O_1 Component of Geomagnetic Lunar Daily Variation at Alibag and Its Contamination by M_2 Component*

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An analysis of absolute mean hourly values of the three geomagnetic elements H , D and Z at Alibag for the period 1932–1972 has revealed clear and significant $L(O_1)$, the geomagnetic lunar daily variation associated with the lunar atmospheric tide O_1 . Partial tides are small in comparison with the phase law tides, suggesting that the variation resulting from the atmospheric tide O_1 and higher harmonics of ionospheric conductivity contribute little to $L(O_1)$ at Alibag. Separation of $L(O_1)$ into parts of oceanic and ionospheric origin shows that (i) primary source of $L(O_1)$ in Z is the oceanic dynamo; (ii) smaller amplitude of the first harmonic of $L(O_1)$ compared to that of the second, contrary to expectation, is not due to the influence of oceanic dynamo contribution on $L(O_1)$. It is inferred that even yearly $L(O_1)$ contains some contribution from $L(M_2)$, lunar variation associated with the atmospheric tide M_2 . Next, by applying the formulae due to Winch, the seasonal harmonics of $L(O_1)$ and $L(M_2)$ are corrected for the possible contamination of one by the other. It is found that seasonal progression of $L(O_1)$ and $L(M_2)$ prior to and after the removal of the effect of one from the other remains the same.

1. Introduction

In recent years, geomagnetic lunar daily variation, $L(O_1)$, associated with the lunar diurnal declinational tide, O_1 , has been the subject of several studies. The results of WINCH (1970, 1971), TARPLEY (1971), RAO and SASTRI (1974) and SASTRI *et al.* (1975) have indicated that O_1 tide is able to produce detectable variations in the geomagnetic field. The time dependence of $L(O_1)$, which is determined by the product of the terms representing lunar diurnal atmospheric tidal movement and the time variation of electrical conductivity of the ionosphere, restricted to the first two harmonics, is given by:

$$L(O_1) = \sum_{n=-1}^3 c_n \sin(nt - 2\nu - h + \lambda_n) \quad (1)$$

where c_n is the amplitude of the n th harmonic, t is mean solar time increasing

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by 2π per solar day, ν is lunar phase in angular measure and is given by $\nu = s - h = t - \tau$, s and h are the mean longitude of the moon and sun respectively, τ is the mean lunar time. Substituting $t - \tau$ for ν and $s - t + \tau$ for h , Eq. (1) can also be written as follows:

$$L(O_1) = \sum_{n=-1}^3 c_n \sin(\tau + (n-1)t - s + \lambda_n). \quad (2)$$

Additional terms corresponding to $n = -2, -3, 4$ and 5 arise from the inclusion of higher harmonics in the expression representing the daily variation of electrical conductivity. For the numerical analysis of the data, the range of summation is taken from $n = -4$ to $n = 4$. Summands $n = 1, 2, 3, 4$ are referred to as phase law tides and summand $n = 0$ as the long period tide. Following SCHNEIDER (1963), summands $n = -1, -2, -3, -4$ are denoted as partial tides. WINCH (1970) generalised the CHAPMAN-MILLER (1940) method, permitting determination of the partial tides and the long period tide in addition to the phase law tides.

The harmonic constituents of transient geomagnetic daily variations associated with atmospheric tide M_2 and O_1 have frequencies which differ by one cycle per year (WINCH, 1971). It is likely that each such transient variation might contribute to the numerical estimate of the other. In fact, the results of WINCH (1970) and TARPLEY (1971) indicated the presence of such contamination. WINCH (1970) suggested that appropriate adjustments made for the effect of $L(O_1)$, may substantially reduce the seasonal variation of $L(M_2)$ and may, thus, offer an explanation for the unduly large seasonal variation observed in global $L(M_2)$. His results for Toolangi appeared to bear out this speculation and he stated that further analyses from many more stations would be very desirable.

RAO and SASTRI (1974) have recently determined clear and significant $L(O_1)$ tides in the Indian equatorial region, whose various features are favourably explained by an ionospheric current like the equatorial electrojet. Later, SASTRI *et al.* (1975) have found that trends of changes in the magnitude of $L(O_1)$ in H at Alibag (Geog.-Lat. $18^\circ 38'N$; Long. $72^\circ 52'E$) with solar and magnetic activities are similar to those observed for $L(M_2)$. In this communication the prominent features of $L(O_1)$ in all the three geomagnetic elements H , D and Z at Alibag and their seasonal variations are discussed. Finally the influence of the contribution of $L(O_1)$ to $L(M_2)$ and *vice versa* for seasonal subdivisions of data are evaluated and their effects discussed.

