On Chapman-Miller Lunar Variation Computations

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Chapman and Miller (1940) gave a method for computing the lunar daily variation in geomagnetic and meteorological elements. Wilkes (1962) modified this method for the case where the number of days in each lunar-age group is not the same. A method by which the computations of the lunar daily variation can be shortened is here presented.

A common procedure (see Tschu (1949)) in order to determine the lunar variation components is to compute the quantities \( U_p \) and \( V_p \). Wilkes (1962) showed that if average group sum sequences are used then the lunar variation computations are simplified for then

\[
U_p = A_p A - B_p B \quad (1)
\]
\[
V_p = B_p A + A_p B \quad (2)
\]

The number of calculations required to compute \( U_p \) and \( V_p \) as given by (1) and (2) can be further reduced if \( U_p \) and \( V_p \) are expressed in the form

\[
U_p = u_p + C_A - D_p B \quad (3)
\]
\[
V_p = v_p + C_B + D_p A \quad (4)
\]

where

\[
C_A = \sum_r c_r \cos(2\pi r/R), \quad C_B = \sum_r c_r \sin(2\pi r/R) \quad (6)
\]
\[
D_{pA} = \sum_r d_{pr} \cos(2\pi r/R), \quad D_{pB} = \sum_r d_{pr} \sin(2\pi r/R) \quad (7)
\]

with \( c_r = \frac{1}{2} (g_{sr} - g_{0r}) \quad \) and with \( d_{pr} = \frac{1}{2} (g_{sr} - g_{0r}) \cot(\pi p/S) \).

Here \( S \) and \( R \) are taken to have the values 24 and 12 respectively; the summation for \( r \) and \( s \) are from 0 to 11 and from 0 to 23 respectively; the average group sum sequence for the \( r \)th lunar-age group is \( g_{sr} (s=0, 1, \ldots, S) \); \( p \) usually takes the value 1, 2, 3 and 4.

In effect, by use of (3) and (4) instead of the equation of Tschu (1949), about half the number of summations need to be done. Also since \( c_r \) does not depend on \( p \) equations (6) results in only two quantities \( C_A \) and \( C_B \) instead of eight quantities when using the method of Tschu (1949). However there is a multiplication by a more complicated expression for the sine and cosine in (3) and (4) due to \( u_p \) and \( v_p \) but it will be shown that the computation can be simplified further in the case of sequences of hourly values. If the noncyclic variation has been removed from the average group sum sequences \( g_{sr} (s=0, 1, \ldots, S) \) then \( g_{SR} - g_{0R} = 0 \) and

\[
C_A = C_B = D_{pA} = D_{pB} = 0.
\]
In the case of hourly values the computation of \( u_p \) and \( v_p \) can be shortened by appropriate rearrangement of the \( g_{sr} \) values. This rearrangement is different for different values of \( p \) and will be given for \( p=1, 2, 3 \) and \( 4 \) but can be extended to other values of \( p \). When \( p=1 \) start with the average group sum sequences as for the normal computation of the lunar variation. Then leave the sequence \( g_{sr} (s=0, 1, \ldots, 23) \) for \( r=0 \) unchanged. For \( g_{s1} (s=0, 1, \ldots, 23) \) place the values for \( s=22, 23 \) as \( s=0, 1 \) and the previous values for \( s=0 \) to 21 as \( s=2 \) to 23. For \( g_{s2} (s=0, 1, \ldots, 23) \) place values for \( s=20 \) to 23 as \( s=0 \) to 3 and the previous values for \( s=0 \) to 19 as \( s=4 \) to 23. Continue in a similar manner for \( r=3, 4, \ldots, 11 \) so that for \( g_{s11} (s=0, 1, \ldots, 23) \) place values for \( s=2 \) to 23 as \( s=0 \) to 21 and the previous values for \( s=0, 1 \) as \( s=22, 23 \).

Now form the sequence
\[
g_s = \sum_r g_{sr} \quad (s=0, 1, \ldots, 23).
\]

Then
\[
u_1 = \sum_s g_{s0} \cos(2\pi s/24) \quad (8)
\]
\[
v_1 = \sum_s g_{s0} \sin(2\pi s/24). \quad (9)
\]

When \( p=2 \) start with the average group sum sequences as for the normal computation of the lunar variation. Then leave \( g_{s0} (s=0, 1, \ldots, 23) \) unchanged. For \( g_{s1} (s=0, 1, \ldots, 23) \) place value for \( s=23 \) as \( s=0 \) and the previous values for \( s=0 \) to 22 as \( s=1 \) to 23. For \( g_{s2} (s=0, 1, \ldots, 23) \) place values for \( s=22, 23 \) as \( s=0, 1 \) and the previous values for \( s=0 \) to 21 as \( s=2 \) to 23. Continue in a similar manner for \( r=3, 4, \ldots, 11 \) so that for \( g_{s11} (s=0, 1, \ldots, 23) \) place values for \( s=13 \) to 23 as \( s=0 \) to 10 and the previous values for \( s=0 \) to 12 as \( s=11 \) to 23.

