Enhancements in Geomagnetic Power Spectra 
in the Frequency Band 1.6 to 6.8 MHz

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Irregular pulsations with periods from 150 to 600 seconds have been ob-
served as enhancements above the average background in the geomagnetic
power spectra computed over two-hour intervals from magnetic records taken
at Siple, Antarctica (L≈4); Durham, New Hampshire (L≈3.2); Lac Rebours,
Quebec (L≈4); and Girardville, Quebec (L≈4.5). A study is made of the
morphological characteristics of the power enhancements corresponding to
these pulsations. The irregular pulsations are compared with the more regular
but short-lived (typically 5-10 oscillations) sinusoidal pulsations with periods
of 150 to 600 seconds, usually known as Pc5's, which have been studied pre-
viously. Some comment is made on the theoretical models proposed for Pc5's
as models for the generation of the irregular pulsations.

1. Introduction

Certain types of pulsations in the earth's magnetic field have been exten-
sively studied. Sinusoidal pulsations with periods from 150 to 600 seconds are
called Pc5's. These usually appear in chart records of the magnetic field as
regular, sine-like waves, persisting for five to ten oscillations. The maximum
peak-to-peak amplitude is commonly on the order of 100 gamma (SAITO, 1969;
SAMSON, 1972). Hirasawa's study (HIRASAWA, 1970) includes only Pc5's with
maximum amplitude over 10 gamma.

The amplitudes of Pc5 events vary strongly with latitude; they are greatest
in the auroral zone, around L≈6, and fall off rapidly at lower latitudes. Am-
plitudes at L≈4 are typically lower than at L≈6 by roughly a factor of three
(SAITO, 1969). Pc5's occur most frequently and have largest amplitudes from
about 0600 to 0900 local time (LT). There is a smaller maximum of equal
sharpness in the afternoon or early evening (OHL, 1962; SAITO, 1969). The
average period of Pc5's becomes shorter at lower latitudes (SAITO, 1969; HIRA-
sawa, 1970; SAMSON and ROSTOKER, 1972). However, an individual event ob-
served simultaneously at many stations has the same period at all latitudes
(WILSON and Sugiura, 1961; SAMSON and ROSTOKER, 1972). Pc5's tend to
have shorter periods in the morning, on the average, and gradually longer

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periods throughout the day (Hirasawa, 1970).

Sato (1964) proposed that periodic injections of particles from the tail into the auroral zone cause Pc5's. Others have hypothesized that an instability in the plasma sheet or tangential stress by the solar wind on the boundary of the magnetosphere causes standing oscillations of the field lines, as these oscillations would have their largest amplitudes at high latitudes.

The diurnal variation in occurrence and amplitude of Pc5's strongly supports the torsional model of field-line oscillation discussed in some detail in Saito's review paper (Saito, 1969). Tangential stress on the field lines caused by interaction with the solar wind should be greatest on the morning and evening sides of the earth, and virtually absent during the middle of the night. It should cause left-hand polarized waves on the morning side and right-hand polarized waves on the evening side, as shown in Fig. 1 (Nagata et al., 1963). The left-hand sense corresponds to an Alfvén wave of non-isotropic propagation and the right-hand sense to a wave of isotropic propagation. Kato and Sato (1965) suggest that an anisotropy of the ionosphere causes waves rotating in the left-hand sense to have larger amplitudes than waves rotating in the right-hand sense when observed on the ground, and indeed, more and larger waves are seen during the morning maximum than during the evening.

