Fluctuations in the Earth’s Rate of Rotation Related to Changes in the Geomagnetic Dipole Field

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Relation between changes in the dipole field and those in the earth’s rate of rotation has been examined for three different periods, 8000, 400 and 65 years, based on such various kind of data as those of archaeomagnetism, observation of moon’s longitudes and recent instrumental observations. It has been confirmed that fluctuation exists in the rate of rotation corresponding to the dipole change for these periods. The magnitude of the variation in the rotation to be related to the dipole change increases as the period decreases.

Time variation in the drift velocity has also been examined for several features of the geomagnetic secular variation. It is very probable that a change in the drift velocity accompanied the 400 year period fluctuation in the earth’s rotation.

The results thus obtained are compared with a theoretical result. Except for the phase of the angular velocity of the mantle for 65 year period and the magnitude of the geomagnetic westward drift for 400 year period, the observed results are well accounted for by an electromagnetic coupling caused by a dipole change at the core-mantle boundary, provided that the electrical conductivity of the lower mantle is as large as $10^{-8}$ emu.

1. Introduction

In a previous paper (Yukutake, 1972), a change in the dipole moment was suggested as a possible cause of irregular fluctuations in the rate of the earth’s rotation. When the dipole moment changes, toroidal fields within the core and the conducting lower mantle are supposed to vary. Then the electromagnetic couple acting on the mantle and the core will also change and lead to fluctuations in the rotational speed of both the mantle and the outer part of the core.

In this paper, it is attempted to correlate the observed variations in the dipole field with those of the earth’s rotation. Time variations in the drift rate of the geomagnetic field are also investigated, since they can be an important information of the movement of the outer part of the core.

As the dipole field is observable, unlike the toroidal field, at the surface of the earth, no insurmountable difficulty will be met in the examination of relation between changes in the dipole field and those in the earth’s rate of rotation.
It is, on the other hand, extremely difficult to extract real change in the drift velocity of the geomagnetic field as a whole. Existence of close correlation was pointed out between movement of the eccentric dipole of the geomagnetic field and fluctuation in the length of the day (Vestine, 1953; Vestine and Kahle, 1968; Kahle et al., 1969). The apparent correlation has been accepted as a strong evidence for the electromagnetic coupling of the mantle to the core, since the movement of the eccentric dipole has been regarded as representing that of substantial part of the geomagnetic field which is frozen in the outer part of the electrically conducting liquid core. However, it was shown that, with the present values of the spherical harmonic coefficients of the magnetic potential, the westward movement of the eccentric dipole is determined almost entirely by the westward drift of only one term, that for \( n=2, m=1 \), of the magnetic potential (Yukutake, 1973). Therefore, it is not justified to consider that the eccentric dipole is eligible for representing the movement of the geomagnetic field as a whole only because it contains substantial part of the magnetic energy.

If the main field is taken to consist of standing and drifting parts, movement of any particular feature of the main field does not indicate real variation in the velocity of the drifting field (Yukutake and Tachinaka, 1969; Yukutake 1970). From the main field data, detection of the real change is only possible when a simultaneous change is confirmed in the drift velocities of various features.

Provided that both standing and drifting fields are constant in time, the velocity of the drifting field is obtainable not from main field itself but from the rate of change in the field, the secular variation. Accordingly the secular variation data are preferable for examining the velocity change to the main field data. When scrutinized, however, intensities of both fields are suspected to change with time. This may add another complexity in detection of change in the drift velocity. In order to avoid this effect from standing field, examination of the spherical harmonic term, for \( n=2, m=2 \) of the secular variation is most desirable, because the term, \( n=2, m=2 \), has the highest content of drifting field among the spherical harmonic components of the main field. The amplitude of the drifting part is 4.7 times larger than that of the standing part for this component (Yukutake and Tachinaka, 1969).

