On the Electrical Conductivity of the Upper Atmosphere

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Abstract

The paper deals with the problem of the electrical conductivity supported by the upper atmospheric ionization, to which the diurnal variation of terrestrial magnetism is attributed in the dynamo theory. Number densities of free electrons and ions and their collision frequencies are substantially important for the evaluation of the vertical distribution and integrated value of the electrical conductivity. For this purpose various elementary processes governing the ionospheric characteristics, that is, recombination, attachment, detachment and collision, are discussed on the basis of ionospheric observation with the aid of some theoretical data by other scientists. Although our present knowledges had to face some contradictions on the way of numerical estimation of necessary elements, it is indicated that the total conductivity will be about 2×10^{-7} e.m.u. during the daytime, with the maximum conductivity of $(1.5 \sim 2.0) \times 10^{-11}$ e.m.u. in the *E*-region.

I. Introduction

Although many important points on the physical relations between the ionosphere and the terrestrial magnetism have been found up to this time, there still remains the problem of the electrical conductivity of the ionosphere, which has a significant bearing to the physical nature of diurnal variation of the terrestrial magnetism. From the analysis of diurnal variations observed in the terrestrial magnetism it was shown that the total conductivity of the upper atmosphere should be of the order of 10^{-6} e.m.u. Recently, however, T. Nagata and others⁽¹⁾ reported the results of study on ionospheric induction effect, and according to their conclusions it will be reasonable to accept that the total conductivity will be less than 10^{-7} e.m.u. Sir Edward Appleton⁽²⁾ gave a formula with a table to evaluate the total conductivity based on the recombination theory and we can calculate the total conductivity by assuming collision frequencies, number densities of electrons and ions and scale height. As to the evaluation of density ratio of electron to ion, which is very important to our present problem, we can refer to the reports by G. Goubau,⁽³⁾ F.L. Mohler,⁽⁴⁾ D.R. Bates and H.S.W. Massey⁽⁵⁾ and others.

The writer discusses here some problems necessary to find out the density ratio of electron to ion and collision frequencies on the basis of ionospheric measurements together with some theories of other scientists, and the vertical distribution and the integrated value of the electrical conductivity and their changes from day to night are presented.

II. Mean collision frequencies

1. Theoretical values

By theoretical calculation of T. Yamanouchi⁽⁶⁾ the mean collision frequency (ν) of free electron in F_2 -layer, where Oxygen atom is the controlling particle, is given as

$$\nu \approx 3 \times 10^3 \text{ sec}^{-1} \quad (h = 250 \text{ km}).$$

In this case the number density of Oxygen atoms is assumed to be about 6×10^9 /c.c. and the temperature as about 1000°K.

2. Observed values

According to the experiments by Farmer, Ratcliffe and others⁽⁷⁾ we have the values for electron collision as follows.

$$\nu \approx 10^5 \text{ sec}^{-1}$$

for E -layer (100 km),

$$\nu \approx 1.5 \times 10^3 \text{ sec}^{-1}$$

for F_2 -layer (250 km).

As to the F_2 -layer we consider the contribution of electronion collision according to T. Yonezawa⁽⁸⁾ and adopt the value of ν a little less than the above value, that is, $(3\sim 10)\times 10^8$ for F_2 -region.

3. Height dependence

Height dependence of ν for electron collision is given by the following relation,⁽⁶⁾

$$\nu \propto N\sqrt{T}, \quad (1)$$

where N is the number density of neutral particles and T the electron temperature. For the purpose we have to know the vertical distributions of N and T , but it is very difficult at present stage to show them exactly. The following table is presented to show the values of ν for each layer, together with other various values, with which we consider the problems as our model ionosphere.

