into the atmosphere (Tohmatsu and Nagata, 1960; Oguti and Nagata, 1961; Oguti, 1963; Rees, 1963; Maeda, 1965; Kamiyama, 1966a). A great number of x-ray bursts have been observed at balloon-heights, accompanying aeronomic disturbances at high latitudes. These bremsstrahlung x-rays associated with precipitating electrons have been shown to be effective in ionizing the D region of the ionosphere (Aikin and Maier, 1963; Rees, 1964; Kamiyama, 1966b). In most of the studies referred to above, a certain energy spectrum of precipitating electrons has been assumed to estimate the ionizing effect. However, the energy spectrum varies extremely from event to
event (O'Brien, 1963), and the evidence has been shown for the significant difference between the spectra during the day and the night (Oguti, 1963).

Because of the variability in the energy spectrum, this paper directs greater attention to the characteristic features of the synthetic effects of a flux of mono-energy electrons. A considerable flux of electrons with energies exceeding 40 kev has been detected in observations by satellites and rockets during geomagnetic disturbances. Electrons with energies at about 10 kev are also known to be responsible for producing aurora. This paper will choose as representative for the incident energy of such electrons, the values of 10 kev and 100 kev. The satellite observation (Freeman, 1964) has indicated that in the magnetospheric tail beyond the distance of 8 earth's radii, there exists a hot plasma with the mean energy of the order of a kev which could possibly precipitate on occasion into the ionospheric region. Therefore, an additional description is given of the ionization caused by 1 kev electrons. A comparison of the theoretical results for the representative incident energies with the observational facts would then lead to a better understanding of the auroral-zone phenomena.

2. The energy dissipation of precipitating electrons

According to satellite observations of precipitating electrons at about the 1000 km-level (O'Brien, 1964), the electron flux decreases sharply towards higher incident energy, and the flux at relativistic energies is found to be about $10^3 \text{cm}^{-2} \text{sec}^{-1}$ during an auroral display (Paulikas and Freden, 1964) which is about one-millionth of the flux of lower energy electrons. Hence, the only concern of this paper will be with calculations for electrons having non-relativistic energies.

As an electron penetrates into the atmosphere, spiraling along a line of magnetic force, the kinetic energy possessed is dissipated in inelastic collisions with atmospheric particles and in the emission of bremsstrahlung. According to Heitler (1954), the energy loss per unit length of a path traversed through inelastic collisions is expressed, in the energy range concerned, by

$$-\left(\frac{dE}{dx}\right)_e = N(z)Z\phi_0 \frac{3\mu}{4E} \left[ \ln \frac{E}{IZ\sqrt{2}} + \frac{1}{2} \right],$$  \hspace{1cm} (1)

where $E$ denotes the kinetic energy of an electron, $x$ the path length in cm traversed by an electron, $N(z)$ the number of atmospheric particles per cm$^3$ at an altitude $z$, $Z$ the atomic number, $\phi_0$ the cross section for Thomson scattering ($6.65 \times 10^{-26} \text{cm}^2$), $\mu$ the rest energy of an electron, and $I$ the mean ionization potential of an atmospheric particle. On the other hand, the energy change through the bremsstrahlung emission is expressed by

$$-\left(\frac{dE}{dx}\right)_{\text{br}} = N(z)E\phi_r,$$  \hspace{1cm} (2)

in which the total cross section $\phi_r$ for the radiation of bremsstrahlung is given by integrating the cross section $\phi(E, h\nu)$ for the emission of a photon with a particular energy $h\nu$. For an electron with energy $E$, the cross section for the emission of a photon in an energy range
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\( h\nu \sim h\nu + d(h\nu) \) is given, in the non-relativistic energy range, by

\[
\phi(E, h\nu)d(h\nu) = \frac{3}{8} \frac{m}{E} \ln \left( \frac{\sqrt{E + \sqrt{E - h\nu}}^2}{h\nu} \right) \cdot \frac{d(h\nu)}{h\nu},
\]

where

\[ \phi = ro^{2}/137, \]

in which \( ro = e^2/(mc^2) \), \( e \) and \( m \) being the charge and mass of an electron, respectively, and \( c \) the light velocity in a vacuum.

The total change in the energy of an electron is then given by

\[
-\frac{dE}{dx} = -\left( \frac{dE}{dx} \right)_e - \left( \frac{dE}{dx} \right)_s.
\]

However, in the energy range below a few hundred kilovolts, the energy loss due to the radiation of bremsstrahlung is negligible compared with that due to inelastic collision. The energy of a precipitating electron thus calculated as a function of altitude in the previous paper (Kamiyama, 1966a) is then applicable to the study in this paper. The computation was based on a model atmosphere illustrated in Fig. 1 which is essentially identical with that given by Harris and Priester (1962) for one of the conditions during the night. The computed results for electrons with various initial energies and pitch angles are reproduced in Fig. 2. Conclusions from the figure are that 1-kev electrons are stopped in

![Graph](image_url)
the height range from about 140 km to 250 km and that 10-kev electrons are effective in producing ionization and excitation in the E region, while 100-kev electrons penetrate into the atmosphere to about the 80km-level.

