Absolute Sensitivity of a High- μ Metal Core Solenoid as a Magnetic Sensor

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An experiment was carried out for a precise calibration of the absolute sensitivity of a high- μ metal core solenoid used as a sensor of an induction magnetometer. The high- μ metal core induction magnetometer was operated simultaneously with an air-core loop antenna induction magnetometer system which has a known absolute sensitivity. The two systems were run at Churchill, Manitoba in February, 1977. A moderately intense quasi-sinusoidal Pc 1 event recorded by both systems on the vertical component was made use of as a calibration signal. Comparison of the outputs from the two induction magnetometers made it possible to determine the sensitivity of the high- μ metal core solenoid. A theory was also presented to calculate the absolute sensitivity of a high- μ metal core solenoid. Agreement between the theory and the experiment is good. The theory shows us how to design a reliable and portable high-sensitivity solenoid.

1. Introduction

High- μ metal core solenoids are used most commonly as a head sensor of an induction magnetometer. However, absolute calibration of this type of sensor is hard to perform in a laboratory because of the reason mentioned later. To avoid this difficulty, the authors have carried out an experiment for calibration in a field. A detailed description of this experiment is presented in this paper. Also presented is a theory which enables us to estimate approximately the absolute sensitivity of a high- μ metal core coil sensor. The agreement between the theory and the experiment is good. The theory, endorsed with an experiment in this way, gives us a guide as to the design of reliable and portable high-sensitivity sensors. Reliability of high- μ metal core coil sensors is often questioned, because of some undesirable properties of core materials such as non-linearity and hysteresis in the B - H relation and instability in permeability

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due to thermal and/or mechanical stress. Many high- μ metal core coils are also heavy, inconvenient for field work.

Air core loop antennae are another type of sensors often used as a head sensor. The absolute sensitivity of an air core loop antenna can be determined easily, knowing the geometry of the loop and the coil's total turn number. The absolute sensitivity is proportional to the total coil area. However, it is not so easy to determine the absolute sensitivity of a high- μ metal core solenoid. It can be determined theoretically as will be described later. However, the theory is not a rigorous one at this moment. It is approximate, though the approximation is good. It is also difficult to determine the sensitivity through a laboratory experiment. A high- μ metal core solenoid, placed within an externally applied uniform magnetic field, disturbs the field over a considerable extent in space because of a large effective permeability of the core which is several hundreds to several thousands and of its fair length which is one to several meters usually. The disturbance field at points far away from the core is of a dipolar type. The calibration field has to be uniform in a volume of space comparable to the extension of the disturbance field. It is hard to create such an extensive uniform field in a laboratory. To circumvent the difficulty of this nature, the authors carried out an experiment in a field, using natural magnetic pulsations as calibration signals. Two induction magnetometers were operated in the field, one having an air core loop antenna with a known absolute sensitivity and the other a high- μ metal core solenoid whose sensitivity is unknown. The experiment was carried out at Twin Lake of the Research Range of National Research Council of Canada, Churchill, Manitoba in February 1976. The two sensors were separated at a distance of 100 m approximately. The air core loop antenna was placed horizontally and the other vertically so that both sensors detected field changes on the vertical component. Comparison of the outputs from the two systems made it possible to determine the absolute sensitivity of the high- μ metal core coil sensor.

2. Experimental Results

The air core loop antenna used for the experiment is circular in geometry; the inner is 149.0 ± 0.2 cm and the outer diameter 151.4 ± 0.2 cm. The total turn number is $5,000 \pm 1$. Then, the total coil area amounts to

$$A = 8.86 \times 10^3 \text{ m}^2 \pm 0.3\% . \quad (1)$$

* A sensor of 1 Caner in sensitivity, placed within a uniform external sinusoidal magnetic field with a frequency of 1 Hz and with an amplitude of $1 \text{ m}\gamma$ (10^{-12} Tesla), gives rise to an electromotive force of an amplitude of 1 microvolt.