Table of Model Ionosphere

Region	Height (km)	Gas	Tem- perature (deg. Kelvin)	Pressure (mmHg)	Gas density N(cm ⁻³)	Mean collision frequency ν (sec ⁻¹)	Attachment coefficient β (sec ⁻¹)	Apparent re- combination coefficient α' (cm ⁻³ sec ⁻¹)
<i>E</i>	90	O ₂ & O	400	2×10^{-3}	1.4×10^{14}	4×10^5	7×10^{-2}	10^{-8}
	100			5×10^{-4}	3×10^{13}	1×10^5	2×10^{-2}	
	110			2×10^{-4}	1×10^{13}	4×10^4	8×10^{-3}	
	120			1×10^{-4}	5×10^{12}	2.3×10^4	5×10^{-3}	
	200	O	800	2.8×10^{-6} (1.6×10^{-5})*	4×10^{10} (2×10^{11})**	3×10^3	5×10^{-4}	10^{-9}
	220			1.5×10^{-6} (1.0×10^{-5})*	2×20^{10} (1.3×10^{11})**	2×10^3	3×10^{-4}	
	250		1,000	6×10^{-7} (5.0×10^{-6})*	7×10^9 (6×10^{10})**	1×10^3	1.4×10^{-4}	10^{-10}
	400			2×10^{-7} (1.5×10^{-6})*	2.2×10^9 (2×10^{10})**	3×10^2	4×10^{-5}	

()* Values by P.O. Pedersen.⁽⁹⁾

()** Values corresponding to the above.

4. Collision of ion with neutral particles

As we have no exact ground for the estimation of mean collision frequency of ion with neutral particles, except that we know the ratio of mean collision frequencies of electron and ion at the normal pressure, we assumed that the above ratio may hold up to the height of F_2 -layer. The ratio is taken as about 40.⁽⁹⁾

III. Ionic density

1. Mathematical relations of electronic and ionic densities

As is easily shown, the following relations governing the densities of electron, negative and positive ions are given, taking attachment and detachment as well as recombination processes into account.

$$\frac{dn}{dt} = q - \alpha n^+ n - \beta n + \gamma n^- + \delta n^-, \quad (2)$$

$$\frac{dn^-}{dt} = \beta n - \gamma n^- - \delta n^- - \alpha_i n^+ n^-, \quad (3)$$

$$n^+ = n^- + n, \quad (4)$$

where

$n, n^-, n^+ \dots$ number densities (cm^{-3}) of electron, negative and positive ions respectively,

$q \dots \dots \dots$ ion production rate ($\text{cm}^{-3} \cdot \text{sec}^{-1}$) by solar radiation,

$\alpha \dots \dots \dots$ recombination coefficient ($\text{cm}^3 \cdot \text{sec}^{-1}$) of electron and positive ion.

$\alpha_i \dots \dots \dots$ recombination coefficient ($\text{cm}^3 \cdot \text{sec}^{-1}$) of negative and positive ions,

$\beta \dots \dots \dots$ attachment coefficient (sec^{-1}) of electron to neutral particle,

$\gamma \dots \dots \dots$ photo-detachment coefficient (sec^{-1}) of electron from negative ion,

$\delta \dots \dots \dots$ detachment coefficient (sec^{-1}) of electron from negative ion by collision.

During the night we may put $q = \gamma = 0$.

2. Attachment coefficient (β)

Concerning β for F -region, some theoretical results⁽⁹⁾⁽¹⁰⁾ on attachment cross section of Oxygen atom and many experimental data⁽¹¹⁾ are available, but as for E -region, the value is speculative. Considering simply the pressure dependence of β we use the values in the above table for our present work.

3. Recombination coefficient (α and α_i)

Besides the theoretical formulas, a great many experimental data⁽¹²⁾ are available. Considering that the observed value of α is actually the apparent value α' , which is equal to $(1 + \lambda)\alpha$ ($\lambda = n^-/n$), we must be careful to utilize the experimental values.