2. Data Analysis

The absolute hourly mean values of the geomagnetic elements *H*, *D* and *Z* at Alibag during the period 1932–1972 are used in the analysis. Days on which even a single hourly value was missing as well as the days with daily magnetic character *A_p* > 100 are excluded from the analysis. While the data of *H* and *Z* are in units of nano Tesla (1 nT = 1 γ = 10^{−5} Gauss), basic data for *D* are expressed in degrees and minutes west. Declination at Alibag is westerly throughout the period of analysis and, therefore, is treated as negative quantity so that results of the analysis directly refer to east declination. Following the generalised method of WINCH (1970), the yearly and seasonal harmonics of geomagnetic lunar *O*₁ tide, *L*(*O*₁), in the three elements *H*, *D* and *Z* of Alibag are obtained and are given in Table 1. The vector probable error, *pe*, of each of the harmonics is also included in the Table. Amplitudes and probable errors

Table 1. Seasonal and yearly harmonics of *L*(*O*₁) in *H*, *D* and *Z* at Alibag. Amplitudes (*c_n* and *c_n*′) and probable errors (*pe*) are in units of 0.01 nT. Phase angles (*λ_n* and *λ_n*′) in degrees are given at epoch of local midnight (*t*=0) and also when $-2\nu-h=0$.

Season	Harmonic ' <i>n</i> '	<i>H</i>					<i>D</i>					<i>Z</i>				
		Phase		Partial		<i>pe</i>	Phase		Partial		<i>pe</i>	Phase		Partial		<i>pe</i>
		<i>c_n</i>	<i>λ_n</i>	<i>c_n</i> ′	<i>λ_n</i> ′		<i>c_n</i>	<i>λ_n</i>	<i>c_n</i> ′	<i>λ_n</i> ′		<i>c_n</i>	<i>λ_n</i>	<i>c_n</i> ′	<i>λ_n</i> ′	
<i>y</i>	0	180	319			125	49	128			30	25	114			48
	1	45	248	30	18	09	25	168	09	320	02	16	274	09	28	02
	2	32	65	07	243	06	54	342	11	259	03	35	30	12	332	03
	3	13	242	03	334	03	22	162	11	102	02	26	198	09	151	02
	4	06	20	04	75	03	09	238	05	326	01	08	311	04	15	01
<i>j</i>	0	298	337			344	128	142			100	226	100			132
	1	14	55	54	69	15	46	175	21	264	07	51	272	29	313	05
	2	29	240	26	237	10	91	1	11	329	05	56	47	07	5	04
	3	14	77	15	355	08	60	180	14	118	02	58	242	07	165	03
	4	08	19	14	68	08	07	236	08	347	02	01	237	08	24	02
<i>e</i>	0	183	135			251	61	306			68	131	252			50
	1	30	304	34	275	17	27	269	11	6	07	19	350	28	117	06
	2	12	36	17	65	11	53	116	20	249	05	40	166	18	318	04
	3	10	190	11	196	06	36	321	05	75	04	26	11	06	166	03
	4	11	320	08	234	04	11	152	03	339	02	08	226	02	321	02
<i>d</i>	0	451	305			148	76	105			64	58	321			68
	1	134	236	65	11	13	47	124	10	337	09	18	167	17	22	06
	2	115	67	14	259	07	134	312	12	208	05	94	2	12	339	05
	3	49	255	05	07	05	52	128	15	96	04	75	163	15	142	03
	4	12	79	07	67	05	21	263	05	291	02	26	328	03	16	02

of harmonics for D have been multiplied by 0.000291 H, to convert them to force units; here the numerical factor is the value of $1'$ of arc. Most of the harmonics are found to be significantly determined as their amplitudes are greater than 2.08 times the corresponding probable errors. As mentioned earlier the seasonal $L(O_1)$ harmonics are likely to be biased by $L(M_2)$ harmonics, therefore the discussion of the primary features of $L(O_1)$ that follows is confined to the results obtained from the data of 1932–1972 without any subdivision into seasons, denoted by 'y' in Table 1.