The simple torsional model does not explain the constant period of individual Pc5's over a wide range of latitudes. Unless there is some effective filtering system, interaction with the solar wind should cause each field line to oscillate at its own natural resonant frequency (neglecting mode-coupling; cf. Radoski, 1971). The natural period of a line is dependent upon its length (Jacobs, 1970):

$$T = 2 \int_{\theta_0}^{\pi/2} \frac{d\theta}{V_A} = \frac{8\pi^{1/3}a^1}{M \sin^b \theta_0} \int_{\theta_0}^{\pi/2} \rho^{1/3} \sin^7 \theta \, d\theta,$$

where the first integration is over the length of the field line, $V_A$ is the Alfvén velocity, $\theta$ is the colatitude along the field line, $\theta_0$ is the colatitude of the field line at the earth's surface, $M$ is the dipole moment of the earth, $a$ is the radius of the earth, and $\rho$ is the plasma density along the field line. As noted above,
the period of an individual event does not change with latitude as predicted by Eq. (1) although, statistically, greater numbers of higher-frequency events are seen at lower than at higher latitudes.

HIRASAWA (1970) has proposed that a sharp discontinuity in the Alfvén wave velocity in the magnetosphere acts as a filter for ULF waves. That is, waves with frequencies corresponding to the eigenfrequencies of the field lines through the discontinuity are predominantly selected. He assumed a coincidence between the inner edge of the plasma sheet and the trapping boundary for radiation belt particles, and proposed that the trapping boundary might be this discontinuity.

HIRASAWA (1970) explained the diurnal variation of the average Pc5 period as an effect of changes in the position of the trapping boundary. His argument rests on the assumption that the trapping boundary is coincident with the inner edge of the plasma sheet. VASYLIUNAS (1968) has observed that this inner edge is closest to the earth at dawn and moves outward from morning until early evening. Throughout the day, as the field lines that constitute the boundary become longer, the natural period of vibration of the field lines should also lengthen. However, it should be noted that this argument assumes similar plasma distributions along all of the field lines, cf. (1).

We have computed power spectra in the frequency band from $4 \times 10^{-4}$ to $10^{-1}$ Hz for the geomagnetic fluctuations observed at four magnetometer stations near $L=4$. The stations are Siple, Antarctica, located at 84.0°W, 76°S; Durham, New Hampshire, at 70.9°W, 43.1°N; Lac Rebours, Quebec at 72.45°W, 47.87°N; and Girardville, Quebec at 72.53°W, 49°N (all geographic coordinates). Lac Rebours and Siple are conjugate points, as they are located at the northern and southern ends of a field line at $L=4.0$. Durham is at $L=3.2$, and Girardville is at $L=4.5$. The latter two stations are along a geomagnetic longitude ($\sim 3.5°W$) through Lac Rebours. Local time of the stations is equal to universal time (UT) minus five hours.

The power spectra often contained enhancements, visible as broad peaks above the average background, at frequencies between 1.67 and $6.67 \times 10^{-3}$ Hz; i.e., in the frequency range normally associated with Pc5 oscillations. As the power spectra were computed over two-hour intervals, an enhancement at a particular frequency means that during the two-hour period an usually large proportion of the power was at that frequency. A Pc5 wave (\~12\gamma amplitude) lasting for fifteen minutes would not produce a power enhancement above the background in a typical power spectrum taken over two hours. Neither would an enhancement necessarily correspond to an unusually long-lived or large Pc5. The enhancements usually corresponded to enhanced Fourier components at a certain frequency throughout the time interval. Often during an enhancement, the magnetic record showed a few oscillations at the enhancement frequency, but these oscillations recurred intermittently throughout the time period and
were irregularly shaped, in contrast to the more regular, sinusoidal-shaped Pc5's. One hour of H-component record taken at Siple Station on 17 January 1972 is plotted in Fig. 2. This hour of data is one-half of the data record used in computing the power spectra of Fig. 6, below. A typical Pc5 wave reproduced from HIRASAWA (1970) is plotted as an insert to Fig. 2. The irregular appearance of the oscillations corresponding to spectral enhancements suggested calling such waves Pi's. In keeping with the classification system for other irregular pulsations, the pulsations with periods from 150 to 600 seconds corresponding to our enhancements shall be referred to as Pi3's. For the purposes of a rough comparison of Pc5 and Pi3 amplitudes, it should be noted that the peak-to-peak amplitude of the largest Pi3 oscillation was usually from ~3 to ~13 gamma.