From experiments we may take α' as follows.

$$\alpha' \simeq 1 \times 10^{-8} \text{ cm}^3 \text{ sec}^{-1} \text{ for } E\text{-region,}$$

$$\alpha' \simeq 1 \times 10^{-10} \text{ cm}^3 \text{ sec}^{-1} \text{ for } F\text{-region.}$$

Concerning α_i we have little knowledges,⁽⁹⁾ and in the following considerations we touch this quantity only little, though in the more detailed study α_i should be taken into account.

4. Criterion from the increase of electron density of F_2 -layer near the sunrise at ground.

When we consider the rapid increase of F_2 -layer electron density from the sunrise at F -layer height (Ca. 250~300 km) to near the one at ground, which is very remarkable especially in Winter (from November to February), we can explore rough values of $\gamma + \delta$ (detachment) and then n^+ and n^- .

As q is zero for the above period of day, we obtain the following equations from equation (2).

$$\frac{dn}{dt} = (\gamma + \delta)n^- - \alpha n^+ n - \beta n, \quad (5)$$

$$\simeq (\gamma + \delta)n^+ - (\gamma + \delta + \beta)n, \quad (5')$$

where $\alpha n^+ n$ is neglected in comparison to βn ,

If we consider n^+ as constant for the period under consideration, the above equation is solved as follows.

$$n \simeq \frac{\gamma + \delta}{\beta + \gamma + \delta} n^+ \{1 - e^{-(\beta + \gamma + \delta)t}\} + n_0 e^{-(\beta + \gamma + \delta)t}, \quad (6)$$

where n_0 is the value of n at the sunrise of F -region height.

The above equation is rewritten in the following form.

$$n/n_0 \simeq \frac{\gamma + \delta}{\beta + \gamma + \delta} (1 + \lambda) \{1 - e^{-(\beta + \gamma + \delta)t}\} + e^{-(\beta + \gamma + \delta)\lambda}. \quad (7)$$

When we use the value 10^{-4} as β , and 3×10^{-3} as γ , which is reported by some theoretists,⁽⁹⁾ (δ is taken as less than β), we get for n/n_0 after 333 seconds (about 5.5 min.),

$$n/n_0 \simeq 0.63(1 + \lambda) + 0.37.$$

This value is too much, unless λ is taken as very small compared with unity. But with such small λ the actual observed curve of n/n_0 deviates greatly from the equation (7) for the later part of the period. Then it may be supposed that γ (including δ) should be as low as of the order of 10^{-4} , but in this case the constancy of λ or n^+ is violated.

In treating the equation (5'), we may proceed on another way as follows. The equation (5') can be written in the following form,

$$\frac{dn}{dt} = (\gamma + \delta)\lambda n - \beta n. \quad (5'')$$

Considering λ as variable with time we get the solution,

$$\frac{n}{n_0} = e^{(\gamma + \delta) \int \lambda dt - \beta t}, \quad (8)$$

and for n^- the solution (n_0^- is the initial value of n^-),

$$\frac{n^-}{n_0^-} = e^{\beta \int \frac{1}{\lambda} dt - (\gamma + \delta)t}. \quad (8')$$

When we compare the above equation (5'') or (8) with the actual increase of n in F_2 -layer in Winter by taking β as 10^{-4} , we find that the value of $\gamma + \delta$ as the order of 10^{-4} with λ of the order of unity will be the most probable, but $n^+ (= n + n^-)$ varies considerably in this case and this is unreasonable for the period under consideration.

Then it may need, so far as the coincidence of the theoretical equations and the actual observed curves is concerned, that both $\gamma + \delta$ and λ are considered to vary for the period, subject to constancy on n^+ . In any case the coincidence is more or less insufficient, but it is recommendable that γ (including δ) should be taken as $10^{-4} \sim 3 \times 10^{-4}$ with $\lambda \simeq 1 \sim 3$.

And in these estimations we find also that λ just before the sunrise at F_2 -layer height will be of the order of $3 \sim 10$, which may be the night value for λ .

In these considerations the heights of maximum electron density of F_2 -layer for day and night are taken as 260 and 300 km respectively and the values of electron density at constant height are used for the calculation.

