Current System for S_D -Field and the Bay Disturbance*

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Abstract

The dynamo-theory for S_q has been proved to be applicable also for S_D or the bay disturbance in the geomagnetic field. The disturbance field of S_D or bay is generally larger in the dark hemisphere compared with that in the sunlit hemisphere. This fact can be interpreted as a result of the condition that the electric conductivity of the auroral zones is markedly large, especially in the dark hemisphere. In this paper, the current system derived from the dynamo-theory with such conditions is numerically computed for various probable cases with the aid of the perturbation method. The calculated current system, such as given in Fig. 5, is in fairly good agreement with the observed one.

§ 1. Introduction

It has been shown by T. Rikitake¹⁾ and further extended by the present author²⁾ that the current system corresponding to S_D -field and the bay disturbance can be derived from the dynamo-theory under the assumption that the conductivity above the so-called auroral zones is much higher compared with that in the other regions. In the calculation hitherto made, however, the conductivity in each region was assumed to be invariant with longitude as well as latitude for the sake of mathematical simplicity. Although the calculated current system well agrees, as a first approximation, with that derived from the actual analysis of the geomagnetic data, there remain some discrepancies between them; For example, the current intensity is large in the dark hemisphere compared with that in the sunlit hemisphere. Such a discrepancy might be removed by adopting the more plausible assumption on the distribution of the conductivity of the upper atmosphere. In this paper, the calculated current system under the appropriate assumption of the conductivity was compared with the observed result. The method used here is a similar one with that used by A. Schuster³⁾ in the case of examining S_q current system, that is, the perturbation method.

* Contribution from the Division of Geomagnetism and Geoelectricity, Geophysical Institute, Tokyo University. Series II, No. 12 (1950)

Fig. 1

§ 2. Assumptions on the distribution of the conductivity in the upper atmosphere.

As shown in Fig. 1, the conducting layer of radius R_0 , which is concentric with the earth, is divided into 5 parts with respect to geomagnetic latitude, namely

the polar caps (regions I and V)

($\theta = 0^\circ \sim 20^\circ, 160^\circ \sim 180^\circ$),

the auroral zones (regions II and IV)

($\theta = 20^\circ \sim 25^\circ, 155^\circ \sim 160^\circ$),

and the equatorial zone (region III)

($\theta = 25^\circ \sim 155^\circ$).

In the equatorial zone, the total conductivity of the conducting layer K is assumed to be

$$K = K_0 (1 + \gamma' \cos \chi),$$

where K_0 is a constant, χ denotes the zenith distance of the sun, and γ' a constant less than unity. If the sun's declination is zero,

$$\cos \chi = -\sin \theta \cos \varphi$$

where φ is longitude reckoned from the midnight meridian.

As for the conductivity in the higher latitude regions, its dependence on χ could be ignored, because $\gamma' \cos \chi \ll 1$. According to the result of the actual analysis of the geomagnetic data in the auroral zones, such as by M. Hasegawa⁴⁾, L. Harang⁵⁾, H.C. Silsbee and E.H. Vestine,⁶⁾ the conductivity there can be considered to be very large especially in the dark hemisphere. It will be recognized that its maximum value takes place at midnight in the first approximation. Hence, the conductivity of the auroral zones is assumed to have a form

$$K = bK_0 (1 + \gamma \cos \varphi),$$

where b is a constant much larger than unity, and $\gamma < 1$. In the polar caps, the conductivity assumes

$$K = aK_0,$$

where a is a constant.

Summarizing the assumptions on the distribution of the conductivity in this calculation, the total conductivity in each regions is

$$\left. \begin{array}{ll} \text{in the polar caps} & K = aK_0, \\ \text{in the auroral zones} & K = bK_0 (1 + \gamma \cos \varphi), \\ \text{and in the equatorial zone} & K = K_0 (1 - \gamma' \sin \theta \cos \varphi). \end{array} \right\} \quad (1)$$

§ 3. General theory

As in the case of the ordinary dynamo-theory, the air motion in the conducting shell of radius R_0 has a velocity $\mathbf{v}(u, v)$, which can be derived from velocity potential

ψ , and ψ is assumed to be expanded by surface spherical harmonics, namely

$$\psi = \sum_N \sum_M k_N^M P_N^M(\cos \theta) \sin(M\varphi + \alpha_N^M). \quad (2)$$