In Section 2, quantitative estimate is attempted of the relation between the dipole change and the fluctuation in the rotation for three different periods. They are 8000 year period observed in the dipole change as obtained from archeomagnetism, several hundred years period characteristic to the “Great Empirical Term” in the longitude variation of the moon, and the decade variation in the earth’s rotation. Possibility of change in the drift velocity of the geomagnetic field is also investigated in association with the several hundred year period change and the decade variation, with particular weight on the harmonic component for \( n=2, m=2 \) of the secular variation. The results thus
obtained show that the variation in the rotation associated with the change in the dipole is highly dependent on period. In Section 3, the observed results are compared with the theory presented in the previous paper (Yukutake, 1972), where it is shown that such period dependence as observed in the variation in the rotation is possible to arise from two sources. One is the effect of the electrical conductivity of the lower mantle. With the increase in the conductivity a large amplitude variation in the rotation is caused for short periods. The other source is the effect of the toroidal field from within the core. Magnitude of the toroidal field diffusing out into the mantle has also large effect on the period dependence of the variation in the rotation. The results obtained from three different periods are explained sufficiently well by taking the conductivity of the lower mantle to be $10^{-8}$ emu.

2. Geomagnetic Secular Variations to Be Related to Fluctuations in the Earth's Rate of Rotation

2.1 Acceleration of the earth's rotation during the past few thousand years

From ancient observations of solar eclipses and some other events such as lunar occultations, lunar eclipses and equinox observations, the earth's rate of rotation is known to have been undergoing secular acceleration during the past few thousand years besides the steady retardation due to tidal frictional source (see for example Munk and MacDonald, 1960; Newton, 1969, 1970). For the data before 500 A.D. Newton (1969) obtained the non-tidal acceleration to be $4.63 \times 10^{-22}$ rad/sec$^2$ with an effective epoch of about 200 B.C. For the data after 500 A.D. he obtained $6.02 \times 10^{-22}$ rad/sec$^2$ with an effective epoch of 1000 A.D. A remarkable fact indicated by these results is that, during the past few thousand years, the earth's rotation has been being accelerated due to a non-tidal mechanism.

As was discussed in the previous paper (Yukutake, 1972), some parts of the non-tidal acceleration of this time scale can be produced by an exchange of angular momentum between the mantle and the core through electromagnetic coupling associated with a large change in the geomagnetic dipole moment with a period of about 8000 years. It is well established that the geomagnetic dipole moment about 2000 years ago was approximately 1.5 times as large as the present one. Further extension of archeomagnetic investigation over the past 9000 years has revealed that the dipole field has been changing approximately periodically with a large amplitude amounting to about 50% of the present dipole moment, and its period being about 8000 years (Bucha, 1967; Smith, 1968; Cox, 1968; Kitazawa, 1970; Bucha, 1970). The last maximum of the dipole moment was probably sometime between 0 and 500 A.D. Since then the moment has been decreasing, and the electromagnetic strength that couples the
mantle to the core has been diminishing. Accordingly, acceleration of the earth’s rotation is expected during the past 2000 years.

For the variation with such a long period as 8000 years, the theory suggests the phase difference of almost $\pi$ between the angular velocity of the mantle and the dipole change. Consequently, so far as the electromagnetic process as in YUKUTAKE (1972) is assumed as the source of non-tidal fluctuation in the rotation, negative acceleration is to be expected before the magnetic moment reaches its maximum and zero acceleration at the time of its maximum. Observation suggests, however, positive acceleration amounting to $4.63 \times 10^{-22}$ rad/sec$^2$ just before the dipole moment reached the last maximum. Some other sources must be sought for the positive acceleration of this amount. The difference in the acceleration between 1000 A.D. and 200 B.C., that is $1.39 \times 10^{-22}$ rad/sec$^2$, might be ascribed to the effect of the periodical variation in the dipole moment. Since the decrease in the dipole moment is still continuing up to the present, covering further 1000 years after the last estimation of acceleration, $2.8 \times 10^{-22}$ rad/sec$^2$ may be roughly taken as the maximum acceleration corresponding to the dipole variation. This leads to fluctuation of angular velocity of $1.1 \times 10^{-11}$ rad/sec, which is the same order of magnitude as suggested theoretically in the previous paper (YUKUTAKE, 1972), when the dipole term changes with an amplitude of 50% of the present value, $\delta g^0_1 = 0.15$ gauss. This is equivalent to the variation of the angular velocity of $7.5 \times 10^{-11}$ rad/sec for the change in the dipole term of $\delta g^0_1 = 1$ gauss.