Therefore, the absolute sensitivity in units of Caner* is given as follows (UEDA and WATANABE, 1975),

$$S_a = 2\pi A \times 10^{-6} = 0.0557 \text{ Caner} \pm 0.3\% . \quad (2)$$

If a uniform sinusoidal magnetic field variation with a frequency f (in Hz) and with an amplitude B (in mT) took place in nature, the voltage output, more specifically, the amplitude of the output from an induction magnetometer, expressed in volts, would be

$$V_o = S_a B f \frac{R_i}{R_i + R} G F(f) \times 10^{-6} \quad (3)$$

where R is the d.c. resistance of the sensor coil, R_i is the input impedance of the amplifier, G is the d.c. gain of the amplifier and $F(f)$ is the amplitude response function of the whole detection system (sensor and amplifier) normalized to the d.c. value. The ratio of the outputs from the two systems would then be

$$\frac{V_o^{(2)}}{V_o^{(1)}} = \frac{S_a^{(2)}}{S_a^{(1)}} \cdot \frac{G^{(2)}}{G^{(1)}} \cdot \frac{R_i^{(2)}}{R_i^{(2)} + R^{(2)}} \cdot \frac{R_i^{(1)} + R^{(1)}}{R_i^{(1)}} \cdot \frac{F^{(2)}(f)}{F^{(1)}(f)} . \quad (4)$$

For convenience, let us put subscripts (1) and (2) to the air core loop antenna system and to the high- μ metal core coil system, respectively. The coil resist-

Fig. 1. Amplitude vs. frequency response function $F(f)$ for the amplifier of the air-core loop system (a) and for that of the high- μ metal core system (b).

ance, the input impedance and the gain of each system were measured in a laboratory;

$R^{(1)} = 5.130 \text{ k}\Omega \pm 0.2\%$

$R_1^{(1)} = 4.000 \text{ k}\Omega \pm 0.2\%$

and

$G^{(1)} = 5.21 \times 10^4 \pm 1\%$

$R^{(2)} = 1.831 \text{ k}\Omega \pm 0.2\%$

$R_1^{(2)} = 7.21 \text{ k}\Omega \pm 4\%$

$G^{(2)} = 4.01 \times 10^4 \pm 1\%$

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(5)

The frequency response $F(f)$ of each system was also calibrated in a laboratory (Fig. 1).

On February 17th, both induction magnetometers detected a moderately intense quasi-sinusoidal *Pc 1* event (Fig. 2). Period and amplitude measurements were made from a microfilm record using a travelling microscope. The *Pc 1* signal was found to have a very narrow band in frequency. It was found to be $0.245 \text{ Hz} \pm 1.6\%$. Around this frequency, the amplitude response function of each system is flat, as seen from Fig. 1. The phase response function of each is proportional to frequency and no wave form distortion occurs. Peak-to-peak amplitude measurements yielded

Fig. 2. *Pc 1* event recorded at Twin Lake, Research Range of National Research Council of Canada, Manitoba on February 17, 1977.

$$V_o^{(2)}/V_o^{(1)} = 1.41 \pm 0.8\% . \quad (6)$$

Combining Eqs. (4) and (6), we obtain the absolute sensitivity of the high- μ metal core coil sensor as follows

$$S_a^{(2)} = 0.0560 \text{ Caner} \pm 7\% . \quad (7)$$

3. Theoretical Consideration

The absolute sensitivity of a high- μ metal core solenoid can be estimated theoretically, if the core, usually a rod circular or rectangular in cross section, is approximated with a prolate spheroid. A ferromagnetic spheroid is magnetized uniformly in a homogeneous external magnetic field H which is parallel to the major axis of the spheroid. The flux density B_i in the spheroidal core is related to the external field as follows

$$B_i = \mu_0 \mu_{\text{eff}} H \quad (8)$$

where μ_0 is the permeability of free space,

$$\mu_0 = 4\pi \times 10^{-7} \text{ henry/meter} \quad (9)$$

and μ_{eff} , the effective relative (dimensionless) permeability, is given by

$$\mu_{\text{eff}} = \frac{\mu \mu_G}{\mu + \mu_G - 1} . \quad (10)$$

In this expression, μ is the intrinsic permeability of the core material and μ_G is the inverse of the demagnetization factor. The demagnetization factor for a prolate spheroid has been calculated theoretically (STRATTON, 1941; OSBORN, 1945; BOZORTH, 1951; see Appendix also). It is expressed as follows (STRATTON, 1941, p. 214)