The earth's magnetic field is replaced by the magnetic field of a centred dipole of moment M , with its axis coincident with the rotation axis of the earth. Vertical component of the earth's magnetic field H_z can be given by

$$H_z = -\frac{2M}{R_0^3} \cos \theta \equiv -2G \cos \theta. \quad (3)$$

Then, the current function J for the stationary state must satisfy the following equations:

$$\left. \begin{aligned} \frac{\partial J}{R_0 \sin \theta \partial \varphi} &= K v H_z - K \frac{\partial S}{R_0 \partial \theta}, \\ -\frac{\partial J}{R_0 \partial \theta} &= -K u H_z - K \frac{\partial S}{R_0 \sin \theta \partial \varphi}, \end{aligned} \right\} \quad (4)$$

where S denotes the electrostatic potential, and K is the total conductivity in each region given by the relation (1).

3. 1. Solution in the auroral zones

Substituting the expression for the conductivity in the auroral zones into (4), and neglecting the higher order terms of γ , (4) becomes

$$\left. \begin{aligned} 2bK_0 G \cot \theta \frac{\partial \psi}{\partial \varphi} &= bK_0 \frac{\partial S}{\partial \theta} + (1 - \gamma \cos \varphi) \frac{\partial J}{\sin \theta \partial \varphi}, \\ -2bK_0 G \cos \theta \frac{\partial \psi}{\partial \theta} &= bK_0 \frac{\partial S}{\sin \theta \partial \varphi} - (1 - \gamma \cos \varphi) \frac{\partial J}{\partial \theta}. \end{aligned} \right\} \quad (5)$$

Let S_0 and J_0 denote the expressions for S and J when $\gamma=0$ respectively. Then, in general expression,

$$\left. \begin{aligned} S &= S_0 + S', & S' &= O(\gamma), \\ J &= J_0 + J', & J' &= O(\gamma), \end{aligned} \right\} \quad (6)$$

Where S' and J' satisfy the following equations:

$$\left. \begin{aligned} bK_0 \frac{\partial S'}{\partial \theta} - \gamma \cos \varphi \frac{\partial J_0}{\sin \theta \partial \varphi} + \frac{\partial J'}{\sin \theta \partial \varphi} &= 0, \\ bK_0 \frac{\partial S'}{\sin \theta \partial \varphi} + \gamma \cos \varphi \frac{\partial J_0}{\partial \theta} - \frac{\partial J'}{\partial \theta} &= 0. \end{aligned} \right\} \quad (7)$$

Eliminating S' from (7), the following equation can be derived:

$$\frac{\partial^2 J'}{\sin \theta \partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial J'}{\partial \theta} \right) - \gamma \left\{ \frac{\partial}{\partial \varphi} \left(\cos \varphi \frac{\partial J_0}{\sin \theta \partial \varphi} \right) + \cos \varphi \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial J_0}{\partial \theta} \right) \right\} = 0. \quad (8)$$

On the other hand, the equation for J_0 is

$$\frac{\partial^2 J_0}{\sin \theta \partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial J_0}{\partial \theta} \right) + 2bK_0 G \left\{ \cot \theta \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \cos \theta \frac{\partial \psi}{\partial \theta} \right) \right\} = 0, \quad (9)$$

and the solution in these regions becomes

$$\left. \begin{aligned} J_0 &= -2bK_0 G \sum_N \sum_M S_N^M(\theta) \sin(M\varphi + \alpha_N^M) - 2bK_0 G \sum_m \{ C_2^m p_m(\theta) \sin(m\varphi + \alpha_{r,II}^m) + C_3^m q_m(\theta) \\ &\quad \times \sin(m\varphi + \alpha_{q,II}^m) \} \\ &\quad \text{for the northern auroral zone,} \\ J_0 &= -2bK_0 G \sum_N \sum_M S_N^M(\theta) \sin(M\varphi + \alpha_N^M) - 2bK_0 G \sum_m \{ C_6^m p_m(\theta) \sin(m\varphi + \alpha_{r,IV}^m) + C_7^m q_m(\theta) \\ &\quad \times \sin(m\varphi + \alpha_{q,IV}^m) \} \\ &\quad \text{for the southern auroral zone,} \end{aligned} \right\} \quad (10)$$