It is not at all clear when the acceleration of the rotation of the mantle was at its minimum. The time of the minimum acceleration is theoretically expected to coincide with that of the maximum dipole moment. However, if 200 B.C. is tentatively taken as an extreme date for the time of minimum acceleration, the difference between this and the time of theoretical expectation, that is sometime between 0 and 500 A.D., is equivalent to the phase difference of $0.2\pi$ for the period of 8000 years. This gives a rough estimate of the uncertainty of the phase difference between the rotation and the dipole change.

2.2 Several hundred years variation

Since the early 19th century, when spherical harmonic analysis of the earth’s magnetic field started, the magnitude of the dipole component $g^0_1$ has been decreasing. However, a recent analysis for the 17th and the 18th century data (YUKUTAKE, 1971) suggests that $|g^0_1|$ increased during that period, and reached a maximum towards the end of the 18th century, as shown at the top of Fig. 1. This maximum is a fluctuation superposed on the general trend described in the previous section, the gradual decrease since about 2000 years ago. Then there must be a minimum in the dipole moment sometime around 1600 A.D. The peak to trough amplitude of this variation amounts to about 1750 year$^{-1}$ on a time scale of approximately 400 years.
During this time there was a large fluctuation in the observed moon’s longitudes which is ascribed to a change in the earth’s rate of rotation. The fluctuation is so remarkable that it may be approximated by so called Newcomb’s
"Great Empirical Term," with its maximum around 1790 A.D., as is shown in Fig. 1, curve B (Jones, 1939; Brouwer, 1952). If the fluctuation in the moon's longitude is entirely due to the non-tidal change in the earth's rotation, rate of change in the annual means of the longitudes gives fluctuation in the length of the year. The rate of change is calculated by taking the gradient of the curve B in Fig. 1 on the data picked up roughly at an interval of 40 years from Jones' (1939) and Brouwer's (1952) and shown in Fig. 1. It suggests that during the period concerned the length of the year was the longest around 1700 A.D. and then becomes shorter, arriving at its minimum sometime around the end of the 19th century. The peak to trough variation of the length of the year amounts to 1.1 sec/year, which corresponds to the fluctuation of the angular velocity of the earth, $\delta \omega/\omega = 3.4 \times 10^{-8}$. The length of the year curve leads the dipole curve by about one hundred years, a phase difference of about $\pi/2$.

2.3 Variation in the drift velocity of the geomagnetic field

The westward drift of the geomagnetic field is clearly exhibited by tracing some particular features of the geomagnetic secular variation. During the period from the 16th to the early 20th century at many places in North America, magnetic eastward declination reached its maximum and turned to change westwards (Bauer, 1902; Herbert, 1926; see also Yukutake, 1967; Dawson and Dalgetty, 1967). Figure 2 shows how the longitudes, where the maximum deviation of declination was observed, varied with time. The east longitudes seem to have decreased fairly rapidly during the period from 1540 A.D. to 1625 A.D., suggesting perhaps very fast westward drift. Until 1800 A.D. the variation in the longitude is rather small. Since then, however, the decrease became remarkable again. This indicates that the westward drift of the maximum devia-

![Figure 2](image-url)
Fluctuations in the Earth's Rotation Related to Dipole Change

tion of declination which was fast over 1540 A.D. to 1625 A.D. became very slow during the 18th century and it resumed its rapid velocity at the beginning of the 19th century. Open circles in Fig. 2 show the longitudes where time derivative of the east component of the geomagnetic field becomes zero along a parallel circle 40°N, namely, the longitudes where the east component reaches its maximum. They were read from charts of geomagnetic secular variation synthesized from spherical harmonic coefficients (Yukutake and Tachinaka, 1968). They fit in well with the maximum declination data.

The same tendency is also seen in the variation in the longitudes of intersection of the equator with the west agonic line on which the magnetic declination is zero (Bauer, 1895; Yukutake, 1967). Slowness of the westward drift in the 18th century is not likely to be a local phenomenon peculiar to North America but a phenomenon which took place on a worldwide scale. This may be more clearly shown from examination of other features of the geomagnetic secular variation outside American continent. There are three other places along 40°N parallel where the time derivative of the east component becomes zero. At one of them the east component is at its maximum like in North America, and at other two localities it is at minimum. All these four longitudes are averaged and shown in Fig. 3 by $\phi_y$. $\phi_y$ decreases with time. However, the rate of decrease is small in the 18th century in contrast to the rapid decrease since 1800 A.D., indicating the slowness of the drift velocity in the 18th century.