3. Bremsstrahlung photon flux

The emission of bremsstrahlung photons in an energy range from \( h\nu \) to \( h\nu + d(h\nu) \) caused by an electron with energy \( E \) is given by

\[
P(z, E, h\nu) d(h\nu) = N(z) \phi(E, h\nu) d(h\nu),
\]

where the cross section \( \phi(E, h\nu) \) is already defined by (3).

Assuming an isotropic incidence of electrons at the 1000 km-level, the emissivity of bremsstrahlung photons with various energies has been computed in the preceding paper (Kamiyama, 1966b) for a unit flux of incident electrons (1 electron cm\(^{-2}\)sec\(^{-1}\)ster\(^{-1}\)) with the representative energies. The maximum emissivity has been found in the region from 170 km to 190 km for \( E_0 = 1 \) kev, at about 110 km for \( E_0 = 10 \) kev, and at about 90 km for \( E_0 = 100 \) kev.

In calculating the photon flux at any point in the atmosphere, the assumption is made that photons are equally radiated in all directions from a volume element. The error introduced by this assumption is expected to be within a factor of 2 at most for \( E_0 = 100 \) kev, and much less for lower incident energies. Assuming also the uniform precipitation of electrons over a sufficiently large area, and taking into account the absorption on a path traversed in the atmosphere, the preceding paper (Kamiyama, 1966b) calculated the photon flux which is given by the integration of a flux contributed from the emission from a volume element with respect to space. At a point at any level \( z_p \), the upward and downward photon fluxes in an energy range from \( h\nu \) to \( h\nu + d(h\nu) \) which are caused by a unit flux of electrons with an incident energy \( E_0 \) are expressed as
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\[ J_d(E_0, h\nu, z_p) d(h\nu) = \frac{1}{2} \int_0^{z_p} P(z, E_0, h\nu) d(h\nu) \cdot \varphi(h\nu, z, z_p) dz, \quad (6) \]

and

\[ J_d(E_0, h\nu, z_p) d(h\nu) = \frac{1}{2} \int_{z_p}^{\infty} P(z, E_0, h\nu) d(h\nu) \cdot \varphi(h\nu, z, z_p) dz, \quad (7) \]

where

\[ \varphi(h\nu, z, z_p) = \int_1^t e^{-At} \frac{dt}{t} = \ln A - 0.5722 + A - \frac{A^2}{2!} - \cdots - \frac{(-A)^r}{r \cdot r!}, \quad (8) \]

and \( A \) is the optical thickness, at frequency \( \nu \), defined by

\[ A = \left| \sigma \int_{z_p}^z N(z) dz \right|. \quad (9) \]

The values of the absorption cross section (Nawrocki and Papa, 1963) are listed in Table 1 in the following section. The photon emissivity \( P(z, E_0, h\nu) d(h\nu) \) is given by Eq. (5), because the energy of a precipitating electron is given as a function of altitude for the incident energy \( E_0 \). The results computed in the preceding paper show that the downward flux maximizes in the region between 160 km and 210 km for \( E_0 = 1 \) kev, between 110 km and 130 km for \( E_0 = 10 \) kev, and between 40 km and 200 km for \( E_0 = 100 \) kev, depending on photon energy, while the upward fluxes are almost constant and are much more intense than the corresponding downward fluxes in the region above about 300 km.

### 4. Ionization rate

Ionization of atmospheric particles occurs in two different ways. One is the ionization caused by the impact of energetic particles, and the other is the photoionization through the absorption of photons. First, the photoionization by the bremsstrahlung photons associated with energetic electrons will be considered. In a photoionization process, photon energy is absorbed by an atom which then releases an electron with a certain kinetic energy. When

<table>
<thead>
<tr>
<th>Photon Energy (kev)</th>
<th>Cross Section (cm(^2))</th>
<th>Ionization Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>5.5 \times 10^{-24}</td>
<td>1.4 \times 10^{9}</td>
</tr>
<tr>
<td>20</td>
<td>1.9 \times 10^{-23}</td>
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<tr>
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<td>1.0 \times 10^{-22}</td>
<td>2.7 \times 10^{2}</td>
</tr>
<tr>
<td>5</td>
<td>1.5 \times 10^{-21}</td>
<td>1.3 \times 10^{2}</td>
</tr>
<tr>
<td>2</td>
<td>1.3 \times 10^{-20}</td>
<td>4.5 \times 10^{3}</td>
</tr>
<tr>
<td>1</td>
<td>9.0 \times 10^{-20}</td>
<td>1.5 \times 10^{4}</td>
</tr>
<tr>
<td>0.5</td>
<td>3.3 \times 10^{-19}</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>2.1 \times 10^{-19}</td>
<td>1</td>
</tr>
<tr>
<td>0.1</td>
<td>6.9 \times 10^{-19}</td>
<td>1</td>
</tr>
<tr>
<td>0.05</td>
<td>3.9 \times 10^{-18}</td>
<td>1</td>
</tr>
<tr>
<td>0.02</td>
<td>1.1 \times 10^{-17}</td>
<td>1</td>
</tr>
</tbody>
</table>
the photon energy absorbed is high, the ejected electron possesses a sufficient kinetic energy to ionize other molecules in collisions. Therefore, it is convenient to adopt an ionization efficiency $M(h\nu)$ which is equal to the total number of electrons created through absorption of a photon with energy $h\nu$. For the air, the ionization efficiencies estimated at various photon energies are given as listed in Table 1, together with the values of the absorption cross section. The production rate of electrons in a unit volume at any level $z_p$ through the absorption of photon flux, $(J_u(E_0, h\nu, z_p) + J_d(E_0, h\nu, z_p)) d(h\nu)$ is then given by

$$Q_p(E_0, h\nu, z_p)d(h\nu) = \sigma_z M(h\nu) N(z) \{ J_u(E_0, h\nu, z_p) + J_d(E_0, h\nu, z_p) \} d(h\nu).$$