5. Ionic density of F_2 -region

In the daytime, we get the following relation for around noon, which the

electronic density reaches near its maximum value.

$$n = \frac{q + (\gamma + \delta)n^-}{\beta + \alpha n^+}. \quad (9)$$

And as a preliminary approximation we have the relation,

$$n \simeq \frac{\gamma + \delta}{\beta + \gamma + \delta} n^+. \quad (9')$$

Using the values

$$\begin{aligned} \beta &= 1.0 \times 10^{-4} & (\text{for } 260 \text{ km}), \\ &= 4 \times 10^{-5} & (\text{for } 300 \text{ km}), \\ \gamma + \delta &= 10^{-4} \sim 3 \cdot 10^{-4}, \end{aligned}$$

we get

$$\begin{aligned} n &\simeq \frac{1}{2} n^+ \sim \frac{3}{4} n^+ & (\text{for } 260 \text{ km}), \\ &\simeq \frac{2}{3} n^+ \sim n^+ & (\text{for } 300 \text{ km}), \end{aligned}$$

and then

$$\begin{aligned} n^+ + n^- &= 3n \sim \frac{5}{3} n & (\text{for } 260 \text{ km}), \\ &= 2n \sim n & (\text{for } 300 \text{ km}). \end{aligned}$$

For the above values the equation (9) holds roughly with the magnitude of $q \simeq 100$.

In the night, when the electronic density decreases to its final value, we get with neglect of $\alpha n^+ n$,

$$\lambda = \frac{n^-}{n} = \frac{\beta}{\delta}. \quad (10)$$

Although δ is not exactly known, δ must be less than 10^{-4} by the consideration of the preceding paragraph, and we assume n^-/n as $2 \sim 10$ (the former for 300 km and the latter for lower than 250 km.) and consequently $n^+ + n^-$ as about $5 \sim 20$ times of n during night for F -layer.

6. Ionic density of E -region

For E -region β was given in the table. In the daytime we have from various experiments the following relation of electron density (n) around noon, ($\frac{dn}{dt}$ is put to zero)

$$q \simeq \alpha' n^2 = (1 + \lambda) \alpha n^2. \quad (11)$$

As the above relation is fairly accurate, the following relation must hold from the equations (2) and (11).

$$\begin{aligned} \beta n &\simeq (\gamma + \delta) n^-, \\ \therefore \frac{n^-}{n} &= \frac{\beta}{\gamma + \delta}. \end{aligned} \quad (12)$$

If we take γ as independent of height ($10^{-4} \sim 3 \times 10^{-4}$) and δ as 10^{-3} , we get following results.

$$\begin{aligned} \lambda = \frac{n^-}{n} &\simeq 20 \sim 15 & (\text{for } 100 \text{ km}), \\ &\simeq 7 \sim 6 & (\text{for } 110 \text{ km}). \end{aligned}$$

And for the night, if we neglect δ , we get the following equation from (3), considering $\frac{dn^-}{dt} = r = 0$,

$$n/n^+ n^- \simeq \frac{\alpha_i}{\beta}, \quad (13)$$

and then

$$\lambda = \frac{1}{\sqrt{n}} \sqrt{\frac{\beta}{\alpha_i}}. \quad (14)$$

Putting $\alpha_i \simeq 10^{-9}$ and n as less than 10^{-1} , we get

$$\lambda > 45 \quad (\text{for } 100 \text{ km}),$$

$$\lambda > 28 \quad (\text{for } 110 \text{ km}).$$

And when only δ is taken into account (α_i ; neglected), λ will decrease from the above values to about 20 and 8 respectively, and therefore the above values of λ will be the upper limits.