Fig. 3. Variation of $\phi^2$ with time, the phase angle for a spherical harmonic term, $n=m=2$ of the secular variation potential and that of $\phi_y$, the mean longitude of the maximum and the minimum east component of the magnetic field.
Among the spherical harmonic coefficients of the main field, the term \(n=2\) and \(m=2\) has the highest content of drifting part. In order to envisage fluctuation in the drift rate it is most reliable to examine the phase angle for the term \(n=2, m=2\) of the geomagnetic potential of the secular variation. \(\phi_2^2\) is calculated from \(g_2^2\) and \(h_2^2\) in YUKUTAKE (1971), by \(\phi_2^2 = (1/2) \tan^{-1}(h_2^2/g_2^2)\), and shown in Fig. 3. Variation of \(\phi_2^2\) with time is very similar to that of \(\phi_Y\). Drift rate of \(\phi_2^2\) is obtained by calculating the gradient of successive points and listed on Table 1. It was 0.20\(^\circ\)/year before 1700 A.D. but decreased to 0.10\(^\circ\)/year around 1724 A.D. Then it increased again to approximately 0.4\(^\circ\)/year in the early 20th century as is shown in Fig. 1. In Fig. 1 fluctuation in the drift rate of \(\phi_2^2\) is compared with those of geomagnetic dipole term \(g_1^0\) and the length of the year. With increase in the dipole intensity the westward drift velocity of the secular variation decreases. When the eastward direction is taken positive, the variation in the drift velocity leads the dipole change by about 50 years, equivalent to 0.125\(\pi\) phase lead. Figure 1 suggests that, associated with the change in the dipole moment with its peak around 1800 A.D., variations are likely to have taken place both in the earth's rate of rotation and in the drift velocity of the geomagnetic field.

### 2.4 Variation in the length of the year on decade time scale

It is well known that the earth’s rotation undergoes large irregular fluctuations on decade time scale. Let \(\Delta t\) be difference between the ephemeris time and the universal time, namely, \(\Delta t = ET - UT\). Then the rate of change in \(\Delta t\) gives fluctuation in the length of the year. Approximating five successive annual values of \(\Delta t\) with a quadratic equation by a least squares method, we have as the time derivative at the central point

\[
\frac{d(\Delta t(0))}{dt} = \frac{1}{10}[\Delta t(1) - \Delta t(-1) + 2(\Delta t(2) - \Delta t(-2))].
\]

Based on \(\Delta t\) in BROUWER (1952), McBAIN (1948, 1951, 1952, 1953) and the data obtained by Hydrographic Office of Maritime Safety Board, Japan (see also

<table>
<thead>
<tr>
<th>Epoch</th>
<th>(\phi_2^2)</th>
<th>Epoch</th>
<th>(d\phi_2^2/dt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1650</td>
<td>41°35'</td>
<td>1680</td>
<td>-0.20°/yr</td>
</tr>
<tr>
<td>1710</td>
<td>29 45</td>
<td>1724</td>
<td>-0.10</td>
</tr>
<tr>
<td>1738</td>
<td>27 05</td>
<td>1772</td>
<td>-0.14</td>
</tr>
<tr>
<td>1805</td>
<td>17 35</td>
<td>1831</td>
<td>-0.37</td>
</tr>
<tr>
<td>1857</td>
<td>-1 50</td>
<td>1879</td>
<td>-0.37</td>
</tr>
<tr>
<td>1900</td>
<td>-17 48</td>
<td>1921</td>
<td>-0.53</td>
</tr>
<tr>
<td>1942.5</td>
<td>-40 12</td>
<td>1954</td>
<td>-0.34</td>
</tr>
<tr>
<td>1965</td>
<td>-47 44</td>
<td></td>
<td></td>
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</tbody>
</table>
The rate of change has been calculated by the above formula. The results are shown in Fig. 4, which is essentially the same as that already obtained by many investigators (Brouwer, 1952; Vestine, 1953). The length of the year varies irregularly with fairly large amplitude. For 1891 to 1901 A.D. it was lengthened about 1.5 sec/year. In 1909 it started shortening and reached a minimum around 1930. The decrement amounts to 1.5 sec/year which is a fluctuation in the angular velocity \( \frac{\Delta \omega}{\omega} \) of \( 4.6 \times 10^{-8} \).
2.5 Variations in the dipole field and apparent changes in the drift rate of the non-dipole field