where

$$S_N^M(\theta) = k_N^M \left[\frac{N \{(N+1)^2 - M^2\}^{\frac{1}{2}}}{(N+1)(2N+1)} P_{N+1}^M(\cos \theta) + \frac{(N+1)(N^2 - M^2)^{\frac{1}{2}}}{N(2N+1)} P_{N-1}^M(\cos \theta) \right], \quad (11)$$

$$p_m(\theta) = \sin^m \theta \left[1 + \frac{m(m+1)}{2!} \cos^2 \theta + \dots + \frac{m(m+1)\{2(3+2m)+m(m+1)\} \dots \{(2S-2)(2S-1+2m)+m(m+1)\}}{(2S)!} \cos^{2S} \theta + \dots \right], \quad (12)$$

$$q_m(\theta) = \sin^m \theta \left[\cos \theta + \frac{2+2m+m(m+1)}{3!} \cos^3 \theta + \dots + \frac{\{2+2m+m(m+1)\} \{3(4+2m)+m(m+1)\} \dots \{(2S-1)(2S+2m)+m(m+1)\}}{(2S+1)!} \cos^{2S+1} \theta + \dots \right], \quad (13)$$

and C_2^m , C_3^m , C_6^m , C_7^m , $a_{p, II}^m$, $a_{p, IV}^m$, $a_{q, II}^m$ and $a_{q, IV}^m$ are the arbitrary constants. The first term in the right hand side of (10) is the particular solution, while the remaining two terms are the general solutions. The particular solution of J_0 should be substituted into (8), and then J' can be obtained.

3. 2. Solution in the equatorial region

Substitution of the expression for the conductivity in this region into (4) gives the equations:

$$\left. \begin{aligned} 2K_0 G \cot \theta \frac{\partial \psi}{\partial \varphi} &= K_0 \frac{\partial S}{\partial \theta} + (1 + \gamma' \sin \theta \cos \varphi) \frac{\partial J}{\sin \theta \partial \varphi}, \\ -2K_0 G \cos \theta \frac{\partial \psi}{\partial \theta} &= K_0 \frac{\partial S}{\sin \theta \partial \varphi} - (1 + \gamma' \sin \theta \cos \varphi) \frac{\partial J}{\partial \theta}, \end{aligned} \right\} \quad (14)$$

which correspond to (5) in the above case. After a similar treatment as before, the equations for J' in this region will be written as follows:

$$\left. \begin{aligned} K_0 \frac{\partial S'}{\partial \theta} + \gamma' \cos \varphi \frac{\partial J_0}{\partial \varphi} + \frac{\partial J'}{\sin \theta \partial \varphi} &= 0, \\ K_0 \frac{\partial S'}{\sin \theta \partial \varphi} - \gamma' \sin \theta \cos \varphi \frac{\partial J_0}{\partial \theta} - \frac{\partial J'}{\partial \theta} &= 0, \end{aligned} \right\} \quad (15)$$

or

$$\begin{aligned} -\frac{\partial^2 J'}{\sin \theta \partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial J'}{\partial \theta} \right) + \gamma' \sin^2 \theta \cos \varphi \frac{\partial^2 J_0}{\partial \theta^2} + 2\gamma' \sin \theta \cos \theta \cos \varphi \frac{\partial J_0}{\partial \theta} - \gamma' \sin \varphi \frac{\partial J_0}{\partial \varphi} \\ + \gamma' \cos \varphi \frac{\partial^2 J_0}{\partial \varphi^2} = 0. \end{aligned} \quad (16)$$

The solution of the equation for J_0 is

$$J_0 = -2K_0 G \sum \sum S_N^M(\theta) \sin(M\varphi + a_N^M) - 2K_0 G \sum \{C_4^m p_m(\theta) \sin(m\varphi + a_{p, III}^m) + C_5^m q_m(\theta) \sin(m\varphi + a_{q, III}^m)\}, \quad (17)$$

where C_4^m , C_5^m , $a_{p, III}^m$ and $a_{q, III}^m$ are the arbitrary constants.

3. 3. Solution in the polar caps.