As the consequence we have

$$\left. \begin{aligned} n^+ + n^- &\simeq 15 \sim 13, \quad n \text{ (average } 14) \quad (\text{for } 110 \text{ km}); \\ &\simeq 41 \sim 31, \quad n \text{ (average } 36) \quad (\text{for } 100 \text{ km}), \end{aligned} \right\} \text{ for daytime,}$$

$$\left. \begin{aligned} &\simeq 60 \quad n \text{ at most (average } 30) \quad (\text{for } 110 \text{ km}), \\ &\simeq 90 \quad n \text{ at most (average } 60) \quad (\text{for } 100 \text{ km}). \end{aligned} \right\} \text{ for night.}$$

IV. Vertical distribution curves

1. n and $n^+ + n^-$

From the above considerations we reached the stage to draw vertical distribution curves of necessary values. For the distribution of electron density n (lower than the maximum density) we used the observational data and for the other part the square root curve of ion production q by Chapman's theory was used. In the figure shown the maximum electron densities of E and F_2 -layers for daytime are taken as 10^5 at 110 km and 10^6 at 260 km respectively. From this curve we draw the curve of $n^+ + n^-$, where the height dependency of necessary elements is taken into account, and the curve is shown in the figure.

2. Electrical conductivity (σ)

The electrical conductivity σ of the ionosphere is given by the following equation.

$$\sigma = \frac{e^2 n}{m} \frac{\nu}{\omega_{Hl}^2 + \nu^2} + \frac{e^2 (n^+ + n^-)}{M} \frac{\nu_i}{\omega_{Hl}^2 + \nu_i^2} \text{ esu}, \quad (15)$$

where

- ν_imean collision frequency of ion with neutral particle,
- ω_Hgeomagnetic gyrofrequency for electron (sec^{-1}),
- ω_Hgeomagnetic gyrofrequency for ion (sec^{-1}),
- m, Mmass of electron and ion respectively,
- echarge of electron.

In the calculation of σ we used the following values.

$$\begin{aligned} M/m &= 3 \times 10^4, \\ \omega_H &= 6.6 \times 10^6 \quad (H=0.4 \text{ Gauss}), \\ \omega_{Hh} &= 2.3 \times 10^2 \quad (\text{ ,, } \text{ ,, }). \end{aligned}$$

The other values necessary for the calculation of σ can be seen in the figure and the table. The vertical distribution of σ for daytime is calculated and shown in the same figure. As the electronic conductivity is very small in both layers, only *E*-layer daytime conductivity of electron is shown.

According to these curves and some numerical calculations based on the curves the followings can be said. For the daytime the maximum conductivities of *E* and *F*-layers are 1.7×10^{-14} e.m.u. and 1.3×10^{-14} e.m.u. respectively and the integrated values for *E* and *F*-layers are 1.1×10^{-7} e.m.u. (60%) and 0.7×10^{-7} e.m.u. (40%) respectively, the boundary between the two layers being taken as 190 km. The total is about 1.8×10^{-7} e.m.u. The conductivity is rather sensible for the height, and the decrease of *F*-layer height and the increase of *E*-layer height make the conductivity increase.

For the night the total conductivity of *E*-layer decreases roughly to less than one fifth of the daytime value and that of *F*-layer to one eighth, because the increase of height at night decrease σ greatly. The over all conductivity is about 3.15×10^{-8} e.m.u. at most, i.e. one sixth of the daytime value. The situation (maximum condition) during night is shown by the dotted curves in the same figure. (The explanation is omitted.)

V. Concluding remarks

It is to be noticed that the conductivity during daytime depends upon the ionic density of *E* and *F*-layers instead of electron density. The contribution of *E*-layer is greater than that of *F*-layer. During night the contribution of *E*-layer is much greater than that of *F*-layer. The diurnal change of the conductivity is represented partly by the electron density variation of *E*-layer, to which the ionic density of *E*-layer is closely connected, and partly by the height and electron density variation of *F*-layer.

Although it must be said that some dangerous speculations are involved in this estimation of the electrical conductivity, the study will show an example of the efforts that can be done if one aims to estimate the electrical conductivity directly from the knowledge of ionosphere. We expect further studies on this line and the consideration of calm and disturbed time conditions of ionosphere and terrestrial magnetism.

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