On Chapman-Miller Lunar Variation Computations

(Received December 11; 1969)

Chapman and *Miller* (1940) gave a method for computing the lunar daily variation in geomagnetic and meteorological elements. *Wilkes* (1962) modified this method for the case where the number of days in each lunar-age group is not the same. A method by which the computations of the lunar daily variation can be shortened is here presented.

A common procedure (see *Tschu* (1949)) in order to determine the lunar variation components is to compute the quantities U_p and V_p . *Wilkes* (1962) showed that if average group sum sequences are used then the lunar variation computations are simplified for then

$$U_p = A_{pA} - B_{pB} \quad (1)$$

$$V_p = B_{pA} + A_{pB} \quad (2)$$

The number of calculations required to compute U_p and V_p as given by (1) and (2) can be further reduced if U_p and V_p are expressed in the form

$$U_p = u_p + C_A - D_{pB} \quad (3)$$

$$V_p = v_p + C_B + D_{pA} \quad (4)$$

$$\text{where } u_p = \sum_r \sum_s g_{sr} \cos[2\pi(p_s + 2r)/24], \quad v_p = \sum_r \sum_s g_{sr} \sin[2\pi(p_s + 2r)/24] \quad (5)$$

$$C_A = \sum_r c_r \cos(2\pi r/R), \quad C_B = \sum_r c_r \sin(2\pi r/R) \quad (6)$$

$$D_{pA} = \sum_r d_{pr} \cos(2\pi r/R), \quad D_{pB} = \sum_r d_{pr} \sin(2\pi r/R) \quad (7)$$

$$\text{with } c_r = \frac{1}{2}(g_{Sr} - g_{0r}) \quad \text{and with } d_{pr} = \frac{1}{2}(g_{Sr} - g_{0r}) \cot(\pi p/S).$$

Here S and R are taken to have the values 24 and 12 respectively; the summation for r and s are from 0 to 11 and from 0 to 23 respectively; the average group sum sequence for the r th lunar-age group is g_{sr} ($s=0, 1, \dots, S$); p usually takes the value 1, 2, 3 and 4.

In effect, by use of (3) and (4) instead of the equation of *Tschu* (1949), about half the number of summations need to be done. Also since c_r does not depend on p equations (6) results in only two quantities C_A and C_B instead of eight quantities when using the method of *Tschu* (1949). However there is a multiplication by a more complicated expression for the sine and cosine in (3) and (4) due to u_p and v_p but it will be shown that the computation can be simplified further in the case of sequences of hourly values. If the noncyclic variation has been removed from the average group sum sequences g_{sr} ($s=0, 1, \dots, S$) then $g_{Sr} - g_{0r} = 0$ and

$$C_A = C_B = D_{pA} = D_{pB} = 0.$$

In the case of hourly values the computation of u_p and v_p can be shortened by appropriate rearrangement of the g_{sr} values. This rearrangement is different for different values of p and will be given for $p=1, 2, 3$ and 4 but can be extended to other values of p . When $p=1$ start with the average group sum sequences as for the normal computation of the lunar variation. Then leave the sequence g_{sr} ($s=0, 1, \dots, 23$) for $r=0$ unchanged. For g_{s1} ($s=0, 1, \dots, 23$) place the values for $s=22, 23$ as $s=0, 1$ and the previous values for $s=0$ to 21 as $s=2$ to 23 . For g_{s2} ($s=0, 1, \dots, 23$) place values for $s=20$ to 23 as $s=0$ to 3 and the previous values for $s=0$ to 19 as $s=4$ to 23 . Continue in a similar manner for $r=3, 4, \dots, 11$ so that for $g_{s,11}$ ($s=0, 1, \dots, 23$) place values for $s=2$ to 23 as $s=0$ to 21 and the previous values for $s=0, 1$ as $s=22, 23$.

Now form the sequence

$$g_s = \sum_r g_{sr} \quad (s=0, 1, \dots, 23).$$

Then

$$u_1 = \sum_s g_s \cos(2\pi s/24) \quad (8)$$

$$v_1 = \sum_s g_s \sin(2\pi s/24). \quad (9)$$

When $p=2$ start with the average group sum sequences as for the normal computation of the lunar variation. Then leave g_{s0} ($s=0, 1, \dots, 23$) unchanged. For g_{s1} ($s=0, 1, \dots, 23$) place value for $s=23$ as $s=0$ and the previous values for $s=0$ to 22 as $s=1$ to 23 . For g_{s2} ($s=0, 1, \dots, 23$) place values for $s=22, 23$ as $s=0, 1$ and the previous values for $s=0$ to 21 as $s=2$ to 23 . Continue in a similar manner for $r=3, 4, \dots, 11$ so that for $g_{s,11}$ ($s=0, 1, \dots, 23$) place values for $s=13$ to 23 as $s=0$ to 10 and the previous values for $s=0$ to 12 as $s=11$ to 23 .

Now form the sequence

$$g_s = \sum_r g_{sr} \quad (s=0, 1, \dots, 23)$$

Hence obtain

$$w_r = g_r + g_{r+12} \quad (r=0, 1, \dots, 11)$$

Then

$$u_2 = \sum_r w_r \cos(\pi r/6)$$

$$v_2 = \sum_r w_r \sin(\pi r/6).$$

When $p=3$ start with the average group sum sequences as for the normal computation of the lunar variation. Then leave g_{s0} ($s=0, 1, \dots, 23$) unchanged. For g_{s3} ($s=0, 1, \dots, 23$) place values for $s=22, 23$ as $s=0, 1$ and the previous values for $s=0$ to 21 as $s=2$ to 23 . For g_{s6} ($s=0, 1, \dots, 23$) place values for $s=20$ to 23 as $s=0$ to 3 and the previous values for $s=0$ to 19 as $s=4$ to 23 . For g_{s9} ($s=0, 1, \dots, 23$) place values for $s=18$ to 23 as $s=0$ to 5 and the previous values for $s=0$ to 17 as $s=6$ to 23 .