The total rate of photoionization produced by the photon fluxes in the entire energy range caused by unit flux electrons with incident energy $E_0$ is given by the integration

$$Q(E_0, z_p) = \int_0^{E_0} Q_p(E_0, h\nu, z_p)d(h\nu).$$

The integration has been performed numerically, assuming that the absorption cross section is constant in a small energy range. The total photoionization rate, thus computed, caused by a unit flux of electrons with the incident energies, 1 kev, 10 kev, and 100 kev are illustrated in Fig. 3.

Next, calculation for the rate of the ionization caused by electron impact will be concerned. The energy released in a unit length of the path from a precipitating electron with energy $E$ is $-dE/dx$ which is well approximated by Eq. (1). A part of the energy absorbed in air may be available for ionizing atmospheric molecules. According to Valentine and Curran (1958), one ion pair is produced by the absorption in air of an average of 35 ev through inelastic collisions of electrons. Hence, the ion production rate $Q_i(E, \alpha, z)$ at a level $z$ due to flux $i(E, \alpha, z)$ of electrons of energy $E$ and pitch angle $\alpha$ is given by

![Fig. 3 Height distributions of the production rate of electrons through impact ionization and through photoionization by the bremsstrahlung photons resulting from a unit flux of electrons (1 electron cm$^{-2}$ sec$^{-1}$ ster$^{-1}$) with a particular incident energy.](image-url)
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\[ Q(E, a, z) = \frac{N(z)}{35 \varepsilon} Z \phi(Z) \frac{3 \mu}{4E} \left[ \ln \frac{E}{IZ \sqrt{2}} + \frac{1}{2} \right] i(E, a, z), \quad (12) \]

where
\[ \varepsilon = 1.602 \times 10^{-12} \text{erg} \cdot \text{eV}^{-1}. \]

Since the energy of a precipitating electron is given as a function of altitude and incident energy \( E_0 \) for the different pitch angles, the production rate given above may be regarded as the function of altitude and incident energy \( E_0 \) at the top of the atmosphere, \( z_t \). Here, \( Q(E_0, a, z) \) denotes the rate of impact ionization resulting from an incident flux \( i(E_0, a, z) \). Integrating \( Q(E_0, a, z) \) with respect to pitch angle, the total rate of the electron production by impact of all electrons with the incident energy \( E_0 \) is given by
\[ Q(E_0, z) = 2\pi \int_0^{\pi/2} \sin \alpha \cdot Q(E_0, a, z) d\alpha. \quad (13) \]

The computed results for the isotropic incidence of a unit flux of mono-energy electrons (1 electron cm\(^{-2}\) sec\(^{-1}\) ster\(^{-1}\)) are shown in Fig. 3, compared with the results for photoionization by the bremsstrahlung flux.

The figure shows for the incident energy \( E_0 = 1 \) kev, the diffuse distribution of the photoionization which is exceeded greatly, by a factor of \( 10^6 \), by the impact ionization which maximizes at about the 160 km-level. For \( E_0 = 10 \) kev, the maximum of the impact ionization is found to be at about 120 km, while the broad maximum of the photoionization appears at about 100 km. The tendency of the increasing difference between the heights of the maxima of the impact ionization and the photoionization becomes much more pronounced at higher incident energies. For \( E_0 = 100 \) kev, the impact ionization shows the sharp maximum slightly below the 90 km-level, while the photoionization is significant in the wide region between 30 km and 85 km, being maximum at about 55 km. The general conclusion reached is that the photoionization by bremsstrahlung photons is greatly exceeded by the impact ionization, but is effective, especially at higher incident energies, in ionizing the lower region of the atmosphere.

So far in this paper only calculations for mono-energy electrons have been concerned. In practice, however, a flux of precipitating electrons is characterized by an energy spectrum which may vary significantly from event to event. Therefore, as an example, the total effect of an electron flux having a certain form of energy spectrum will be presented below. According to the measurement made by McIlwain (1960), an energy spectrum is assumed so that the number of electrons incident on the top of the atmosphere is expressed as a function of incident energy \( E_0 \) by
\[ i(E_0, a, z_0) dE_0 = I_0(a, z_0) \exp(-E_0/\beta) dE_0 (\text{cm}^{-2} \text{sec}^{-1} \text{ster}^{-1}), \quad (14) \]

with \( \beta = 5 \) kev. The constant \( I_0 \) varies in a wide range with relation to the auroral activity and may attain to the order of \( 10^6 \) cm\(^{-2}\) sec\(^{-1}\) kev\(^{-1}\) ster\(^{-1}\) when a bright aurora appears. Integrating Eqs. (11) and (13) with respect to the incident energy \( E_0 \), the total production rates of the photoionization and impact ionization are shown in Fig. 4 as a function of altitude for \( I_0 = 10^6 \) cm\(^{-2}\) sec\(^{-1}\) kev\(^{-1}\) ster\(^{-1}\). The result shows that the photoionization is exceeded by
the impact ionization by a factor of about $10^4$ in the region above 90 km, but predominates in the D region. The effect of a change in the energy spectrum might be inferable from a reference to the results for mono-energy electrons.