Since the conductivity in the polar caps assumes a constant everywhere, $J' = 0$ or $J = J_0$. The differential equation for J is

$$\frac{\partial^2 J}{\sin \theta \partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial J}{\partial \theta} \right) + 2aK_0 G \left\{ \cot \theta \frac{\partial^2 \psi}{\partial \varphi^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \cos \theta \frac{\partial \psi}{\partial \theta} \right) \right\} = 0, \quad (18)$$

and the solution is

$$\left. \begin{aligned} J &= -2aK_0 G \sum_N \sum_M S_N^M(\theta) \sin(M\varphi + \alpha_N^M) - 2aK_0 G \sum_m C_1^m u_m(\theta) \sin(m\varphi + \alpha_1^m) \\ &\quad \text{for the northern cap,} \\ J &= -2aK_0 G \sum_N \sum_M S_N^M(\theta) \sin(M\varphi + \alpha_N^M) - 2aK_0 G \sum_m C_8^m v_m(\theta) \sin(m\varphi + \alpha_8^m) \\ &\quad \text{for the southern cap,} \end{aligned} \right\} \quad (19)$$

where

$$\begin{aligned} u_m(\theta) &= \sin^m \theta \left[1 + \frac{m(m+1)}{2 \cdot 1! (m+1)} (\cos \theta - 1) + \dots \right. \\ &\quad \left. + (-1)^s \frac{m(m+1)(2+2m+m(m+1)) \dots ((S-1)(S+2m)+m(m+1))}{2^s \cdot S! (m+S)!} (\cos \theta - 1)^s + \dots \right] \quad (20) \\ v_m(\theta) &= u_m(\pi - \theta) \quad (21) \end{aligned}$$

and C_1^m , C_8^m , α_1^m and α_8^m are the constants.

3. 4. Boundary Condition

The solution of the differential equation for the current function J is completely obtained in the above treatment. The groups of arbitrary constants C_1^m , C_2^m , ..., C_8^m and phase angles in the expression for J must be determined by the conditions that the normal component of the current and the tangential component of the electric field given respectively by

$$\frac{\partial J}{R_0 \sin \theta \partial \varphi}, \quad \frac{1}{K} \frac{\partial J}{R_0 \partial \theta}$$

are continuous at the four boundaries of the five regions.

3. 5. Separation of Sq field and S_D field

I will be convenient to consider that the expression for the current function J consists of two parts, that is,

$$J = J_q + J_D, \quad (22)$$

where the current system deduced from J_q corresponds to Sq -field, while that from J_D causes S_D -field. Further, if J_{q0} and J_{D0} denote the expressions for J_q and J_D when $\gamma = \gamma' = 0$, J_q and J_D could be considered to have the composition as follows:

$$\left. \begin{aligned} J_q &= J_{q0} + \gamma' J' q \gamma' \\ J_D &= J_{D0} + \gamma' J' D \gamma' + \gamma J' D \gamma \end{aligned} \right\} \quad (23)$$

where $\gamma' J' q \gamma'$ and $\gamma' J' D \gamma'$ the perturbation terms corresponding to $\gamma' K_0 \cos \chi$, and $\gamma J' D \gamma$ is that corresponding to $\gamma b K_0 \cos \varphi$.

§ 4. Examples of numerical calculation and the comparison of the results with the current system derived from the analysis of the geomagnetic data

In the actual analysis of the diurnal variation of the geomagnetic field, the term of $N=1$ and $M=1$ in the velocity potential is the largest and consequently the most important one. Hence, only the term of $N=1$, $M=1$ will be hereafter dealt with, that is

$$\psi = k_1^1 P_1^1 (\cos \theta) \sin (\varphi + \alpha_1^1). \quad (24)$$

By applying the method described in § 3, the current function J can be obtained as follows:

