The Effect of a Field of External Origin on Spherical Harmonic Analysis Using Only Internal Coefficients

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(Received September 17, 1976)

ALLDREDGE and STEARNS (1974) discussed the effect on spherical harmonic analysis of the geomagnetic field of omitting the $n=1$ terms of external origin (i.e. those associated with the ring current). In a numerical test they added a 100 nT uniform field to a field synthesised from a set of (internal) IGRF coefficients, and then reanalysed by least squares fitting to (X, Y, Z) component data at points on a $10^\circ \times 10^\circ$ grid. Using only internal coefficients they were surprised to find that using $\sin \theta$ weighting they reproduced the IGRF coefficients almost exactly, but that without this weighting there were significant changes in the zonal ($m=0$) coefficients of odd degree.

Their result is in fact to be expected. With the $\sin \theta$ weighting, the least squares' process is to a very good approximation equivalent to determining the coefficients by integrating over the sphere. Now, over the sphere, *any* harmonic field of external origin is orthogonal to *any* harmonic field of internal origin (see e.g. JAMES, 1973); similarly the field of any one harmonic is orthogonal to the field of any other harmonic, and each harmonic contributes independently to the mean square vector field (LOWES, 1966). So the best the least squares can do is to reproduce the internal coefficients and leave the 100 nT external field unfitted.

However without the $\sin \theta$ weighting the polar regions are overweighted and there is no longer orthogonality. The least squares will do its best to reduce the (polar weighted) residual field by changing the internal coefficients, and it does in fact manage to reduce the total (polar weighted) residual field to 96 nT (vector) r.m.s. by introducing a spurious internal field of 24 nT r.m.s. (Properly averaged over the surface the r.m.s. residual field will in fact be $(100^2 + 24^2)^{1/2} = 103$ nT.) That the errors are in the zonal harmonics of odd degree follows from the geometry of the uniform field.

It is interesting that when they analysed the total intensity F they obtained the much larger spurious internal fields of 92 and 94 nT respectively with and

without the $\sin \theta$ weighting (i.e. total residual fields of 136 and 137 nT), another example of the inefficiency of an F analysis. Here, of course, there is no question of orthogonality, individual harmonics do *not* contribute independently to the m.s. value of F , and the residual F values can be very much smaller than the lengths of the residual vectors (LOWES, 1975).

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