5. Electron density

In the previous paper (Kamiyama, 1966a), the electron density was estimated in a rather simple manner for the flux with an assumed energy spectrum, and too high a value of the ratio of the electron density to the auroral luminosity was derived compared with the observed results (Omholt, 1955; Oguti, 1960). In this paper, calculations are carried out with the reliable rate coefficients in the ionic processes, for the fluxes of mono-energy electrons and for the flux with the energy spectrum assumed by Eq. (14).

In the region between the 100 km- and the 200 km-levels, diffusion may be ignored, and the attachment of electrons to neutral particles is assumed to be negligible compared with the dissociative recombination with positive ions. Then, the condition of volume neutrality assesses that

$$n_e = \sum n_i,$$

where $n_e$ denotes the electron density and $n_i$ the density of the $i$th species of ion. In order to estimate the electron density, only the major ionic constituents such as $O^+$, $O_2^+$, $N_2^+$, and NO$^+$ will be considered here. Atomic oxygen ions produced are lost through the processes,

$$O^+ + O_2 \rightarrow O_2^+ + O \quad (k_1)$$

and

![Graph showing height distribution of the rates of impact ionization and photoionization caused by the electron flux having the energy spectrum assumed by Eq. (14) with $I_o = 10^9\text{cm}^{-2}\text{sec}^{-1}\text{kev}^{-1}\text{ster}^{-1}$ and $\beta = 5\text{kev}$.]
in which the rate coefficients are denoted by $k_1$ and $k_2$, respectively. The above ion-atom interchange processes are so dominant in the region concerned that the radiative recombination of $O^+$ with an electron may be ignored. The concentration of atomic oxygen ion, $n(O^+)$, then satisfies

$$\frac{dn(O^+)}{dt} = q(O^+) - \{k_1n(O_2) + k_2n(N_2)\}n(O^+), \tag{18}$$

where $q(O^+)$ is the production rate of $O^+$, and $n$'s are the concentrations of the molecules indicated in parentheses. In equilibrium,

$$n(O^+) = \frac{q(O^+)}{k_1n(O_2) + k_2n(N_2)}. \tag{19}$$

The molecular nitrogen ions react with oxygen through

$$N_2^+ + O = NO^+ + N \quad (k_3) \tag{20}$$

and

$$N_2^+ + O_2 = O_2^+ + N_2. \quad (k_4) \tag{21}$$

The charge transfer process, $N_2^+ + O = O^+ + N_2$, may be ignored, because its reaction rate is very slow compared with that of (20). Molecular oxygen ions produced in the various manners react with molecular nitrogen through

$$O_2^+ + N_2 = NO^+ + NO. \quad (k_5) \tag{22}$$

Although the rate coefficient for the reaction, $O_2^+ + NO = NO^+ + O_2$, is considerably higher than $k_5$, this reaction is assumed to be negligible, because of the much lesser abundance of NO compared with that of $N_2$. The molecular ions, thus formed through the various collision processes and through the ionization processes, are subject to the dissociative recombinations:

$$O_2^+ + e = O^+ + O', \quad (\alpha_1) \tag{23}$$

$$N_2^+ + e = N^+ + N', \quad (\alpha_2) \tag{24}$$

and

$$NO^+ + e = N^+ + O', \quad (\alpha_3) \tag{25}$$

which will balance, in equilibrium, the total of each ionization rate of $O$, $O_2$, and $N_2$. Hence, in equilibrium

$$n(N_2^+) = \frac{q(N_2^+)}{\alpha_2 n_e + k_3 n(O) + k_4 n(O_2)}, \tag{26}$$

$$n(O^+) = \frac{q(O^+) + k_1 n(O^+)n(O_2) + k_2 n(N_2^+)n(O_2)}{\alpha_1 n_e + k_5 n(N_2)}. \tag{27}$$

and
\[ n(\text{NO}^+) = \frac{(k_3 n(O^+) + k_4 n(O_{3}^+)) n(N_2) + k_5 n(N_2^+) n(O)}{n_e}. \]  

The substitution of (19), (26), (27), and (28) into (15) leads to
\[ n_e k + U(z) n_e^4 + V(z) n_e^3 + W(z) n_e = X(z), \]
where
\[ U(z) = n_5 + n_7 - n(O^+), \]
\[ V(z) = n_5 n_7 - ((n_1 + n_2 + n_5 + n_7) n(O^+) + \nu_1 + \nu_2), \]
\[ W(z) = -(n_4 + n_5 + n_9) \nu_2 - n_5 \nu_3, \]
and
\[ X(z) = n_3 n_7 (n_2 + n_3) n(O^+) + n_5 (n_6 \nu_2 + n_7 \nu_3). \]