Now form the sequence
\[
g_s = \sum_r g_{sr} \quad (s=0, 1, \ldots, 23)
\]

Hence obtain
\[
w_r = g_r + g_{r+12} \quad (r=0, 1, \ldots, 11)
\]

Then
\[
u_2 = \sum_r w_r \cos(\pi r/6)
\]
\[
v_2 = \sum_r w_r \sin(\pi r/6). \]

When \( p=3 \) start with the average group sum sequences as for the normal computation of the lunar variation. Then leave \( g_{s0} (s=0, 1, \ldots, 23) \) unchanged. For \( g_{s1} (s=0, 1, \ldots, 23) \) place values for \( s=22, 23 \) as \( s=0 \) and the previous values for \( s=0 \) to 21 as \( s=2 \) to 23. For \( g_{s2} (s=0, 1, \ldots, 23) \) place values for \( s=20 \) to 23 as \( s=0 \) to 3 and the previous values for \( s=0 \) to 19 as \( s=4 \) to 23. For \( g_{s9} (s=0, 1, \ldots, 23) \) place values for \( s=18 \) to 23 as \( s=0 \) to 5 and the previous values for \( s=0 \) to 17 as \( s=6 \) to 23.

Now form the sequence
\[
g_s = \sum_n g_{s3n}, \quad j=3n, (s=0, 1, \ldots, 23)
\]

and obtain \( w_3 \) from
\[ w_i = g_i + g_{i+6} + g_{i+12} + g_{i+18} \quad (i=0, 1, \ldots, 7) \]

Then compute \( x_1, y_1 \) from
\[ x_1 = \sum_{i=0}^{7} w_i \cos(\pi i/4) \]
\[ y_1 = \sum_{i=0}^{7} w_i \sin(\pi i/4) \]

Obtain \( w_1 \) from the average group sum sequences for \( r=1, 4, 7, 10 \) using the same procedure as for \( r=0, 3, 6, 9 \). Then compute \( x_2, y_2 \) from
\[ x_2 = \sum_{i=0}^{7} w_i \cos((\pi i/4) + \pi/6) \]
\[ y_2 = \sum_{i=0}^{7} w_i \sin((\pi i/4) + \pi/6) \]

Obtain \( w_t \) from the average group sum sequences \( r=2, 5, 8, 11 \) using the same procedure as for \( r=0, 3, 6, 9 \).

Then compute \( x_3, y_3 \) from
\[ x_3 = \sum_{i=0}^{7} w_i \cos((\pi i/4) + \pi/3) \]
\[ y_3 = \sum_{i=0}^{7} w_i \sin((\pi i/4) + \pi/3) \]

Finally \( u_3, v_3 \) are given by
\[ u_3 = x_1 + x_2 + x_3 \]
\[ v_3 = y_1 + y_2 + y_3 \]

When \( p=4 \) start with the average group sum sequences as for the normal computation of the lunar variation. Then leave \( g_{s0} \) \((s=0, 1, \ldots, 23)\) unchanged. For \( g_{s2} \) \((s=0, 1, \ldots, 23)\) place value for \( s=23 \) as \( s=0 \) and the previous values for \( s=0 \) to 22 as \( s=1 \) to 23. For \( g_{s4} \) \((s=0, 1, \ldots, 23)\) place values for \( s=22, 23 \) as \( s=0, 1 \) and the previous values for \( s=0 \) to 21 as \( s=2 \) to 23. Continue in a similar manner for \( r=6, 8, 10 \) so that for \( g_{s10} \) \((s=0, 1, \ldots, 23)\) place values for \( s=19 \) to 23 as \( s=0 \) to 4 and the previous values for \( s=0 \) to 18 as \( s=5 \) to 23.

Now compute
\[ g_s = \sum_{n=0}^{5} g_{s,j}, \quad j=2n, \quad (s=0, 1, \ldots, 23) \]
and obtain \( w_t \) from
\[ w_t = g_s + g_{s+6} + g_{s+12} + g_{s+18} \quad (i=0, 1, \ldots, 5) \]

Then compute \( x_1, y_1 \) from
\[ x_1 = \sum_{i=0}^{5} w_i \cos(\pi i/3) \]
\[ y_1 = \sum_{i=0}^{5} w_i \sin(\pi i/3) \]
Obtain \( w_i \) from the average group sum sequences for \( r = 1, 3, \ldots, 11 \) using the same procedure as for \( r = 0, 2, \ldots, 10 \).

Then compute

\[
x_2 = \sum_{i=0}^{5} w_i \cos[(\pi i/3) + \pi/6]
\]
\[
y_2 = \sum_{i=0}^{5} w_i \sin[(\pi i/3) + \pi/6]
\]

Finally \( u_4, v_4 \) are given by

\[
u_4 = x_1 + x_2\]
\[
v_4 = y_1 + y_2.
\]

Also starting from the average groups sum sequences as for the normal computation of the lunar variation one can achieve the same results by summing the appropriate values together. In this way the rearranging required in the previous section is avoided. This method would be more suited for calculations using electronic computers where values in different columns can be readily summed.

For example the \( g_s \) for \( p = 1 \) can be obtained from the following equations:

\[
g_0 = \sum_{r=0}^{10} g_{sr} + \sum_{r=\frac{10}{2}+1}^{11} g_{sr}, \quad j = s - 2r, \quad k = 24 - 2r + s \quad (s = 0, 2, \ldots, 20)
\]
\[
g_1 = \sum_{r=0}^{\frac{10}{2}} g_{sr} + \sum_{r=\frac{10}{2}+1}^{11} g_{sr}, \quad j = s - 2r, \quad k = 24 - 2r + s \quad (s = 1, 3, \ldots, 21)
\]
\[
g_2 = \sum_{r=0}^{11} g_{sr}, \quad j = s - 2r \quad (s = 22, 23)
\]

A method for dealing with equations such as (8) and (9) was given by Goertzel (1958).

References


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