Other comparisons of Pi3 and Pc5 behavior are made below. Both the torsional and Hirasawa's models would be as likely to give the persistent, small jolts needed to cause Pi3's as the large, single impulses needed to cause Pc5's. If the Pi3's conform to the same patterns of diurnal and latitudinal variations as the Pc5's, it would seem likely that the causes of Pi3’s and Pc5’s are the same.

2. Techniques

Power spectra of H-component magnetic records were computed for the period from 2000 LT on January 5 to 1800 LT on January 17, 1972. A prolate-spheroidal data window (THOMSON, 1971; personal communication) was used before obtaining the Fourier coefficients of the time series with a fast Fourier transform algorithm. The power spectrum for frequencies from $4 \times 10^{-4}$ to $10^{-1}$ Hz was performed over two-hour intervals centered around each hour of the day. $>95\%$ of the spectral estimates of the instrument noise are less than $\sim 1 \times 10^{-17}/\text{Hz}$. All spectral points occurring below this spectral power were ignored. Figure 3 shows the power spectra at all four stations for the hours 2100 to 2300 LT on 5 January 1972. Assuming a power-law dependence to the background
Geomagnetic spectrum, we have least-squares-fitted lines to the power spectra over both the entire frequency window and the window with frequencies less than $10^{-3}$/sec. The high-frequency limit for each of the windows was taken as the point where the power first fell to the $>95\%$ spectral estimate of instrument noise. The variance of the spectral points about the fit to the entire window was usually about 0.2.

The spectra of Fig. 2 contain a representative Pi3 enhancement. The power is above the fitted line by a factor of roughly 25 at the maximum; the frequency of the maximum is about $5.5 \times 10^{-3}\text{Hz}$ at all stations. To be counted as a Pi3 event, an enhancement had to have a maximum frequency between 1.67 and $6.67 \times 10^{-3}\text{Hz}$, and a maximum power at Siple of at least a factor of eight above the fitted background. A partial check on the correctness of the second (power) criterion was made by considering separately all Pi3's with power at their maximum above the fitted background by a factor of 10 or more, and also by a factor of 15 or more. For a typical variance of 0.2 about the fitted line, 96.4\% of all the spectral points fall within a factor of 8 above or below the line and therefore would not be counted as enhancements under this criterion; 97.5\% of all the spectral points fall within a factor of 10 above or below the line, and 99.2\% of all the spectral points fall within a factor of 15 above or below the line. 139, 91, and 52 events qualified as enhancements in the Pi3 range with power factors above the fitted background of at least 8, 10, and 15, respectively. All the morphological trends discussed below were similar for the three criteria.

The size of each enhancement was measured as the power factor above the fitted geomagnetic background power, rather than the

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**Fig. 3.** Power spectra containing a Pi3 enhancement which occurred during 2100-2300 LT, January 5, 1972. The enhancement above the fitted background is largest at Durham.
absolute power at the peak of the enhancement. We chose this as the best way to measure the importance of the event at one station for ready comparison with different locations, because use of the fitted line subtracts out the effects of differences in conductivity of the ionosphere, the frequency dependence of the general background power level, and other background influences on the absolute size of the wave. Differences in the background power at different stations are noticeably smaller during magnetically quiet periods than during more active periods (this will be discussed in a future publication). The days for which we computed the power spectra were relatively quiet; $K_p$ was never above 6+, and usually stayed below 4.