$$\begin{aligned}
&\text{in region I} \quad J = -2aK_0G k_1 \left[\left\{ \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_1 u_1(\theta) \right\} \sin(\varphi + a_1^1) \right. \\
&\quad \left. + \{\gamma D_1 + \gamma' D_1'\} u_2(\theta) \sin(2\varphi + a_1^1) + \{\gamma B_1 + \gamma' B_1'\} \sin a_1^1 \right], \\
&\text{in region II} \quad J = -2bK_0G k_1 \left[\left\{ \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_2 p_1(\theta) + C_3 q_1(\theta) \right\} \sin(\varphi + a_1^1) \right. \\
&\quad + \left\{ \gamma \left(q_1(\theta) + \frac{1}{2} \left(\frac{\pi}{2} - \theta \right) p_2(\theta) + \frac{1}{8} \sin 2\theta \right) + (\gamma D_2 + \gamma' D_2') p_2(\theta) \right. \\
&\quad \left. + (\gamma D_3 + \gamma' D_3') q_2(\theta) \right\} \sin(2\varphi + a_1^1) \\
&\quad \left. + \left\{ \gamma \frac{1}{8} \sin 2\theta + (\gamma B_2 + \gamma' B_2') p_0(\theta) + (\gamma B_3 + \gamma' B_3') q_0(\theta) \right\} \sin a_1^1 \right], \\
&\text{in region III} \quad J = -2K_0G k_1 \left[\left\{ \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_4 p_1(\theta) + C_5 q_1(\theta) \right\} \sin(\varphi + a_1^1) \right. \\
&\quad + \left\{ -\gamma' \left(\frac{1}{6} \sin^2 \theta \cos \theta + \frac{4}{3} q_2(\theta) \right) + (\gamma D_4 + \gamma' D_4') p_2(\theta) + (\gamma D_5 + \gamma' D_5') q_2(\theta) \right\} \\
&\quad \times \sin(2\varphi + a_1^1) \\
&\quad + \left\{ \gamma' \frac{1}{24} (\cos 3\theta - 3 \cos \theta) + (\gamma B_4 + \gamma' B_4') p_0(\theta) + (\gamma B_5 + \gamma' B_5') q_0(\theta) \right\} \sin a_1^1 \right], \\
&\text{in region IV} \quad J = -2bK_0G k_1 \left[\left\{ \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_6 p_1(\theta) + C_7 q_1(\theta) \right\} \sin(\varphi + a_1^1) \right. \\
&\quad + \left\{ \gamma \left(q_1(\theta) + \frac{1}{2} \left(\frac{\pi}{2} - \theta \right) p_2(\theta) + \frac{1}{8} \sin 2\theta \right) + (\gamma D_6 + \gamma' D_6') p_2(\theta) \right. \\
&\quad \left. + (\gamma D_7 + \gamma' D_7') q_2(\theta) \right\} \sin(2\varphi + a_1^1) \\
&\quad + \left\{ \gamma \frac{1}{8} \sin 2\theta + (\gamma B_6 + \gamma' B_6') p_0(\theta) + (\gamma B_7 + \gamma' B_7') q_0(\theta) \right\} \sin a_1^1 \right], \\
&\text{in region V} \quad J = -2aK_0G k_1 \left[\left\{ \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_8 v_1(\theta) \right\} \sin(\varphi + a_1^1) \right. \\
&\quad \left. + \left\{ \gamma D_8 + \gamma' D_8' \right\} v_2(\theta) \sin(2\varphi + a_1^1) + \left\{ \gamma B_8 + \gamma' B_8' \right\} \sin a_1^1 \right],
\end{aligned} \tag{25}$$

wherein C_i^1 , C_i^0 and C_i^2 ($i=1, 2, \dots, 8$) are replaced by C_i , $\gamma B_i + \gamma' B_i'$ and $\gamma D_i + \gamma' D_i'$ respectively, and

$$\left. \begin{aligned}
u_0(\theta) &= 1, & u_1(\theta) &= 2 \tan \frac{\theta}{2}, & u_2(\theta) &= 4 \tan^2 \frac{\theta}{2}, \\
p_0(\theta) &= 1, & p_1(\theta) &= \operatorname{cosec} \theta, & p_2(\theta) &= 2 \operatorname{cosec}^2 \theta - 1, \\
q_0(\theta) &= -\log \tan \frac{\theta}{2}, & q_1(\theta) &= \cot \theta, & q_2(\theta) &= \cot \theta \cdot \operatorname{cosec} \theta, \\
v_0(\theta) &= 1, & v_1(\theta) &= 2 \cot \frac{\theta}{2}, & v_2(\theta) &= 4 \cot^2 \frac{\theta}{2}.
\end{aligned} \right\} \tag{26}$$