Here \( n's \) and \( \nu's \) denote the following:
\[ n_1 = k_1 n(O_2) / \alpha_1, \]
\[ n_2 = k_2 n(N_2) / \alpha_3, \]
\[ n_3 = k_1 n(O_2) / \alpha_3, \]
\[ n_4 = k_4 n(O_2), \]
\[ n_5 = k_5 n(N_2) / \alpha_1, \]
\[ n_6 = k_4 n(O_2) / \alpha_3, \]
\[ n_7 = (k_3 n(O) + k_4 n(O_2)) / \alpha_2, \]
\[ n_8 = k_3 n(N_2) / \alpha_3, \]
\[ n_9 = k_3 n(O) / \alpha_3, \]
\[ \nu_1 = q(O_{3}^+) / \alpha_1, \]
\[ \nu_2 = q(O_{2}^+) / \alpha_2, \]
and
\[ \nu_3 = q(O_{3}^+) / \alpha_3. \]

With the above notations, Eqs. (26), (27), and (28) may be rewritten as
\[ n(N_2^+) = \frac{\nu_2}{n_e + n_7}, \]  

\[ n(O_2^+) = \frac{\nu_1 + n_e n(O^+) + n_e n(N_2^+)}{n_e + n_5}, \]
and
\[ n(\text{NO}^+) = \frac{n_e n(O^+) + n_5 n(O_{3}^+) + n_3 n(N_2^+)}{n_e}. \]

The values of the rate coefficient for the various processes are listed in Table 2.

The ionization rate for each atmospheric constituent is assumed to be proportional to the fraction of the total of each constituent at the altitude under consideration, both for the impact ionization and for the photoionization by bremsstrahlung x-rays. For oxygen, how-
ever, Fite and Brackman (1959) show that about one third of the ionization of molecular oxygen produced by electron impact leads to the process

\[ O_2 + e^{(p)} \rightarrow O^+ + O + e^{(s)} + e^{(p)}, \]  

(30)

where \( e^{(p)} \) and \( e^{(s)} \) denote a primary and secondary electron, respectively. The production rate of atomic oxygen ions through impact ionization is, therefore,

\[ q_1(O^+) = \frac{n(O^+)(1/3)n(O_2)}{N} Q_0, \]  

(31)

and that of molecular ions is

\[ q_2(O_2^+) = \frac{2}{3} \frac{n(O_2)}{N} Q_0. \]  

(32)

The total of the production rates resulting from the impact ionization and the photoionization is shown by the solid lines for each ionic species in Figs. 5a, 5b, and 5c. The rate of the total production of all the ions is drawn as the dashed-and-dotted line.

Table 2. Rate Coefficients

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Notation</th>
<th>Coefficient ((\text{cm}^3\text{sec}^{-1}))</th>
<th>Reference</th>
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<tr>
<td>( O^+ + O_2 \rightarrow O_2^+ + O )</td>
<td>( k_1 )</td>
<td>( 4.7 \times 10^{-12} )</td>
<td>Ferguson,</td>
</tr>
<tr>
<td>( O^+ + N_2 \rightarrow NO^+ + N )</td>
<td>( k_2 )</td>
<td>( 3.0 \times 10^{-12} )</td>
<td>Fehsenfeld, Goldan,</td>
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<td>( N_2^+ + O \rightarrow NO^+ + N )</td>
<td>( k_3 )</td>
<td>( 2.5 \times 10^{-10} )</td>
<td>and Schmeltekopf (1965)</td>
</tr>
<tr>
<td>( O_2^+ + N_2 \rightarrow NO^+ + NO )</td>
<td>( k_4 )</td>
<td>( 1.0 \times 10^{-10} )</td>
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<tr>
<td>( O_2^+ + e \rightarrow O^+ + O'' )</td>
<td>( \alpha_1 )</td>
<td>( 1.7 \times 10^{-7} )</td>
<td>Biondi (1964),</td>
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<td>( N_2^+ + e \rightarrow N^+ + N'' )</td>
<td>( \alpha_2 )</td>
<td>( 2.8 \times 10^{-7} )</td>
<td>Kasner and Biondi (1965),</td>
</tr>
<tr>
<td>( NO^+ + e \rightarrow N^+ + O' )</td>
<td>( \alpha_3 )</td>
<td>( 3.2 \times 10^{-7} )</td>
<td>Gunton and Shaw (1965)</td>
</tr>
</tbody>
</table>

Fig. 5a  The production rate of each ionic species and the equilibrium electron density distribution resulting from the isotropic incidence of \( 10^7\text{cm}^{-2}\) sec\(^{-1}\) ster\(^{-1}\) of mono-energetic 10 kev-electrons.
Below 100 km, numerous processes involving various kinds of negative ions are of greater significance in estimating the electron density. However, all the ways in which these negative ions are formed and lost have not yet been clearly understood; further, the rates at which the processes concerned take place are quite uncertain. Therefore, the electron density is deduced from the relation

$$\frac{1}{2} n_e = \frac{1}{\alpha_{\text{eff}}}$$

where $\alpha_{\text{eff}}$ denotes the effective recombination coefficient for which the experimental result presented by Witten, et al. (1965) is adopted. The extrapolation of their curve is made in this paper up to the 95 km-level so that the electron density given by Eq. (33) is in
agreement with the result derived from Eq. (29).