Pi3's appearing at Siple were also measured at the three northern stations. In order to be included in the comparisons of relative power of the Pi3's at the three northern stations, each event was required to satisfy two additional criteria: a spectral enhancement of at least a factor of two above the fitted background had to be present at two or more of the northern stations, and all of the enhancements, including the original enhancement at Siple, had to fall within a frequency range of $0.5 \times 10^{-3}$ Hz. Fourteen of the 139 events were excluded from the table of relative power in the north because the enhancements in the north were too small, or because there were no enhancements at the frequency of the Siple maximum. Eight additional events were excluded from further analysis because data was unavailable from one of the stations at the time of the Siple Pi3. Finally, the power of each Pi3 at Siple was compared with the powers in the north. The same criteria were applied in the north and south for the events included in this comparison: the Pi3 enhancement had to be above the fitted background by a factor of eight or more at both Siple and Lac Rebours, the northern conjugate point to Siple.

The enhancements in Fig. 3 illustrate some common features of Pi3's. It is evident that the shape of the enhancement is different at the different stations. As there were often several neighbouring spectral peaks, the peak measured for each of the northern stations was the one that occurred within $0.5 \times 10^{-3}$ Hz of the frequency of maximum amplitude at Siple. This ensured that what was measured at the northern station was not a local effect but a magnetospheric one, associated with the lines of force near $L=4$ and appearing at both the northern and southern ends of the field line. This method of choosing the peak would have been misleading if the frequencies of Pi3's had been systematically dependent on latitude. But we observed no overall, large-scale trend in frequency as a function of latitude. The envelopes of the enhancements usually showed negligible shift in frequency between stations, and the differences in shape of the enhancements probably resulted from random local perturbations. For about two thirds of the Pi3's, the peak nearest the frequency of maximum amplitude at Siple was also the maximum peak of the enhancement for all the northern stations.
3. Results and Discussion

The numbers of Pi3's observed at Siple for each hour of the day are plotted against local time in Fig. 4. The lined area corresponds to events with power factors of 15 or more, the dotted area to events with power factors of 10 or more, and the cross-hatched area to events with power factors of 8 or more. There is a distinct maximum in the occurrence of events during the early morning hours, similar to the morning maximum reported for the occurrence of Pc5's. In fact, the maximum is observed between 0600 and 0900 LT, during the hours which Obertz and Raspopov (1968) cite as the time of excitation of all their Pc5's. However, a second occurrence maximum extends roughly from 1000 until 2000 LT. This maximum is too high and much too broad to agree with the evening maximum observed in the occurrence of Pc5's (Ohl, 1962; Saito, 1969; and Hirasawa, 1970).

The frequency at Siple of each Pi3 event is graphed in Fig. 5 as a function of local time. A least-squares straight-line fit to the events between 1000 and 2000 LT (the time interval including most of the events) gave a best-fit slope of $-0.0028$, showing an absence of any daily correlation between frequency and local time. However, a fit to the 53 events between 1500 and 2100 LT produced a slope of $-0.13 \pm 0.08$ with a linear correlation coefficient of 0.212. This low correlation coefficient indicates a 10% chance that the trend would arise from 53 uncorrelated points. Such a trend, if real, suggests that the plasmapause might have some effect on the frequency of Pi3's, as it is during the local evening hours that the plasmapause boundary would cross $L \sim 4$ during days of moderate geomagnetic activity (Carpenter, 1966).

Power factors above the average background, measured at Siple for all the enhancements, were also plotted against local time in the hopes of finding a general envelope of maximum power of Pi3's as a function of time. There was no observable trend. The few largest Pi3's tended to be seen during the hours

![Fig. 4. Frequency of occurrence of Pi3's as a function of local time.](image-url)
of maximum occurrence, as would be expected for any randomly picked group of events. The general profile of maximum power factors against local time was flat.

Most of the Pi3's obey the amplitude pattern observed for Pc5's: a strong dependence on latitude, with the greatest power at the highest latitude, close to the auroral zone. In our case, most of the Pi3's were largest at Girardville, next largest at Lac Rebours, and smallest at Durham, in latitudinal order from

![Graph showing frequency of Pi3's observed at Siple as a function of local time. The dashed lines mark the Pi3 frequency limits.](image)

**Fig. 5.** Frequency of Pi3's observed at Siple as a function of local time. The dashed lines mark the Pi3 frequency limits.