Each elements in (23) will be written as follows:

$$\left. \begin{aligned}
Jq_0 &= -2K_0G k_1 \cdot \frac{P_2^1(\cos \theta)}{2\sqrt{3}} \sin(\varphi + a_1^1), \\
Jq_1' &= -2K_0G k_1 \cdot \frac{1}{24} \left\{ (\cos 3\theta - \cos \theta) \sin(2\varphi + a_1^1) + (\cos 3\theta - 3 \cos \theta) \sin a_1^1 \right\},
\end{aligned} \right\} \tag{27}$$

over the whole conducting layer, and

$$\begin{aligned}
 J_{D0} = & \left\{ \begin{array}{ll} -2a K_0 G k_1^1 \left\{ \frac{a-1}{a} \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_1 u_1(\theta) \right\} \sin(\varphi + \alpha_1^1) & \text{in the polar cap,} \\ -2b K_0 G k_1^1 \left\{ \frac{b-1}{b} \frac{P_2^1(\cos \theta)}{2\sqrt{3}} + C_2 p_1(\theta) + C_3 q_1(\theta) \right\} \sin(\varphi + \alpha_1^1) & \text{in the auroral zone,} \\ -2K_0 G k_1^1 C_5 q_1(\theta) & \text{in the equatorial region,} \end{array} \right. \\
 J'_{D\gamma} = & \left\{ \begin{array}{ll} -2a K_0 G k_1^1 \left\{ D_1 u_2(\theta) \sin(2\varphi + \alpha_1^1) + B_1 \sin \alpha_1^1 \right\} & \text{in the polar cap,} \\ -2b K_0 G k_1^1 \left[\left\{ q_1(\theta) + \frac{1}{2} \left(\frac{\pi}{2} - \theta \right) p_2(\theta) + \frac{1}{8} \sin 2\theta + D_2 p_2(\theta) + D_3 q_2(\theta) \right\} \right. \\ \quad \times \sin(2\varphi + \alpha_1^1) + \left. \left\{ \frac{1}{8} \sin 2\theta + B_2 - B_3 \log \tan \frac{\theta}{2} \right\} \sin \alpha_1^1 \right] & \text{in the auroral zone,} \\ -2 K_0 G k_1^1 \left[D_5 q_2(\theta) \sin(2\varphi + \alpha_1^1) - B_5 \log \tan \frac{\theta}{2} \cdot \sin \alpha_1^1 \right] & \text{in the equatorial region,} \end{array} \right\} (28) \\
 J'_{D\gamma'} = & \left\{ \begin{array}{ll} -2a K_0 G k_1^1 \left[\left\{ D_1' u_2(\theta) - \frac{1}{24a} (\cos 3\theta - \cos \theta) \right\} \sin(2\varphi + \alpha_1^1) + \left\{ B_1' - \frac{1}{24a} \right. \right. \\ \quad \times (\cos 3\theta - 3\cos \theta) \left. \right\} \sin \alpha_1^1 \right] & \text{in the polar cap,} \\ -2b K_0 G k_1^1 \left[\left\{ D_2' p_2(\theta) + D_3' q_2(\theta) - \frac{1}{24b} (\cos 3\theta - \cos \theta) \right\} \sin(2\varphi + \alpha_1^1) \right. \\ \quad + \left. \left\{ B_2' - B_3' \log \tan \frac{\theta}{2} - \frac{1}{24b} (\cos 3\theta - 3\cos \theta) \right\} \sin \alpha_1^1 \right] & \text{in the auroral zone,} \\ -2K_0 G k_1^1 \left[\left(D_5' - \frac{4}{3} \right) q_2(\theta) \sin(2\varphi + \alpha_1^1) - B_5' \log \tan \frac{\theta}{2} \cdot \sin \alpha_1^1 \right] & \text{in the equatorial region,} \end{array} \right.
 \end{aligned}$$

in the northern hemisphere. C_4 , D_4 , D_4' , B_4 and B_4' in (25) are all zero.

Fig. 2 The calculated current system for S_D -field or the bay disturbance under the assumption that the conductivity is assumed to be a constant in each regions.

In the numerical calculation here, K_0 is assumed to be 3×10^{-6} e.m.u., M 7.9×10^{25} c.g.s., R_0 6.47×10^8 cm, that is, the conducting layer situates 100km above the earth's surface. The value of a and b adopted here are 1 and 10 respectively. For the determination of the value of k_1^1 in ψ , the result of Chapman's analysis of the geomagnetic data in 1905⁽⁷⁾ is referred, and α_1^1 is taken as $\frac{3}{2}\pi^{(2)}$.