$$\frac{1}{\mu_G} = \frac{1}{2} \left(\frac{b}{a} \right)^2 \frac{1}{e^3} \left(-2e + \ln \frac{1+e}{1-e} \right) \quad (11)$$

where a and b represent the semi-major and semi-minor axes of the spheroid and e is the eccentricity,

$$e = \sqrt{1 - \frac{b^2}{a^2}} . \quad (12)$$

It can be also expressed as (OSBORN, 1945)

$$\frac{1}{\mu_G} = \frac{m}{m^2 - 1} \left\{ \frac{m}{\sqrt{m^2 - 1}} \ln (m + \sqrt{m^2 - 1}) - 1 \right\} \quad (13)$$

where

$$m = a/b . \quad (14)$$

If m is much larger than 1, both Eqs. (11) and (13) reduce to (OSBORN, 1945)

$$\frac{1}{\mu_G} = \frac{1}{m^2} \{ \ln(2m) - 1 \} = \frac{b^2}{a^2} \left\{ \ln \left(\frac{2a}{b} \right) - 1 \right\}. \quad (15)$$

In Fig. 3, μ_G according to this formula is plotted as a function of m .

With regards to most high- μ metal solenoids used as a sensor of an induction magnetometer, μ_G is of the order of several thousands or less, whereas μ is of the order of several tens of thousands or more. In many cases, $\mu_G \ll \mu$ and then, the effective permeability is equal to μ_G with good approximation;

$$\mu_{\text{eff}} = \mu_G. \quad (16)$$

Now that the effective permeability of the core has been obtained, the total effective area of the solenoid which will be denoted with A , can be calculated as follows: (effective permeability) $\times$ (area of core's cross section) $\times$ (total turn number),

$$A = \pi \mu_{\text{eff}} N b^2. \quad (17)$$

The absolute sensitivity is then given by

$$S_a = 2\pi A \times 10^{-6} \text{ Caner} \quad (18)$$

where A is measured in units of m^2 (UEDA and WATANABE, 1975).

Fig. 3. Geometrical permeability, or the inverse of the demagnetization factor, of a prolate ellipsoid given as a function of m , ratio of the major axis of the ellipsoid to the minor axis. The graph is based on the formula (15).

The solenoid used for this experiment has a rectangular mumetal* rod core. This particular solenoid is one of twelve similar coils made at the University of B.C. The total turn number of each solenoid is 50,000. Every core is made of 45 to 50 identical mumetal strips, insulated and laminated. One mumetal strip is 3 ft (91.44 cm) long and 14/1,000" (0.03556 cm) in thickness. The area of the normal cross section of the core falls in the range from 3.048 to 3.387 cm². Then, the radius b of an equivalent circular rod core whose normal cross section is equal in area to the rectangular core, amounts to 0.9851–1.038 cm. Therefore, m falls in the range from 44.03 to 46.41. The effective permeability according to Eq. (15) and the sensitivity according to Eq. (18) amount to 557–610 and 0.0584–0.0593 Caner, respectively. The sensitivity obtained theoretically in this way agrees well with the experimentally obtained value.

Combining Eqs. (15) and (17), the absolute sensitivity is expressed as follows (UEDA and WATANABE, 1974),

$$S_a = \frac{2\pi^2 N a^2 \times 10^{-6}}{\ln(2a/b) - 1} \text{ Caner} . \quad (19)$$

The numerator of the right-hand side is the absolute sensitivity of a circular air core loop antenna whose radius is a and turn number is N . The sensitivity of the high- μ metal core coil is obtained by multiplying a correction factor, $1/\{\ln(2a/b) - 1\}$, to the sensitivity of the air core coil. The correction factor is 0.2832–0.2875 in the case of the high- μ metal core solenoid used for this experiment. An interesting point is that the correction factor does not depend too much on the ratio of the length of the core to the diameter of the core. For instance, the correction factor would be about 0.238 if the core length is doubled ($2a=6'$). The change in the correction factor is less than 20%, whereas the numerator of Eq. (19) becomes 4 times greater. The sensitivity of a high- μ metal core solenoid largely depends on the length of the core.