The electron density, thus computed in the whole region below 200 km, is shown by the heavy lines in Figs. 5a, 5b, and 5c, respectively, for the mono-energy isotropic fluxes of $10^7$ cm$^{-2}$sec$^{-1}$ster$^{-1}$ of 10 kev-electrons, and of 100 kev-electrons, and for the isotropic flux having an energy spectrum given by Eq. (14) with $I_0=10^8$cm$^{-2}$sec$^{-1}$kev$^{-1}$ster$^{-1}$. The maximum electron density is found to be 8.4 x 10$^4$ cm$^{-3}$ at 117 km for $E_0=10$ kev, 3.0 x 10$^5$ cm$^{-3}$ at 90 km for $E_0=100$ kev, and 3.2 x 10$^5$ cm$^{-3}$ at 114 km for the flux with the spectrum assumed above. In the case of 100 kev-electrons, the broadening of the D region with the significant electron density results from the photoionization by bremsstrahlung x-rays with energies exceeding 10 kev. Viewing the results, the flux having an energy spectrum, such as expressed by Eq. (14) with a constant somewhat greater than 5 kev for $\beta$, seems to be responsible for the auroral sporadic E ionization and cosmic noise absorption events.

6. Auroral luminosity

The intensities of some of the auroral lines resulting from an electron influx have been calculated theoretically by Tohmatsu and Nagata (1960) for the energy spectrum in which the flux of incident electrons is inversely proportional to their incident energy, and by the author (Kamiyama, 1966a) for the energy spectrum assumed by Eq. (14) with $\beta=5$ kev. In this section, estimates are made of the intensities at $\lambda\lambda$3914, 5577, and 6300 resulting from the fluxes of mono-energy electrons. The comparison of the results characterized by the incident energy with the luminosity resulting from the flux having the assumed energy spectrum would then lead to a better understanding of the problem.

According to Takayanagi and Yonezawa (1961), about 0.3 $n(O)/N$ per secondary electron produced by electron impact are excited to O(1S) through

$$O(3P) + e(e) \rightarrow O(1S) + e(e),$$

and about 0.05 $n(N_2)/N$ per secondary electron are excited to $N_2^*(B^2\Sigma_u^+)$ through

$$N_2(X^1\Sigma_g^+) + e(e) \rightarrow N_2^*(B^2\Sigma_u^+) + e(e),$$

where $N$ is the total atmospheric concentration as denoted before. The cross section for the excitation to O(1D) is larger than that to O(1S) by a factor of about 11, averaged over the energy range concerned (Seaton, 1953). Hence, a major part of the excitation rate to the 1D state is obtained readily from that to the 1S state. However, an additional excitation to the 1D state resulting from the dissociative recombination of $O_2^+$ should be taken into account. The excitation rate to the 1D state through this process is given by $a_1n_e n(O_2^+)$, where $n_e$ and $n(O_2^+)$ are given by Eqs. (27) and (29), respectively. As for the excitation to $N_2^*(B^2\Sigma_u^+)$, the contribution from primary electrons through the process

$$N_2(X^1\Sigma_g^+) + e(e) \rightarrow N_2^*(B^2\Sigma_u^+) + e(e)$$

must be taken into account. The cross section for the excitation of the (0,0) first negative bands of $N_2^+$ by electron impact is adopted from the results of the measurement made by Stewart (1956) and by Hayakawa et al. (1956). Since the production rate of secondary elec-
trons by electron impact has been estimated in Section 4, and the energy of precipitating electrons is given as a function of altitude in Section 2, one can readily obtain the vertical distributions of the rates of the excitations which are shown in Figs. 6a, 6b, and 6c, for the fluxes of 10 kev- and 100 kev-electrons, respectively, and for the flux having the spectrum (14) with $I_0=10^4\text{cm}^{-2}\text{sec}^{-1}\text{kev}^{-1}\text{ster}^{-1}$ and $\beta=5$ kev.

As for the red line ($\lambda 6300\mu\text{m}$) emission from the oxygen atoms in the $^1\text{D}$ state, the collisional deactivation is of considerable importance. According to Seaton (1958), the ratio $r$ of the collisional deactivation probability to the radiative transition ($^1\text{D}$)$-^3\text{P}$ is expressed as

$$r = 3.2 \times 10^{-12} N + \frac{2.0 \times 10^{-9} N(O)}{1 + 3.0 \times 10^{-12} N(O)}.$$  \hspace{1cm} (37)

Fig. 6a Height distributions of the rates of the excitations caused by the isotropic incidence of $10^4\text{cm}^{-2}\text{sec}^{-1}$ ster$^{-1}$ of mono-energetic 10 kev-electrons.

Fig. 6b Height distributions of the rates of the excitations caused by the isotropic incidence of $10^4\text{cm}^{-2}\text{sec}^{-1}$ ster$^{-1}$ of mono-energetic 100 kev-electrons.
Hence, the red line emission is given by multiplying the excitation rate by the factor of $1/(1+r)$, and the results are shown by the dotted lines in the figures.

The heights at which the excitations maximize are estimated to be about 120 km for $E_0 = 10$ keV, and about 90 km for $E_0 = 100$ keV. A slight difference between the heights of the maximum rates of the O(1S) excitation and the $N_2^+(B^2\Sigma^+)$ excitation is essentially due to the change in the relative abundance of O and N$_2$. The dissociative recombination of O$_2^+$ contributes to the O (1D) excitation to a considerable extent in the lower region, where the red line emission, however, is suppressed drastically by the deactivation process. Hence, the luminosities of the representative auroral lines, which are given by integrating the emission over the height range, are remarkably characterized by the incident energy of primary electrons. The flux of $2\pi \times 10^6$ cm$^{-2}$ sec$^{-1}$ gives rise to the luminosities of 210 R (rayleighs) at $\lambda 3914$, 140 R at $\lambda 5577$, and 170 R at $\lambda 6300$ for $E_0 = 10$ keV, and 1.4 kR at $\lambda 3914$, 150 R at $\lambda 5577$, and 46 R at $\lambda 6300$ for $E_0 = 100$ keV.