**Table 1.** Relative Power at Northern Stations

<table>
<thead>
<tr>
<th>Number of Pi3 events with power greatest at</th>
<th>day</th>
<th>night</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girardville</td>
<td>53</td>
<td>30</td>
</tr>
<tr>
<td>Lac Rebours</td>
<td>17</td>
<td>10</td>
</tr>
<tr>
<td>Durham</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

**Table 2.** Power at Siple Relative to Power in the North

<table>
<thead>
<tr>
<th>Power at Siple</th>
<th>Power in the North Greatest at:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Girardville</td>
</tr>
<tr>
<td><strong>Daytime events</strong></td>
<td></td>
</tr>
<tr>
<td>&gt; power at 2 or more northern stations</td>
<td>23</td>
</tr>
<tr>
<td>&lt; power at 2 or more northern stations</td>
<td>16</td>
</tr>
<tr>
<td><strong>Nighttime events</strong></td>
<td></td>
</tr>
<tr>
<td>&gt; power at 2 or more northern stations</td>
<td>7</td>
</tr>
<tr>
<td>&lt; power at 2 or more northern stations</td>
<td>4</td>
</tr>
</tbody>
</table>
north to south. However, a significant number of events were largest at Lac Rebours, and some were even largest at Durham. The event illustrated in Fig. 3 has its greatest power at Durham, with power factors above the fitted line of 31, 22, 16 and 25 at Durham, Lac Rebours, Girardville, and Siple, respectively. The absolute power at both Durham and Lac Rebours is greater than the power at Girardville. The largest wave of all those observed, shown in Fig. 6, was largest at Lac Rebours. It reached a power enhancement of over 100 at Lac Rebours, with enhancements of 80, 9.2, and 52 at Girardville, Durham, and Siple, respectively.

The number of Pi3’s which were largest at each station during the day and night are listed in Table 1. Any Pi3 occurring at or between 0700 and 1700 LT was considered a daytime event. Pi3’s with largest amplitudes at Girardville—the predominant type of Pi3—show a marked diurnal dependence, and the events with largest amplitudes at Durham and Lac Rebours seem to show similar behavior. However, the seven events which were largest at Durham, and perhaps the 27 events which were largest at Lac Rebours as well, are an insufficient number to determine whether there is a diurnal dependence. None of these three subtypes of Pi3’s were concentrated at any one frequency; they all seemed to be evenly and similarly distributed over the range of Pi3 frequencies.

Table 2 contains comparisons of power at Siple with power in the north for events with power factors of 8 or more both at Siple and at Lac Rebours. As Siple is at the same $L$ value as Lac Rebours, but is higher in latitude than Durham and lower than Girardville, the expected variation of power for events largest at Girardville predicts that

![Fig. 6. Power spectra containing a Pi3 enhancement which occurred during 1600–1800 LT, January 17, 1972. The enhancement is largest at Lac Rebours.](image)
power factors should be equal at Siple and at Lac Rebours, on the average. Enhancements should be larger at the northernmost station, Girardville, than at Siple, and smaller at Durham than at Siple. Therefore, the power factor at Siple should be greater than the power factors at two or more northern stations about half the time, and less than the power factors at two or more northern stations about half the time. Although only 50 of the Pi3's which were largest at Girardville qualified for this analysis, the final statistics seem to indicate that this prediction is not verified. The power at Siple is systematically greater than that expected for these Pi3's. This trend appears both during local day, when all the stations are in sunlight, and during the January night, when only the Siple ionosphere is illuminated. The numbers of waves largest at Lac Rebours and Durham which qualify for this analysis are too small to yield any statistically significant results.