There are many spherical harmonic analyses conducted in the 20th century. However, mostly they are based on different set of data. The methods employed for the analysis are also different. This restricts detection of small fluctuation of the magnetic field. In order to examine the variation with high accuracy, it is desirable to base the analysis on the data more homogeneous with time such as annual mean values at fixed observatories, though the reliance can only be placed on relative variations rather than absolute values for respective epochs unless the observatories cover the globe uniformly with high density. Malin (1969) made analysis for annual mean values of 80 observatories at an interval of five years for the period from 1942.5 to 1962.5. This is still insufficient both in the total coverage of period and in the time interval of the analysis to compare with the data of the earth’s rotation. In this paper the results of spherical harmonic analyses both for the main field and for the secular variation based on each annual mean values of selected 21 stations from 1900 to 1965 (Yukutake, 1972, unpublished) are used.

The absolute value of the axial dipole term \( g_i^e \) is decreasing gradually during the period concerned, as has been indicated by many investigators (see, for example, Mauersberger, 1959; Rikitake, 1966; Malin, 1969). Then a straight line was applied to approximate the general tendency and subtracted from the annual \( g_i^e \) values. The results \( \delta g_i^e \) are shown in Fig. 4. To extend \( \delta g_i^e \) before 1900 A.D. the analyses were repeated for six observatories where the data are available for the period from 1890 to 1965. The results are also shown in Fig. 4. Good parallelism is seen between the analyses for the 21 stations data and those for six stations data.

It is clearly seen that \( \delta g_i^e \) has changed in a sense to reduce the absolute value of \( g_i^e \) during the first 30 years of the 20th century and then turned to change in an opposite sense. From around 1955 A.D. the rate of change seems to have diminished, as if to suggest another maximum approaching shortly. When the result of the analysis for the six observatories is examined, there was a maximum around 1900 A.D. The amplitude from the crest to the trough amounts to approximately 260\( \gamma \) and the period is supposed to be about 65 years. Similarity is noted as in Fig. 4 between the variations in \( \delta g_i^e \) and the length of the year which was at its maximum sometime between 1900 A.D. and 1910 A.D. and at a minimum around 1930 A.D.

The drift rate of the eccentric dipole has been calculated from the annual mean value in a way similar to the length of the year calculation, and is also shown in Fig. 4. Variation in the drift rate is considerably different from that obtained by Vestine (1953), and no clear correlation is seen to the variation in the length of the year. The drift rate of the harmonic component \( n=2 \) and \( m=2 \) for the secular variation was similarly calculated by \( V_2^e=(1/2) \tan^{-1}(h_2^e) \).
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Being about 0.55°/year at the beginning of the 20th century, it decreased to 0.17°/year in 1934 A.D. and resumed its previous large value, about 0.5°/year, in 1950's. Gross tendency appears to resemble the variation in $\delta g_2^0$ and that of the length of the year. However, it should not be concluded hastily that the variation obtained here is straightforward representation of fluctuation in the drift rate of the drifting part of the geomagnetic field, because as shown in Fig. 5 there is a hump in $g_2^0$ for the period 1900 A.D. to 1950 A.D. amounting to 250γ, while $h_2^2$ decreases rather monotonously during the period. If this is due to the change in the drift rate, a similar hump will be also observed in the variation of $h_2^2$. Therefore the fluctuation in the drift rate $V_2^2$ shown in Fig. 4 is more likely due to some other sources rather than the change in the drift rate of the drifting geomagnetic field.

2.6 Summary of the observational data

Though the data are still insufficient to establish definite relationship between the fluctuations in the earth's rate of rotation and those in the dipole moment, the results obtained in the previous sections are summarized here, on the assumption that the variations in the rotation are caused by seemingly corresponding changes in the dipole moment.

Table 2 shows the relation between the angular velocity of the mantle and the change in the dipole term, $\delta g_2^0$. The fourth column indicates the equivalent angular velocity variation ($\delta \omega_m$) reduced to that when a change in the dipole
term of unit intensity ($\delta g_1^0$: gauss) is applied. As the period becomes shorter, the excited angular velocity increases. In the fifth column the phase difference between the angular velocity and $\delta g_1^0$ is listed. It is noted that it is nearly $\pi$ both for $T=8000$ years and for $T=65$ years. However, for an intermediate period $T=400$ years, $\delta \omega_m$ leads $\delta g_1^0$ by $\pi/2$.

