The Reliability of Conductivity Derived from Diurnal Variations

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Global conductivity models can be based on the complex ratio of external to internal diurnal variation fields. If this ratio is derived from the five quiet days of each month, a great variation from month to month is found. This indicates that the results are very sensitive to the group of observatories from which hourly values are derived.

A conductivity anomaly must be defined as a departure from some normal conductivity distribution. This is usually taken as the overall world average of conductivity as a function of depth. The most accurate determination of this, and the oldest, comes from a separation of the diurnal variation field into parts of external and internal origin. The potential of the field can be written as

$$V = \sum_{n,m} a(R^n E_m^n + R^{-n-1} I_m^n) P_n^m(\theta) e^{-im\phi}$$

where a is the radius of the earth

$$R = r/a$$

θ is colatitude

ϕ is longitude relative to solar midnight

and E and I are complex constants expressing the magnitude and phase of the external and internal fields. Most investigators have used either the north and vertical or east and vertical components of the diurnal variation for the separation of the field.

The object of this work is to investigate the reliability of this approach. If a separation of the field is made for each month using only the five quiet days, then the coefficients E and I vary from month to month, but the complex ratio $E/I = Ce^{i\phi}$ should remain constant, because it depends only on the conductivity distribution in the interior of the earth. Such an analysis has been made for the 18 months of the I.G.Y. (PARKINSON, 1971) using all observatories available. Figure 14 of that reference shows a plot of C (ratio of magnitude of E and I) and ϕ (phase difference between E and I) for the principal harmonics of the 24- and 12-hour waves.

The simplest realistic model of conductivity distribution is that of a non-conducting shell overlying a conductosphere of uniform finite conductivity.

Two parameters are required to specify this model, the depth to the top of the conductosphere and its conductivity. In the above reference, lines are drawn forming a grid of various models. The significant fact shown by these graphs is that the points have a wide scatter and those for the 24-hr wave (harmonic P_2^1) cluster around a much lower phase difference than those for the 12-hr wave (harmonic P_3^2).

The most obvious defect from which an analysis like this suffers is that some of the observatories lie on or near conductivity anomalies. The assumption inherent in the analysis is that the conductivity is a function of depth only. More reliable results would be obtained if anomalous observatories were omitted from the analysis. The question then is; what criterion should be used to define an anomalous observatory? The best criterion would be based on a network of temporary observatories operated around each permanent observatory for a period of some months. This has been done around Toolangi (BENNETT and LILLEY, 1973) but in general such data are not available. The alternative method, used here, is to make a preliminary analysis of each element as a function of latitude and local time, and to calculate residuals at each observatory. The residuals of Z in particular are likely to be large at anomalous observatories.

Two different criteria were used to define anomalous observatories. One is that the sum of the squares of the Z -residuals (for 24-, 12- and 8-hour waves) should be greater than 100 gamma². The other is that the total squared residual for all components should be greater than 500 gamma². It is found that the difference between the results using these two criteria is small and of doubtful significance.

Fig. 1. Amplitude and phase of complex ratio of external to internal potential for P_2^1 (24-hour) harmonic with corresponding models.

Fig. 2. Amplitude and phase of complex ratio of external to internal potential for P_3^2 (12-hour) harmonic with corresponding models.

Figures 1 and 2 show the results of this analysis using the first of these two criteria for selecting anomalous observatories. Each model specified by a depth and conductivity of the conductosphere corresponds to a point on these graphs. This is indicated by the grids. Again there is considerable scatter in the points representing E/I . But in both cases the centroid of points is shifted towards greater phase differences, relative to the previous analysis, i.e., to lower conductivity values.

The scatter of points suggests that a statistical approach should be made. Because of the small size of the samples, statistical parameters of the population are estimated from the median and quartiles of the samples rather than from the mean and standard deviation. The 90% confidence region for population mean is taken as:

$$(\text{median}) \pm (\text{difference of quartiles})/5.$$

Results of this and the previous analysis are summarised in the following table:

Period	Criterion	C (mean)	C (range)	ϕ (mean)	ϕ (range)
24 h	all elements	2.40	2.36–2.44	7.0°	6.0–8.0°
24	Z only	2.40	2.35–2.45	7.2	5.8–8.6
12	all elements	2.28	2.26–2.30	17.5	16.3–18.7
12	Z only	2.20	2.13–2.27	16.0	15.0–17.0
8	all elements	2.25	2.19–2.31	20.0	18.0–22.0
8	Z only	2.10	1.99–2.21	17.5	16.1–18.9
24	all observatories	2.10		0.0	
12	all observatories	2.10		12.0	
8	all observatories	2.20		38.0

Fig. 3. Rectangles of 90% confidence in conductivity and depth to conductosphere for 3 harmonics. : anomaly criterion based on Z residuals. : anomaly criterion based on residuals of all elements.

These ranges can be translated into ranges in conductivity and depth in terms of the above model. If this is done for the selective analysis then areas of confidence can be defined in terms of conductivity and depth. For simplicity these can be taken as rectangles, although this slightly exaggerates the size of the areas. The results for the selective analysis are shown in Fig. 3.

In summary, three points emerge from this analysis. First there is a very significant difference in the models that fit the 12- and 24-hour waves. It follows that this type of model is not strictly applicable, for if it were the models should coincide. But a model with a sharp increase in conductivity near 160 km and another near 400 km would fit observations.

Secondly, using 17 months of values, and both X and Y , only rather wide limit can be set on the ratio of internal to external fields. In terms of conductivity models, this indicates a considerable uncertainty in conductivity of the conductosphere.

Thirdly, very different results are obtained if anomalous observatories are included in the analysis, in particular the the phase difference of the 24-hour wave, and therefore the value of conductivity at depth, is very sensitive to the choice of observatories.

REFERENCES

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