In Fig. 2, the calculated current system viewed from above the north pole corresponding to S_D -field when $\gamma = \gamma' = 0$, namely J_{D0} in (28) is given, where the electric current of 2.34×10^3 amp. flows between successive stream lines. As γ becomes larger, the current intensity in the auroral zone becomes more

intense in the dark hemisphere and less in the sunlit hemisphere, while the equatorial current intensity shows only a slight decrease in the dark hemisphere and increase in the sunlit hemisphere. The ratios of the total amount of the current flowing in the auroral zone and the equatorial region of the dark hemisphere to those of the sunlit hemisphere, denoted by $f(A)$ and $f(E)$ respectively, are given in Figs. 3 and 4. At that time, the current intensity in the polar caps does not show remarkable change, its change being less than 10%.

Fig. 3 Change in the ratio of the intensity of the auroral zone current in the dark hemisphere to that in the sunlit hemisphere with γ .

Fig. 4 Change in the ratio of the current intensity of the equatorial region in the dark hemisphere to that in the sunlit side with γ .

In Fig. 5, the current system when $\gamma = \frac{1}{2}$ and $\gamma' = 0$ is shown. The current system for the bay disturbance obtained by H.C. Silsbee and E.H. Vestine⁶⁾ is also shown here for

Fig. 5 The calculated current system for S_D -field and the bay disturbance when $\gamma = \frac{1}{2}$ and $\gamma' = 0$.

Fig. 6 The current system for the bay disturbance obtained by H.C. Silsbee and E.H. Vestine. A total of 5×10^4 amp. flows between successive current-lines.

comparison. There will be seen that the both current systems are qualitatively in good agreement with each other, although there are discrepancies of phase of about

30° and of the current intensity between them. The latter difficulty may be removed in some way, for example, by the author's hypothesis of the double conducting layers.⁽²⁾

In the analysis of the geomagnetic disturbance field at the time of magnetic storms, the *Dst*-component should contain, according to its definition, not only the magnetic field which is attributed to the equatorial ring current, but also the zonal part such as derived from $J_D \gamma$. In other words, the zonal part is to be eliminated from the so-called S_D -field. If this zonal part is eliminated from the current system shown in Fig. 5, the current system shown in Fig. 7 can be obtained. In the latter system, the current intensity in the sunlit hemisphere and that in the dark hemisphere are nearly the same.

Fig. 7 The current system obtained by eliminating the zonal part from the current system in Fig. 5.

Fig. 8 The calculated current system for S_D -field when $\gamma = \frac{1}{2}$ and $\gamma' = \frac{1}{2}$.

In Fig. 8, the current system for $\gamma = \frac{1}{2}$ and $\gamma' = \frac{1}{2}$ is given. As will be seen in this figure, the current flow in the equatorial region is considerably deformed, namely it becomes very intense in the lower latitude in the sunlit hemisphere and the current vortex appears in the low latitude region in the dark hemisphere. On the other hand, the current intensity in both the auroral zones and the polar caps is scarcely affected by the value of γ' . Therefore, the asymmetry of the conductivity in the equatorial region will be not significant in the present problem.

§ 5. Conclusion.

It was shown in this study that the assumption in the dynamo-theory that the conductivity of the auroral zones is much higher, especially in the dark hemisphere, results in the production of the current system which can be identified with that derived from the analysis of geomagnetic data. In this calculation, however, the higher order terms in the harmonic expansion of the velocity potential are neglected. If these higher harmonic terms are also adopted, the resultant current system will become

more complicated, but look more like the observed system.

It must be noted here that the conductivity of the equatorial region K_0 , which is assumed conventionally here 3×10^{-6} e.m.u., should be replaced by the more reliable value of about 5×10^{-8} e.m.u. obtained by T. Nagata.⁸⁾ However, we concern in this calculation under dynamo-theory only the product of the conductivity and the velocity of ionosphere wind, absolute values of conductivity being not significant. In other words, the relative magnitudes of conductivity in respective regions are to be the cause of S_D - or bay-fields under the consideration of the dynamo-theory.

At any rate, the dynamo-action is considered to be the most fundamental process to produce the various kinds of geomagnetic variation fields, although there still remains some difficulty in detail of each variation fields.

In concluding, the writer wishes to express his hearty thanks to Dr. T. Nagata for his direction throughout this study, and to Miss N. Ono for assistance in the numerical computation.

(Read: Oct. 9, 1950)

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