In Table 3 relations inferred from observation are listed between $\delta g_1^0$ and the angular velocity of $n=2$ and $m=2$ term in the geomagnetic secular variation ($\delta \omega_1$), which is regarded as best representing the motion of the drifting part of the geomagnetic field. In the sixth column, phase difference of $\delta \omega_m$ to $\delta \omega_1$ is also shown. For the period $T=400$ years the phase difference is 1.23\(\pi\). A phase difference of $\pi$ would indicate that when the rotational speed of the mantle is accelerated the westward drift of the geomagnetic field is also accelerated. As for such short periods as 65 years, the relation remains obscured. Observation suggests that both the rotation of the mantle and the $n=2$ and $m=2$ term of the geomagnetic secular variation have changed coincidentally in the same phase.

3. Comparison of the Observational Data with a Theory

3.1 Effect of the mantle conductivity

In the previous paper (YUKUTAKE, 1972), electromagnetic coupling of the mantle to the core has been calculated based on rigidly rotating spherical shell model. When the change in the dipole field originates near the surface of the core, and hence the change in the toroidal field is generated only at the core-mantle boundary (case B of the previous paper), the variation in the angular velocity of the mantle caused by a change in $\delta g_1^0$ of unit intensity is nearly con-
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stant \( (=5.5 \times 10^{-11} \text{ rad/sec. gauss}) \) for periods longer than 1000 years. For periods shorter than 1000 years, the angular velocity of the mantle is highly dependent on the period, as is seen in Fig. 6, where the amplitude and the phase

![Graph showing the change in angular velocity of the mantle](image)

Fig. 6. Change in the angular velocity of the mantle produced by a change in the dipole field of unit strength \((\delta g_0 = 1 \text{ gauss})\). The upper diagram shows variation in the absolute value of the angular velocity with period, and the lower diagram that of the phase difference between the angular velocity and the dipole change. The curves are the results calculated in Yukutake (1972) for various mantle conductivities on an earth model where the dipole change is limited near the core surface and the variation in the toroidal field is generated solely at the core mantle boundary. The three points are the observed results reduced to variations for unit change of the dipole field \((\delta g_0 = 1 \text{ gauss})\).
of the angular velocity are shown by solid curves, taking the mantle conductivity ($\sigma_m$) as a parameter. If the electrical conductivity of the lowest several hundred kilometers' thickness of the mantle is assumed to be $10^{-9}$ emu (model B1), the angular velocity of the mantle decreases with a decrease in the period, whereas it increases for the conductivity of $10^{-8}$ emu (model B2). There must be a critical conductivity between $10^{-9}$ and $10^{-8}$ emu for which the angular velocity is independent of period.

The observational data obtained in the previous sections are plotted by solid circles in Fig. 6. These data indicate that the amplitude of the angular velocity increases as the period decreases. This is evidently different from the curve for $\sigma_m=10^{-9}$ emu. The observed three points are between the curves for $10^{-8}$ and $10^{-7}$ emu, rather close to that for $10^{-8}$ emu. Therefore the amplitude data are well accounted for by the present model (case B), so long as the electrical conductivity of the lower mantle is taken to be slightly higher than $10^{-8}$ emu.

The phase difference, however, is not so well explained as the amplitude by the above simple model. As is seen in the lower diagram of Fig. 6, observed points for the periods of 400 and 8000 years are very close to the curve for $\sigma_m=10^{-8}$ emu, rather close to that for $10^{-8}$ emu. Therefore the phase data are well accounted for by the present model (case B), so long as the electrical conductivity of the lower mantle is taken to be slightly higher than $10^{-8}$ emu.

The phase difference, however, is not so well explained as the amplitude by the above simple model. As is seen in the lower diagram of Fig. 6, observed points for the periods of 400 and 8000 years are very close to the curve for $\sigma_m=10^{-8}$ emu, rather close to that for $10^{-8}$ emu. Therefore the phase data are well accounted for by the present model (case B), so long as the electrical conductivity of the lower mantle is taken to be slightly higher than $10^{-8}$ emu.