The theoretical expression for the sensitivity (Eq. (19)) has been derived on two assumptions: (i) the coil wire is wound very thin on the surface of the core and (ii) the wire is wound at the centre of the core. The inner and outer diameters of the coil are 1 1/4" (3.175 cm) and 2 7/8" (7.3025 cm), respectively. The length of the coil is 18" (45.72 cm), half the length of the core. If a coil has a finite thickness, the total magnetic flux crossing the coil should include Φ , the contribution from the flux lines passing between the surface of the core and the outer boundary of the coil. This contribution has been estimated, providing that assumption (ii) holds true. The inner and outer radii of the coil are assumed to be 1 and 4 cm, respectively. The parameter m is taken equal to 45. The intrinsic permeability of the core μ is assumed to be very large. Then,

* Mumetal is the trade name of a high- μ metal product commercially available.

it is found that the total flux Φ is about 0.2% of Φ_c the flux through the core. Therefore, the finite thickness of the coil is negligible. For details of this calculation, reference is made to Appendix of this paper. Next, let us consider the opposite case: assumption (i) holds but (ii) does not. Since we have assumed that the core is a spheroid in shape, we may have to assume that the area of normal cross section of the core is not constant but is dependent on position on the core. If the ratio of the coil's length to the core's length is p and if the coil is wound symmetric with respect to the central position on the core's surface, it is found that the total coil area is given as follows (cf. ONISHI and KATO, 1976)

$$A = \pi \mu_{\text{eff}} N b^2 \left(1 - \frac{p^2}{3} \right). \quad (20)$$

Comparing this with Eq. (17), we see that the last factor on the right-hand side is the correction factor. It is 11/12 in the case of the present solenoid, since $p = 1/2$.

4. Concluding Remarks

The theory developed in the previous section indicates that one can use a thin high- μ metal rod as the core of a sensor. In this way, the weight of the sensor can be reduced without losing much sensitivity. One of the problems of high- μ metal core solenoids is their weight. Usually, they are fairly heavy so that they are rather inconvenient for field work. Of course, we cannot make the diameter of the core's cross section too small. If the diameter is made too small, μ_G becomes so large as seen from Eq. (15) that it may not be negligible compared to the intrinsic permeability μ any longer. Then, the effective permeability would depend on μ as seen from Eq. (10) and consequently become susceptible to thermal and/or mechanical stress as the intrinsic permeability is strongly affected by such stress. This implies an unstable sensitivity which should be avoided. The condition $\mu_G \ll \mu$ has to be satisfied, since under the conditions the effective permeability depends on μ_G , a quantity determined only by the geometry of the core.

The new, convenient systems thus developed have been operated at the station nets in Canada since 1976 for a joint Univ. B.C.-Univ. Tokyo research of ULF during the IMS, proving themselves to be very convenient and efficient for field use. Further improvements are now being undertaken.

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APPENDIX

An analysis is presented of the magnetic field due to a ferromagnetic prolate spheroid magnetized by an externally applied uniform magnetic field, H_0 . The same notations as in the main text will be used except for those which are introduced in this appendix. Every logarithmic symbol means a natural log. The magnetizing field and the major axis of the spheroid are assumed to be parallel. The major and minor axes of the spheroid are represented with $2a$ and $2b$. For the analysis, it is most convenient to use spheroidal coordinates. They are represented with ξ , η and ϕ , the same notations as STRATTON (1941, pp. 52–54 and pp. 56–57). Considering the Cartesian coordinate system in which the x -axis is along the major axis of the spheroid and the origin is at its centre, the coordinates, x , y and z are related to ξ , η and ϕ as follows,

$$x = -c\xi\eta, \quad y = c\sqrt{(\xi^2-1)(1-\eta^2)} \cos \phi, \quad z = c\sqrt{(\xi^2-1)(1-\eta^2)} \sin \phi \quad (\text{A.1})$$

where the spheroidal coordinates are defined in the following intervals,

$$1 \leq \xi < \infty, \quad -1 \leq \eta \leq 1, \quad 0 \leq \phi \leq 2\pi. \quad (\text{A.2})$$

In these equations, c is the distance of a focus from the centre,

$$c = \sqrt{a^2 - b^2}. \quad (\text{A.3})$$

Parametric surfaces given by $\xi = \text{const.}$ form a family of confocal spheroidal surfaces and those defined by $\eta = \text{const.}$ surfaces of confocal hyperboloids of two sheets.