Now form the sequence

$$g_s = \sum_{n=0}^3 g_{sj}, \quad j=3n, \quad (s=0, 1, \dots, 23)$$

and obtain w_i from

$$w_i = g_i + g_{i+8} + g_{i+16} \quad (i=0, 1, \dots, 7)$$

Then compute x_1, y_1 from

$$x_1 = \sum_{i=0}^7 w_i \cos(\pi i/4)$$

$$y_1 = \sum_{i=0}^7 w_i \sin(\pi i/4)$$

Obtain w_i from the average group sum sequences for $r=1, 4, 7, 10$ using the same procedure as for $r=0, 3, 6, 9$. Then compute x_2, y_2 from

$$x_2 = \sum_{i=0}^7 w_i \cos[(\pi i/4) + \pi/6]$$

$$y_2 = \sum_{i=0}^7 w_i \sin[(\pi i/4) + \pi/6]$$

Obtain w_i from the average group sum sequences $r=2, 5, 8, 11$ using the same procedure as for $r=0, 3, 6, 9$.

Then compute x_3, y_3 from

$$x_3 = \sum_{i=0}^7 w_i \cos[(\pi i/4) + \pi/3]$$

$$y_3 = \sum_{i=0}^7 w_i \sin[(\pi i/4) + \pi/3]$$

Finally

$$u_3, v_3 \text{ are given by}$$

$$u_3 = x_1 + x_2 + x_3$$

$$v_3 = y_1 + y_2 + y_3.$$

When $p=4$ start with the average group sum sequences as for the normal computation of the lunar variation. Then leave g_{s0} ($s=0, 1, \dots, 23$) unchanged. For g_{s2} ($s=0, 1, \dots, 23$) place value for $s=23$ as $s=0$ and the previous values for $s=0$ to 22 as $s=1$ to 23. For g_{s4} ($s=0, 1, \dots, 23$) place values for $s=22, 23$ as $s=0, 1$ and the previous values for $s=0$ to 21 as $s=2$ to 23. Continue in a similar manner for $r=6, 8, 10$ so that for $g_{s,10}$ ($s=0, 1, \dots, 23$) place values for $s=19$ to 23 as $s=0$ to 4 and the previous values for $s=0$ to 18 as $s=5$ to 23.

Now compute

$$g_s = \sum_{n=0}^5 g_{sj}, \quad j=2n, \quad (s=0, 1, \dots, 23)$$

and obtain w_i from

$$w_i = g_i + g_{i+6} + g_{i+12} + g_{i+18} \quad (i=0, 1, \dots, 5)$$

Then compute x_1, y_1 from

$$x_1 = \sum_{i=0}^5 w_i \cos(\pi i/3)$$

$$y_1 = \sum_{i=0}^5 w_i \sin(\pi i/3)$$

Obtain w_i from the average group sum sequences for $r=1, 3, \dots, 11$ using the same procedure as for $r=0, 2, \dots, 10$.

Then compute

$$x_2 = \sum_{i=0}^5 w_i \cos[(\pi i/3) + \pi/6]$$

$$y_2 = \sum_{i=0}^5 w_i \sin[(\pi i/3) + \pi/6]$$

Finally u_4, v_4 are given by

$$u_4 = x_1 + x_2$$

$$v_4 = y_1 + y_2.$$

Also starting from the average groups sum sequences as for the normal computation of the lunar variation one can achieve the same results by summing the appropriate values together. In this way the rearranging required in the previous section is avoided. This method would be more suited for calculations using electronic computers where values in different columns can be readily summed.

For example the g_s for $p=1$ can be obtained from the following equations:

$$g_s = \sum_{r=0}^{s/2} g_{jr} + \sum_{r=\frac{s}{2}+1}^{11} g_{kr}, \quad \begin{matrix} j=s-2r, k=24-2r+s \\ (s=0, 2, \dots, 20) \end{matrix}$$

$$g_s = \sum_{r=0}^{\left(\frac{s-1}{2}\right)} g_{jr} + \sum_{r=\frac{(s-1)}{2}+1}^{11} g_{kr}, \quad \begin{matrix} j=s-2r, k=24-2r+s \\ (s=1, 3, \dots, 21) \end{matrix}$$

$$g_s = \sum_{r=0}^{11} g_{jr}, \quad \begin{matrix} j=s-2r \\ (s=22, 23). \end{matrix}$$

A method for dealing with equations such as (8) and (9) was given by Goertzel (1958).

References

- Chapman, S., and J.C.P. Miller, The statistical determination of lunar daily variations in geomagnetic and meteorological elements, *Mon. Not. R.A.S. Geoph. Supp.*, **4**, 649, 1940.
- Goertzel, G., An algorithm for the evaluation of finite trigonometric series, *Amer. Math. Monthly*, **65**, 34, 1958.
- Tschu, K.K., On the practical determinations of lunar and luni-solar daily variations in certain geophysical data, *Austral. J. Sci. Res. A*, **2**, 1, 1949.
- Wilkes, M.W., The solar and luni-solar harmonic components of geomagnetic variation at San Fernando, *Jour. Atmos. Terr. Phys.*, **24**, 73, 1962.

H.F. PETERSONS

Department of Applied Mathematics, School of General Studies,
The Australian National University, Canberra, 2600.