Table 4. Relation between the angular velocity of the mantle ($\omega_m$) and the change in the dipole field ($\delta g_1^0$) calculated for a rigidly rotating spherical shell model.

| Period (yrs) | $|\delta \omega_m|/|\delta g_1^0|$ | Phase | $|\delta \omega_m|/|\delta g_1^0|$ | Phase | $|\delta \omega_m|/|\delta g_1^0|$ | Phase |
|--------------|-----------------|-------|-----------------|-------|-----------------|-------|
| 8000         | $1.46 \times 10^{-10}$ | 0.828$\pi$ | $5.55 \times 10^{-11}$ | 0.992$\pi$ | $5.56 \times 10^{-11}$ | $-0.979\pi$ |
| 400          | $5.46 \times 10^{-8}$ | 0.225$\pi$ | $4.64 \times 10^{-11}$ | 0.856$\pi$ | $8.19 \times 10^{-11}$ | $-0.650\pi$ |
| 70           | $5.43 \times 10^{-4}$ | $-0.793\pi$ | $1.64 \times 10^{-11}$ | 0.818$\pi$ | $4.78 \times 10^{-10}$ | $-0.086\pi$ |
| 60           | $1.95 \times 10^{-3}$ | $-0.358\pi$ | $1.49 \times 10^{-11}$ | 0.840$\pi$ | $6.06 \times 10^{-10}$ | $-0.017\pi$ |

model A: The conductivity of the mantle $\sigma_m=10^{-9}$ emu; Toroidal fields generated both at the core-mantle boundary and at the deep interior interface in the core.

model B1: $\sigma_m=10^{-9}$ emu; Toroidal field generated only at the core-mantle boundary.

model B2: $\sigma_m=10^{-8}$ emu; Toroidal field generated only at the core-mantle boundary.

As for the variation in the drift rate of the magnetic field, we have only one phenomenon that may be associated with the fluctuation in the earth's rotation, that is the variation on the time scale of about 400 years which took place over the 18th to the 19th century. As is summarized in Table 3, the observed phase difference is $0.27\pi$ between the magnetic drift rate and the variation in the dipole moment. The phase difference calculated on the model B2 is $0.35\pi$ for the 400 year period (Table 5). Agreement between the observation and the calculation may be regarded fairly good. However, in the amplitude comparison, there is large discrepancy between them. Comparing Table 3 with
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Table 5. Relation between the angular velocity of the outer core (\(\dot{\omega}_{\text{owl}}\)) and the change in the dipole field (\(\dot{g}^{(i)}_d\)) calculated for a rigidly rotating spherical shell model.

| Period | \(\left|\frac{\dot{\omega}_{\text{owl}}}{\dot{g}^{(i)}_d}\right|\) | Phase \(\langle\dot{\omega}_{\text{owl}} - \dot{g}^{(i)}_d\rangle\) | \(\left|\frac{\dot{\omega}_{\text{owl}}}{\dot{g}^{(i)}_d}\right|\) | Phase \(\langle\dot{\omega}_{\text{owl}} - \dot{g}^{(i)}_d\rangle\) | \(\left|\frac{\dot{\omega}_{\text{owl}}}{\dot{g}^{(i)}_d}\right|\) | Phase \(\langle\dot{\omega}_{\text{owl}} - \dot{g}^{(i)}_d\rangle\) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| yrs    | rad/sec-gauss   | rad/sec-gauss   | rad/sec-gauss   | rad/sec-gauss   | rad/sec-gauss   | rad/sec-gauss   |
| 8000   | 7.91 \times 10^{-10} | 0.158\pi       | 4.44 \times 10^{-10} | -0.009\pi       | 4.45 \times 10^{-10} | 0.021\pi       |
| 400    | 3.36 \times 10^{-7}  | 0.671\pi       | 3.71 \times 10^{-10} | -0.144\pi       | 6.55 \times 10^{-10} | 0.351\pi       |
| 70     | 1.88 \times 10^{-2}  | -0.303\pi      | 1.30 \times 10^{-10} | -0.179\pi       | 3.80 \times 10^{-9}   | 0.917\pi       |
| 60     | 7.88 \times 10^{-2}  | 0.134\pi       | 1.18 \times 10^{-10} | -0.157\pi       | 4.80 \times 10^{-9}   | 0.986\pi       |

Models A, B1, and B2 are the same as in Table 4.

