A Sheath Resonance Observed by a High Frequency Impedance Probe

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A complex impedance of a negative cylindrical probe in a magneto-plasma is calculated using a gradient sheath model, and quantitative values of the sheath resonance frequency for a floating probe are determined for various values of plasma parameters. Calculated results are in good agreement with the experimental data of a rocket borne impedance probe.

A radio frequency impedance probe is usually employed for an in-situ measurement of electron density both in a laboratory and space plasma. It measures a reactive impedance of a metallic probe in a plasma dielectric medium in a high frequency range, detecting the upper hybrid resonance maximum and a sheath resonance minimum which is a series resonance of a capacitive ion sheath-electron depletion layer-formed adjacent to the probe, and an inductive plasma layer. In contrast to the upper hybrid resonance, the sheath resonance is amenable to a sheath structure or a dc potential of a probe along with usual plasma parameters, and theoretical analyses have been worked out by many authors (for example, OYA, 1965; DOTE and ICHIMIYA, 1965; KOSTELNICEK, 1965; BUCKLEY, 1966). Also, we performed the impedance probe experiment on board a rocket vehicle, and obtained some quantitative data of the sheath resonance frequency. In this note, we calculate the impedance of a cylindrical probe at a floating potential in a magneto-plasma using a more realistic sheath model, and determined values of the sheath resonance frequency as a function of electron density, temperature and a static magnetic field with its orientation relative to the probe are compared with these measurement data.

Formulation is performed for a gradient sheath model shown in Fig. 1, after the work of BALMAIN and OKSIUTIK (1969) for a spherical probe. For a vacuum sheath region, the impedance $Z_A$ is expressed as

$$Z_A = \frac{1}{j\omega \varepsilon_0 \varepsilon_s l} \ln \frac{r + s}{r}$$  \hspace{1cm} (1)

where $l$, $r$ and $s$ are the length and radius of the probe and sheath thickness, respectively. An impedance $Z_o$ of an uniform plasma region is given by the
expression derived by Balmain (1964).

\[ Z_c = \frac{1}{j\omega 2\pi \varepsilon_0 l} \frac{a}{S\sqrt{F}} \left( \ln \frac{l}{r+s+t} - 1 - \ln \frac{a + \sqrt{F}}{2F} \right) \]  

(2)

with,

\[ F = \sin^2 \theta + a^2 \cos^2 \theta \]

\[ a^2 = S/P \]

where \( \theta \) is an angle of the probe axis relative to the static magnetic field, and \( S \) and \( P \) are the components of the dielectric tensor (STIX, 1962),

\[ S = 1 - \frac{UX}{U^2 - Y^2}, \quad P = 1 - X/U, \quad U = 1 - jZ, \quad X = \left( \frac{f_p}{f} \right)^2, \]

\[ Y = \frac{f_H}{f}, \quad Z = \frac{\nu}{2\pi f}. \]

For a transition layer of thickness \( t \), an impedance is the sum of the impedance of a sequence of thin annular sheets of linearly increasing density with a cold plasma permittivity given by (OYA, 1966)

\[ K(\rho) = S_\rho(\rho) \cos^2 \theta + \frac{3}{2} \left[ P_\rho(\rho) + S_\rho(\rho) \right] \sin^2 \theta \]

where

\[ S_\rho(\rho) = 1 - \frac{U}{U^2 - Y^2} X_\rho(\rho) \]

\[ P_\rho(\rho) = 1 - X_\rho(\rho)/U \]

\[ X_\rho(\rho) = \frac{\rho - (r+s)}{t} X \quad (r+s < \rho < t). \]

The impedance \( Z_B \) is, after some mathematical manipulations,

\[ Z_B = \frac{1}{j\omega 2\pi \varepsilon_0 l} \frac{t}{t + (1 - Q)(r+s)} \ln \frac{t + r + s}{Q(r+s)} \]

(3)

with

\[ Q = S \cos^2 \theta + \frac{3}{2}(S+P) \sin^2 \theta. \]
Then, the total impedance in the form of a normalized effective capacitance $C_p$ of the cylindrical probe is expressed as

$$C_p = Z_0/(Z_A + Z_B + Z_C),$$

where

$$Z_0 = \frac{1}{j\omega 2\pi \varepsilon_0 l} \left( \ln \frac{l}{r} - 1 \right),$$

and the sheath resonance is to be identified as $\arg (C_p) = -90^\circ$ which corresponds to $|C_p| = \text{max}$ for $Z << 1$. At a floating potential, the sheath thickness is of the order of a few Debye length, and we choose $s$ and $t$ as 3 and 2 times the Debye length.

Fig. 2. A modulus and a phase of an effective capacitance of a probe (denoted by “T”), with those of each region (denoted by “A,” “B,” and “C”) for $f_p = 4.0 \text{ MHz}$, $f_H = 1.15 \text{ MHz}$, $T_e = 1500^\circ \text{K}$ and $\theta = 0^\circ$. 
shielding length, respectively. Also, in the following calculations, we set \( v = 1.0 \times 10^4 \text{ sec}^{-1} \), \( l = 1.6 \text{ m} \) and \( r = 0.01 \text{ m} \) throughout, so as to compare with the rocket measurement data.

Figure 2 shows an example of the modulus and the phase of the effective capacitance \( C_p \) along with an effective capacitance of each region for \( f_p = 4.0 \text{ MHz} \), \( f_H = 1.15 \text{ MHz} \), \( T_e = 1500 \text{ K} \) and \( \theta = 0^\circ \). It is seen that, in contrast to the upper hybrid resonance (UHR) at 4.16 MHz, the gyro-resonance (GR) is modified due to the ion sheath region, and the sheath resonance (SHR) is revealed at 2.3 MHz through the contributions of the vacuum and the gradient sheath regions. In Fig. 3 is illustrated the angular dependence of \( C_p \) for \( \theta = 0^\circ \), 30°, 60° and 90° with the same parameters as in Fig. 2. Below the sheath resonance

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**Fig. 3.** Angular dependence of the effective capacitance of a probe for \( f_p = 4.0 \text{ MHz} \), \( f_H = 1.15 \text{ MHz} \) and \( T_e = 1500 \text{ K} \). Parameter is the angle \( \theta \).
frequency are observed sharp peaks and dips except for the case of $\theta = 0^\circ$, which is associated with the parallel resonance of the gradient transition region, and with the resonance cone under the hyperbolic conditions of the ambient plasma (Oya, 1966). It is noted, in the figure, that the sheath resonance frequency $f_{SHR}$ varies little, and in most of cases, the angular dependence of $f_{SHR}$ can safely be ignored within an error of less than 2%.

Thus, quantitative comparison of the sheath resonance frequency between calculated values and measurement data by an impedance probe on board L-3H-2 rocket is illustrated in Fig. 4 for the case of $f_\nu = 0.8-1.1\,\text{MHz}$ and $T_e = 2000\,\text{K}$. It is understood that fairly good agreement is achieved between theory and experiment: slight discrepancies being due to the fact that accurate values of probe potential and electron temperature are not known, and effects of rocket motion are not taken into account. Thus, the method of calculation is considered to be effective for the analysis of sheath resonance phenomena encountered in the probe impedance experiment for a space and a low density laboratory plasma.

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REFERENCES


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