Planetary Magnetism

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The discovery by space probes that other planets besides the Earth possess magnetic fields (albeit weak) has heightened interest in planetary magnetism and in the means by which such fields are maintained. There have been suggestions that one dynamo model might explain all planetary fields, if only the right scaling law relating fields, rotation, physical properties etc. could be found. DOLGINOV (1975 a, b, 1976 a, b) has attempted to account for the observed intensity of the magnetic field of a number of the planets (including the Moon), based on the assumption that the driving force for the motion is precession. More precisely he assumes that “the inductive interaction between flow and field is proportional to the Poincaré force” i.e. that

$$\nu_m \nabla^2 B \propto (\omega \times \Omega) \times r, \quad (1)$$

where B is the magnetic field, ω the angular rotation, Ω the angular precession velocity, ν_m the magnetic diffusivity, and r the position vector measured from the centre of the planet. There are a number of difficulties with Dolginov’s proposal which has been critically discussed by RUSSELL and GOLDSTEIN (1976). One of the unsatisfactory features of Dolginov’s approach is that the two terms in (1) have different dimensions. The left hand side is an essentially electrical quantity having units e.m.f./length² whilst the right hand side is a purely mechanical quantity having units force/mass. The purpose of this note is to point out that this shortcoming may be rectified and Dolginov’s argument made a little more plausible. In what follows we will work in c.g.s.e.m.u. so that the permeability μ is dimensionless. We will also assume that $\mu=1$, so that $B \equiv H$.

Since the Poincaré force $(\omega \times \Omega) \times r$ is proportional to $\sin \alpha$ where α is the angle between the vectors ω and Ω , suppose that

$$B = f(\rho, R_c, \omega, \Omega, \nu_m) \sin \alpha, \quad (2)$$

where ρ is the density and R_c the radius of the planet’s core. If, in addition, B depends on ω and Ω in the same way, we can write

$$B \propto \rho^a R_c^b \omega^c \Omega^d \nu_m^e \sin \alpha, \quad (3)$$

where the exponents a, b, c, d are to be determined. Considering the dimensions of both sides of (3), it follows that $a=1/2$, although it is not possible to determine b, c , and d uniquely. Following Dolginov if we further assume that the dependence of B on both ω and Ω is linear so that $c=1$, then $d=-1$ and $b=3$.

Hence

$$B \propto \rho^{1/2} R_c^3 \omega \Omega \nu_m^{-1} \sin \alpha. \quad (4)$$

This is the same as Dolginov's formulation except that in his expression $\rho^{1/2}$ is replaced by ρ and, in consequence, is dimensionally incorrect. If SI units are used (in which the permeability μ does have dimensions), then the ratio of the L.H.S. to the R.H.S. of (1) is $(\rho\mu)^{1/2}$ in terms of B , or, $(\rho/\mu)^{1/2}$ in terms of H .

To compare 'measured' values of the ratio $B_{\text{planet}}/B_{\text{Earth}}$ at the core surface with calculated values, Dolginov further assumes, in the absence of any reliable data that ν_m (and hence the electrical conductivity) is the same for all bodies. His calculated values of this ratio for Jupiter, Mars, Venus, Mercury and the Moon are within a factor of two of the measured values. Taking the same values for the basic planetary data, the use of the dimensionally correct formulation (4) instead of Dolginov's expression slightly improves these results.

If we solve the indetermined set of equations for the exponents b, c, d with the further condition that $(R_c \omega \Omega)$ occurs as one quantity, then it follows that the ratio of the L.H.S. to the R.H.S. of (1) is

$$\rho^{1/2} F \left[\frac{R_c^2}{\nu_m} (\omega \Omega)^{1/2} \right]. \quad (5)$$

The argument of the unknown function F is a quite reasonable magnetic Reynold's number and its inclusion could possibly reconcile the difference between the measured and calculated values of $B_{\text{planet}}/B_{\text{Earth}}$. In conclusion it must not be forgotten that it has not been established that precession can power the dynamo, even in the case of the Earth (ROCHESTER, *et al.*, 1975; LOPER, 1975), and that for many of the planets there is considerable uncertainty in some of the basic data. Part of the comparatively good agreement of Dolginov's results could be due to his choice of such data. Moreover it has not been definitely established that all the observed planetary magnetic fields are intrinsic nor that all planets possess a fluid, electrically conducting core.

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