4. Implications

Any theoretical conjectures about Pi3's will have to explain the generation of the events with largest amplitudes at Lac Rebours and Durham, as well as the events with largest amplitudes at Girardville. The torsional model, models using instabilities in the plasma sheet to excite oscillations of the field lines, and other such models (Oguti, 1969) used for Pc5's do not fit the two anomalous subtypes of Pi3's. The standard amplitude pattern predicted by these models allows only Pi3's with largest amplitudes at Girardville. In addition, the torsional model of excitation predicts a Pc5-like pattern of occurrence with sharp morning and evening maxima. The afternoon maximum in Pi3 occurrences is much too broad to agree well with this theory.

The existence of Pi3's which are largest at Durham and at Lac Rebours might be explained by assuming either localized sources or a natural resonant frequency for oscillation of field lines, depending on \( L \). Disturbances generated outside the trapping boundary would tend to produce the greatest oscillations in the field lines which resonate at the frequency of the disturbance, even if these were well inside the trapping boundary. However, both Pi3's greatest at Durham and Pi3's greatest at Lac Rebours cover a wide range of frequencies. Neither type showed any noticeable concentration at a specific frequency. We conclude that localized sources of Pi3's must exist at \( L \) values well below the auroral zone, although the events tend to be generated most often in the higher latitudes. This result will be discussed in more detail in a future publication.

Protons with energies from 0.23 to 3.7 keV have bounce periods in the Pi3 range. The energy density carried by the normal population of these protons in the magnetosphere near \( L = 4 \) is sufficiently high that a Pi3 could conceivably be caused by a semi-monoenergetic group of protons in bounce resonance. If such a semi-monoenergetic group were injected from the tail to roughly \( L = 4 \),
we would expect any protons which diffused inward to $L=3.2$, conserving their first invariant, to have higher energies than those remaining further out. However, in order to cause an enhancement of the same frequency at Durham as at Lac Rebours, the protons at $L=3.2$ would have to be of lower energy than those at $L=4.0$.

This can be seen more clearly in Fig. 7, where the bounce period for a proton is plotted against its energy. The Pi3 in Fig. 3 may be taken as an example. The width of the main peak at Siple was taken as a rough half-width of this Pi3 enhancement. The estimated half-width of 155 to 222 seconds corresponds to a group of protons ranging in energy from 2.1 to 4.6 keV. The two endpoints are plotted along the bounce-period line for $L=4.0$. A group of protons in this energy range would produce a Pi3 with a 125 to 178 second period at Durham, and a 175 to 250 second period at Girardville. But the enhancement in Fig. 2 seems to be located at approximately the same frequency for all the stations. Therefore, if Pi3's are caused by semi-monoenergetic groups of protons in bounce resonance, these groups must be created inside the trapping zone by some process or instability which accelerates protons to different energies depending upon the $L$ value.

The azimuthal drift periods at $L=4$ for protons of 1.15 to 4.6 MeV and electrons of 1.9 to 8.7 MeV are also in the Pi3 range. The enhanced power at the frequency of a particular Pi3 may provide the energy for increased diffusion inward across $L$-shells for particles with the same drift period as the Pi3. These electrons and protons with radiation-belt energies do not carry a sufficient energy density to cause Pi3's, but their temporarily increased diffusion may be of importance in estimating fluctuations in the commonly accepted diffusion coefficients for these particles.

Fig. 7. Bounce period as a function of energy for protons near $L=4$. The dots on the $L=4$ line indicate the energy range for protons with bounce frequencies in the frequency range of the Pi3 event of Fig. 3 (see text).
We would like to thank Ms. M.F. Robbins and C.G. MacLennan for much assistance and advice and for the programs used in computing the power spectra.

REFERENCES


Oguti, T., Distribution of natural waves in the magnetosphere, paper presented to the Japan-U.S. Scientific Cooperation (JUSCO) symposium held in Kyoto, Japan on March 19, 1969.