Table 5, we see the observed amplitude of the magnetic drift rate being 17 times larger than that of the model B1.

If the toroidal field which exerts fluctuating couple on the mantle is solely of a core-mantle boundary origin (case B), the conductivity of \(10^{-8}\) emu for the lowest several hundred kilometers' part of the mantle (model B2) is a fairly good model to account for the period dependent feature of the angular velocities of the mantle and the outer core. However, two problems still remain unsolved in this model. One is regarding the phase difference between the rotation of the mantle and the dipole change for the period of 65 years. There is discrepancy of almost \(\pi\) between observation and theory. The other is about the magnitude of the angular velocity of the outer core. Only one-seventeenths of the observed value is explicable for the 400 year period phenomenon, provided that the detected change in the magnetic drift rate may be regarded as a straightforward representation of the movement of the outer core.

3.2 Effect of the toroidal field from within the core

The difficulties presented in the previous section might perhaps be removed if the toroidal field diffusing out from within the core is taken into account. In the above discussion (case B), the toroidal field associated with the dipole change is assumed to be generated exclusively at the core-mantle boundary. This is, however, an over-simplified concept. However restricted the change in the dipole field may be within a shallow part of the core, shear motions existing in the liquid core will interact with its time varying part to cause changes in the toroidal field. Since the electrical conductivity is more than \(10^2\) times higher in the core than in the mantle, the toroidal field thus generated will have such an intensity as may not entirely be ignored in the process of electromagnetic coupling between the mantle and the core.

An extreme case was studied in the previous paper (YUKUTAKE, 1972), where the core is divided into two parts with equal volumes, a sphere and a surrounding spherical shell. An intense toroidal field is generated in association...
with a large change in the dipole field at the interface between the sphere and the spherical shell (case A). The variation in the angular velocity of the mantle excited in this manner is extremely large and highly dependent on the period of the dipole change. Some of the results for $\sigma_m = 10^{-9}$ emu (model A) are reproduced in Table 4. For the 8000 year period, the magnitude of the angular velocity is about twice as large as the observation in Table 2, whereas it is about 290 times larger than the observation for the 400 year period. This period dependence has the same tendency as that in the model B2, increase in the angular velocity with decrease in the period. The increasing rate is, however, by far steeper than in the model B2. The phase difference also changes very rapidly from 0 to $2\pi$ with a slight decrease in period.

Regarding the motion of the outer spherical shell, the model A also gives large velocity as in Table 5. For 400 year period, variation in the angular velocity to be excited is $3.4 \times 10^{-7}$ rad/sec · gauss, which is about 30 times larger than the observation.

The case A is obviously incompatible with the observation. Any model based on this case gives too large amplitudes and too rapidly changeable phase of the angular velocities both for the mantle and for the outer layer of the core. However, if the interface is much shallower, the toroidal field would not be so intense as in the case A, because the magnitude of the dipole variation by which the toroidal field is generated is greatly reduced at a shallow depth by the shielding effect of the conducting core. Then the period dependence of the angular velocities would become much more moderate than in the case A. The problems presented in the previous section, the phase discrepancy of the angular velocity of the mantle for 65 year period and too large drift velocity of the geomagnetic field observed in association with the 400 year period phenomenon, might possibly be solved by reducing the thickness of the outer layer of the core.

Since the shear motion is supposedly continuous in the core, reduction of the thickness of the outer core in the above is not only restricting the volume of the core fluid involved in the angular momentum transfer, but also extinguishing essentially the variation in the dipole field within the deep interior of the core, only leaving it in a thin surface layer of the outer core.

4. Concluding Remarks

Comparison of data on three different time scales has confirmed that fluctuations in the earth’s rate of rotation are accompanied by changes in the dipole field. The drift velocity of the geomagnetic field is also likely to have changed during the time when a large fluctuation in the rotation took place in the 18th century. Observed data suggest that large magnitude of fluctuation in the rotation is excited by a slight change in the dipole moment as the period becomes shorter. This period dependent feature of excitation of the rotation change is
mostly explained by taking the conductivity of the lower mantle to be somewhat higher than $10^{-5}$ emu and by assuming that the dipole change is limited near the surface within the core. Only a slight contribution of the toroidal field from within the core would be needed to account for the observational data presented in this paper.

REFERENCES