The flux having the energy spectrum assumed as Eq. (14) with $I_0 = 1 \times 10^6$ cm$^{-2}$ sec$^{-1}$ keV$^{-1}$ and $\beta = 5$ keV produces the luminosities of 4.2 kR at $\lambda 3914$, 6.0 kR at $\lambda 5577$, and 13 kR at $\lambda 6300$. The relative intensities of the lines, which are 1:1.4:3.1 for this flux, may change with a variation of the energy spectrum of incident electrons. A flux with a softer energy spectrum will produce the much brighter oxygen lines relative to the first negative bands of nitrogen.

7. Discussion

First, the relation between the maximum electron density in the auroral ionization and the luminosity of aurora will be discussed. Since the rates of the excitations to O(1S) and $N_2^+(B^2\Sigma^+)$ must be proportional to the incident electron flux, in Figs. 7a, 7b, and 7c, straight
lines may be drawn showing the luminosities at $\lambda 3914$ and 5577 as the functions of the incident flux. As for the oxygen red line, its intensity is not strictly in proportion to the incident flux because of the additional excitation through the dissociative recombination of $O_2^+$, as argued in the preceding section. The contribution to the O (1D) excitation from this process is about ten percent in the higher region, and is increased considerably in the region below 110 km, where molecular oxygen is abundant. However, the collisional deactivation of the $^1D$ state dominates over the radiative transition in the lower region. Hence, the red line

![Graph](image_url)

**Fig. 7a** The intensities of some of the auroral lines and the maximum electron density in the ionization as the functions of the incident flux of mono-energetic 10 kev-electrons.

![Graph](image_url)

**Fig. 7b** The intensities of some of the auroral lines and the maximum electron density in the auroral ionization as the functions of the incident flux of mono-energetic 100 kev-electrons.
The electron density distribution in the lower ionosphere may be presumed to be roughly expressed as a linear function of the incident flux. Thus, the straight lines in the figures for λ6300 involve an uncertainty within ten percent.

The electron density, in general, is not expressed as a simple function of the incident flux. In Eq. (29) for the electron density, the fourth term, \( W(z)n_e \), on the left-hand side becomes important as the height is increased. However, in the region below 140 km where the electron density maximizes, only the third term \( V(z)n_e^2 \) becomes essential. Hence, the maximum electron density is expected to be approximately proportional to the square root of the incident flux. The numerical examination has confirmed that the error is within a few percent, in the flux range concerned. Thus, in the same figures straight lines may be drawn showing the maximum electron density as a function of the incident flux.

From the figures, the relation between the maximum electron density and the auroral luminosity can be deduced. Corresponding to a certain value of the maximum electron density, the intensities of some of the auroral lines and the maximum electron density in the auroral ionization as the functions of the incident flux having the energy spectrum given by Eq. (14) with \( \beta = 5 \) kev.

![Figure 7c](image)

Fig. 7c The intensities of some of the auroral lines and the maximum electron density in the auroral ionization as the functions of the incident flux having the energy spectrum given by Eq. (14) with \( \beta = 5 \) kev.

### Table 3 Expected and Observed Luminosity of Aurora

<table>
<thead>
<tr>
<th>Electron energy Flux spectrum</th>
<th>Expected Luminosity (KR)</th>
<th>Obs'd Lum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux spectrum</td>
<td>10 kev Mono-energy</td>
<td>100 kev Mono-energy exp ((-E_0/5))</td>
</tr>
<tr>
<td>Max. Electron density (\lambda)</td>
<td>3914 28 35 42</td>
<td>200*</td>
</tr>
<tr>
<td>(1\times10^6)cm(^{-3})</td>
<td>5577 20 3.6 60</td>
<td>500 #</td>
</tr>
<tr>
<td></td>
<td>6300 23 1.1 130</td>
<td></td>
</tr>
</tbody>
</table>

* Omholt (1955)  # Oguti and Nagata (1961)
density, the luminosities of the auroral lines which would be expected in the equilibrium condition are listed in Table 3, for the fluxes of mono-energy electrons and for the flux with the energy spectrum assumed as Eq. (14).