Let us try a solution, assuming a potential function given as follows,

$$V^{(1)} = H_0 c \xi \eta + D \eta Q_1(\xi) \quad \text{for outside the spheroid,} \quad (\text{A.4})$$

and

$$V^{(2)} = H^{(2)} c \xi \eta \quad \text{for inside the spheroid.} \quad (\text{A.5})$$

In Eq. (A.4), the first term on the right-hand side represents the external uniform magnetic field. In the second term of the same equation, D is a constant to be determined by the boundary conditions and $Q_1(\xi)$ is the first order Legendre function of the second kind,

$$Q_1(\xi) = \frac{\xi}{2} \ln \frac{\xi+1}{\xi-1} - 1. \quad (\text{A.6})$$

The second term in Eq. (A.4) is shown to represent a dipole field at points far away from the spheroid (MORSE and FESHBACH, 1953, pp. 1284–1291). Equation (A.5) means that the field inside the spheroid is uniform, parallel to the magnetizing field H_0 , and has strength $H^{(2)}$.

The boundary conditions at the surface of the spheroid are the continuity of the tangential component of the magnetic field (H) and that of the normal component of the flux density (B). They give

$$H^{(2)}c\xi_1 = H_0c\xi_1 + DQ_1(\xi_1) \quad (\text{A.7})$$

and

$$\mu H^{(2)}c = H_0c + DQ_1'(\xi_1). \quad (\text{A.8})$$

In these equations, ξ_1 represents the inverse of the ellipticity e of the spheroid,

$$1/\xi_1 = e = \sqrt{1 - b^2/a^2} \quad (\text{A.9})$$

and $\xi = \xi_1$ specifies the surface of the spheroid. In Eq. (A.8), μ represents the relative permeability (dimensionless) of the ferromagnetic material of the spheroid. Solving Eqs. (A.7) and (A.8) with respect to $H^{(2)}$ and D , we have

$$H^{(2)} = \frac{1}{(\mu - 1)(\xi_1^2 - 1)Q_1(\xi_1) + 1} H_0. \quad (\text{A.10})$$

and

$$-D = \frac{c\xi_1(\xi_1^2 - 1)(\mu - 1)}{(\mu - 1)(\xi_1^2 - 1)Q_1(\xi_1) + 1} H_0. \quad (\text{A.11})$$

The magnetic flux density inside the spheroid is given as follows, using Eq. (A.10),

$$B^{(2)} = \mu_0 \mu H^{(2)} = \mu_0 \frac{\mu}{(\mu - 1)(\xi_1^2 - 1)Q_1(\xi_1) + 1} H_0. \quad (\text{A.12})$$

We see from this that the effective permeability (dimensionless) is given as follows,

$$\mu_{\text{eff}} = \frac{\mu \mu_G}{\mu + \mu_G - 1} \quad (\text{A.13})$$

where

$$1/\mu_G = (\xi_1^2 - 1)Q_1(\xi_1). \quad (\text{A.14})$$

Using Eqs. (A.6) and (A.9), we can find that Eq. (A.14) is identical to Eq. (11) in the main text.

Concerning the field outside the spheroid, let us consider its strength on the y - z plane of the Cartesian coordinate system defined in the beginning of this appendix. On this plane, $\eta = 0$ and we find that the magnetic field strength is given as follows assuming that the permeability μ is very large;

$$H_{\eta=0}^{(1)} = H_0 \left(1 - \frac{\xi_1}{\xi} \cdot \frac{Q_1(\xi)}{Q_1(\xi_1)} \right). \quad (\text{A.15})$$

On the y - z plane, r , the distance from the major axis of the spheroid, is connected to ξ as follows,

$$\frac{r}{b} = \sqrt{\left(\frac{\xi^2-1}{\xi_1^2-1}\right)}. \quad (\text{A.16})$$

Then, we can find $H_{\eta}^{(1)}=0$ as a function of distance r . A plot of this function is given in Fig. 4, referring to a specific case that $m=45$. This choice is relevant to the experiment reported in the main text. The figure shows that the magnetic field outside the spheroid is disturbed over a distance of $100b$ approximately. It is about 1 m if a value appropriate to our experiment is assigned to b , for instance, $b=1$ cm.