The results shown in the above table suggest that a flux of lower energy electrons leads to an intensification of the oxygen lines relative to the $N_2^+$ bands. Comparing the theoretical results for the flux having an energy spectrum (14) with the observational results presented by Omholt (1955), and by Oguti and Nagata (1961), the luminosities calculated in this paper are too weak by a factor of $5 \sim 8$. Assuming the energy spectrum in which the flux of incident electrons is inversely proportional to their energy, Tohmatsu and Nagata (1960) have obtained the result which is in agreement with the Omholt's observation but is smaller than the observed intensity at $\lambda 5577$ by a factor of about 3. In so far as the paper is based on the rate coefficients listed in Table 2 for the ionospheric processes, the calculated luminosity of aurora is insufficient in relation to the corresponding maximum electron density. Some possible manners in which the auroral luminosity may be intensified without changing the flux density of precipitating electrons are noted below. If the energetic particle flux gives rise to the dissociation of oxygen to a greater extent during the precipitation, as discussed by Maeda (1963), the intensities of $O_1$ lines would be remarkably increased without a significant change in intensity of the $N_2^+$ bands. Megill and Carleton (1964) have shown the possible mechanism for the excitation in the aurora by an electric field which is associated with geomagnetic disturbances in the auroral zone. The present study has shown, in addition to the possibilities mentioned above, that an augmentation of relatively low energy electrons in the flux would lead to the intensification of the auroral lines without changing the maximum electron density. A precipitation of these low energy electrons could also contribute to the ionization in the F region. It should be noted here that the electron density profile is calculated for the condition in static equilibrium. Therefore, when a precipitation of energetic electrons takes place abruptly or in a pulsating manner, the electron density must be lower than that in equilibrium, while luminosity is almost proportional to the instantaneous flux precipitating into the atmosphere. Hence, brighter aurorae might have been observed in relation to the corresponding electron density calculated under the equilibrium condition. A detailed analysis of the problem will be left to the future.

Next, a discussion is given of the electron density profile in the lower ionosphere. The result in Section 5 shows that the significant ionization in the D region is caused only through the photoionization by the bremsstrahlung flux of high energy photons, which are produced by high energy electrons. On the other hand, impact ionization at about 110 km is produced, for the most part, by electrons with energies of about 10 kev. Hence, the auroral zone absorption of radio noise of cosmic origin should depend strongly upon the spectrum of precipitating electrons. The energy spectrum has been measured by a number of investigators, and shown to be quite variable from event to event. In fact, only loose relationships have been found among auroral zone absorption events, auroral sporadic E ionization, auroral luminosity, and x-ray bursts (Ansari, 1964; Barcus, 1965).
8. Summary

Table 4 summarizes the numerical results obtained in this paper. The various characteristic features appearing in consequence of the energetic electron flux indicated are listed in each of the columns.

The conclusions reached in this study are as follows:
1) During a faint aurora, the electron flux is estimated to be about $10^7 \text{cm}^{-2}\text{sec}^{-1}$. This flux produces the maximum electron density of about $1 \times 10^5 \text{cm}^{-3}$ in the auroral E region.
2) If attempts are made to explain the nighttime electron density of the order of $3 \times 10^3 \text{cm}^{-3}$ in the midlatitude E-layer as due to a precipitation of energetic electrons, the total flux required would be $2\pi \times 1.2 \times 10^5 \text{cm}^{-2}\text{sec}^{-1}$ for mono-energy 10 kev-electrons, $2\pi \times 2.3 \times 10^6 \text{cm}^{-2}\text{sec}^{-1}$ for 100 kev-electrons, and $2\pi \times 4.5 \times 10^4 \text{cm}^{-2}\text{sec}^{-1}$ for electrons having the spectrum, $\exp(-E_0/5)$.
3) The electron density profile is characterized by the energy of incident electrons. Only electrons with energies exceeding 50 kev can produce a significant ionization in the D region.
4) The relative intensities of auroral lines are very sensitive to the energy of incident electrons. An incident flux of electrons having a softer energy spectrum leads to an intensification of the OI lines relative to the N$_2^+$ first negative bands.
5) Since the energy spectrum of precipitating electrons is quite variable, only a loose relationship among the maximum electron density in auroral ionization, auroral luminosity, and the cosmic noise absorption would be expected.
6) In this paper the relationship between the maximum electron density and the au-

<table>
<thead>
<tr>
<th>Energy (kev)</th>
<th>Flux (cm$^{-2}$sec$^{-1}$)</th>
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<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>$2\pi \times 10^7$</td>
</tr>
</tbody>
</table>

| Impact ionization | Maximum rate (cm$^{-2}$sec$^{-1}$) | 1.7x10$^3$ | 2.3x10$^4$ | 2.3x10$^4$ |
|                  | Height at which the rate maximizes (km) | 116 | 88 | 117 |

| Photoionization by bremsstrahlung photons | Maximum rate (cm$^{-2}$sec$^{-1}$) | 3.6x10$^{-2}$ | 1.9x10$^1$ | 1.0x10$^0$ |
|                                           | Height at which the rate maximizes (km) | 97 | 45 | 93 |

| Electron density | Maximum density (cm$^{-3}$) | 8.4x10$^4$ | 3.0x10$^5$ | 3.2x10$^5$ |
|                 | Height at which the density maximizes (km) | 117 | 90 | 114 |
|                 | Density at 80 km (cm$^{-3}$) | 1.2x10$^5$ | 2.9x10$^6$ | 8.5x10$^5$ |
|                 | Density at 70 km (cm$^{-3}$) | <1x10$^1$ | 1.9x10$^2$ | 3.4x10$^2$ |

| Auroral luminosity (KR) | N$_2^+$ λ3914 | 0.21 | 1.4 | 4.2 |
|                        | O λ5577 | 0.14 | 0.15 | 6.0 |
|                        | O λ6300 | 0.17 | 0.046 | 13.0 |
roral luminosity was deduced. A comparison of this relationship with the observational results shows a significant discrepancy, which might be attributed to the various possible effects discussed in Section 7. In this respect, an essential may be that the electron density would not be in the equilibrium condition. A more detailed study is awaited before the discrepancy is accounted for.

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