Figure 4 shows that the magnetic field is zero just outside the spheroid and that it increases almost linearly with the logarithm of distance over a considerable length. Within this range of linearity, the field is found to be given by

$$H_{\eta=0}^{(1)} = \frac{\log\left(\frac{r}{b}\right)}{\frac{1}{2} \log\left(\frac{2}{\xi_1^2-1}\right) - 1} H_0. \quad (\text{A.17})$$

Using this equation, we can calculate the magnetic flux φ through an area bounded by two circles with radii, b and r . We find that the flux is given as follows

Fig. 4. Distribution of magnetic flux density on a plane which is perpendicular to the major axis of a prolate ellipsoid and passes through its centre. The flux density is given as a function of distance from the major axis of the ellipsoid. The distances are given in units of the semi-minor axis of the ellipsoid. The ratio of the major axis to the minor axis, m , is assumed to be 45. The intrinsic permeability of the ellipsoid is assumed to be infinitely large.

$$\varphi = \mu_0 \int_b^r 2\pi r H_{\gamma=0}^{(1)} dr = \frac{\mu_0 \pi b^2 H_0}{K} \left[v^3 \log v - \frac{1}{2}(v^2 - 1) \right] \quad (\text{A.18})$$

where

$$v = r/b \quad (\text{A.19})$$

and

$$K = \frac{1}{2} \log \left(\frac{2}{\xi_1 - 1} \right) - 1. \quad (\text{A.20})$$

Now, let us consider a solenoidal coil wound on the spheroid and calculate the total magnetic flux crossing the coil windings Φ . It is obtained as follows,

$$\Phi = \frac{\mu_0 N}{r_1 - b} \int_b^{r_1} \varphi dr = \frac{\mu_0 \pi b^2 N H_0}{K(v_1 - 1)} \left[\frac{1}{3} v_1^3 \log v_1 - \frac{5}{18} v_1^3 + \frac{1}{2} v_1 - \frac{2}{9} \right] \quad (\text{A.21})$$

where r_1 is the outer radius of the solenoid and

$$v_1 = r_1/b. \quad (\text{A.22})$$

In Eq. (A.21), it is assumed that the wire is wound on the y - z plane with a constant linear density in a range of distance from b and r_1 . Now, the total magnetic flux within the spheroid Φ_e equals $\mu_0 \mu_G \pi b^2 N H_0$ so that Eq. (A.21) can be rewritten as follows,

$$\frac{\Phi}{\Phi_e} = \frac{1}{\mu_G} \cdot \frac{1}{K(v_1 - 1)} \cdot \left[\frac{1}{3} v_1^3 \log v_1 - \frac{5}{18} v_1^3 + \frac{1}{2} v_1 - \frac{2}{9} \right]. \quad (\text{A.23})$$

From $m=45$, we have $\mu_G=578$ and if we take $v_1=4$, we obtain

$$\Phi/\Phi_e = 0.22\%. \quad (\text{A.24})$$

This shows that the magnetic flux outside the core can be ignored in many cases when considering the absolute sensitivity of a high- μ metal core solenoid.

REFERENCES

- BOZORTH, R.M., *Ferromagnetism*, Van Nostrand Co., New York, 1951.
 MORSE, P.M. and H. FESHBACH, *Methods of Theoretical Physics*, McGraw-Hill Book Co., New York and London, 1953.
 ONISHI, H. and Y. KATO, Characteristics of an induction magnetometer using a high permeability cylinder core, *Proc. Fac. of Sci., Tokai Univ.*, No. 1, 1976.
 OSBORN, J.A., Demagnetizing factors of the general ellipsoid, *Phys. Rev.*, **67**, 351-357, 1945.
 STRATTON, J.A., *Electromagnetic Theory*, McGraw-Hill Book Co., New York and London, 1941.
 UEDA, H. and T. WATANABE, Sensitivity and frequency response of an induction magnetometer, Fall General Meeting of Society of Terrestrial Magnetism and Electricity of Japan, Gifu, November 1974.
 UEDA, H. and T. WATANABE, Several problems about sensitivity and frequency response of an induction magnetometer, *Sci. Rep. Tohoku Univ., Ser. 5, Geophysics*, **22**, 107-127, 1975.