3. Results and Discussion

3.1 General features of $L(O_1)$ tide

WINCH (1970) suggested that partial tides may play a greater part in $L(O_1)$ than they do in $L(M_2)$ because, unlike in $L(M_2)$, the partial tide term $n = -1$ in the expression for $L(O_1)$, Eq. (1), results from the coupling effect of an atmospheric tide and the first two harmonics of ionospheric conductivity. In $L(M_2)$, the term $n = -1$ arises from the inclusion of the higher harmonics of conductivity. His analysis for Toolangi data revealed that $L(O_1)$ partial tides in D component were greater than those of $L(M_2)$. Examination of partial tide terms in Table 1 shows that amplitudes of various harmonics in H , D and Z are comparable in magnitude (except the first harmonic in H) and their comparison with the corresponding results of $L(M_2)$ for the same period, from an earlier analysis by ARORA (1977), do not show predominance of these over those of $L(M_2)$. This is in conformity with the earlier observation of RAO and SASTRI (1974) for active sun years 1958–1961. The amplitude of the first harmonic of partial tide in H is comparable to that of the first two harmonics of the phase law tide. The other three partial tide harmonics are much smaller and are not significantly determined. From the dominance of $n = -1$ term over the other partial tide terms, SASTRI *et al.* (1975) felt that $L(O_1)$ given by Eq. (1), derived after taking into consideration only the first two harmonics of ionospheric conductivity, would be sufficient to represent adequately the O_1 component of lunar geomagnetic variation at Alibag. It is seen from Table 1, that, in D and Z , the term $n = -1$ does not show any dominance over the other three terms; the amplitude of all the partial tide harmonics are of comparable magnitude and are significantly determined. But they are much smaller than the corresponding phase law harmonics. Thus, exclusion of higher harmonics in the expression representing the daily variation of conductivity may be justified in determining $L(O_1)$ at Alibag. Abnormally high values corresponding to $n = -1$ in H may probably be associated with the location of Alibag with respect to the ionospheric current system responsible for the sevariations.

3.2 Separation of $L(O_1)$ into parts of oceanic and ionospheric origin

Considering the magnitude of the zeroeth and the first harmonic of ionospheric conductivity, TARPLEY (1971) stated that it would be reasonable to expect in the phase law tide the c_1 term to be somewhat larger than c_2 term in magnetic variation produced by ionospheric lunar diurnal tide, O_1 . But the results presented here in Table 1 show contrary feature, i.e. c_1 is smaller than c_2 , except in H . Like the $L(M_2)$ tide, $L(O_1)$ contains contributions not only from the ionospheric dynamo but also from the oceanic dynamo. The oceanic dynamo generates electric currents by the tidal movements of the sea across the lines of force of the earth's main magnetic field. As the conductivity of the ocean does not exhibit appreciable time variation, the dynamo currents resulting from O_1 oceanic tide would produce lunar diurnal term. Hence, the resultant first harmonic of $L(O_1)$ would depend on the relative contributions from these two dynamos. Whether such an effect is responsible for the contrary feature observed here, i.e., c_1 smaller than c_2 , is examined by separating the $L(O_1)$ into parts of ionospheric and oceanic origin. MALIN (1970), based on some reasonable assumptions, has proposed a method for the separation of L associated with M_2 tide into parts of ionospheric and oceanic origin. Taking ocean dynamo variation associated with O_1 tidal effect to be diurnal and incorporating this change in the method of Malin, his equations for oceanic and ionospheric parts under the same assumptions, are modified as follows:

- (i) the oceanic dynamo variation, C_0 , is represented by

$$C_0 = c_0 \sin(\tau - s + \lambda_0)$$

where c_0 and λ_0 are given by

$$c_0 \cos \lambda_0 = \sum_{n=1}^4 c_n \cos \lambda_n$$

$$c_0 \sin \lambda_0 = \sum_{n=1}^4 c_n \sin \lambda_n$$

ρ_0 , the vector probable error of c_0 is

$$\rho_0 = \left[\sum_{n=1}^4 \rho_n^2 \right]^{1/2}$$

- (ii) the ionospheric part, C_I , of C_1 is given by

$$C_I = c_I \sin(\tau - s + \lambda_I)$$

where

$$c_I \cos \lambda_I = -(c_2 \cos \lambda_2 + c_3 \cos \lambda_3 + c_4 \cos \lambda_4)$$

$$c_I \sin \lambda_I = -(c_2 \sin \lambda_2 + c_3 \sin \lambda_3 + c_4 \sin \lambda_4)$$

The vector probable error, ρ_I , of c_I is

$$\rho_I = [\rho_2^2 + \rho_3^2 + \rho_4^2]^{1/2}$$

Table 2. Separation of first harmonic of yearly $L(O_1)$ phase law tide, C_1 , into parts of oceanic, C_0 , and ionospheric, C_I parts (Amplitudes c_0 , c_I , c_1 and probable errors in units of 0.01 nT and phase angles in degrees).

Element	Oceanic part			Ionospheric part			First harmonic of $L(O_1)$		
	c_0	pe	λ_0	c_I	pe	λ_I	c_1	pe	λ_1
H	22	12	260	24	07	236	45	09	248
D	12	04	276	31	04	146	25	02	168
Z	17	04	314	11	04	198	16	02	274

These formulae are applied to the harmonics of yearly $L(O_1)$ phase law tide in H , D and Z and the results are presented in Table 2. Except for the oceanic term in H , all the oceanic and ionospheric terms are significantly determined. In H , both amplitude and phase of both the parts of $L(O_1)$ are nearly equal. In D the ionospheric part dominates with the largest amplitude of all the three elements, but in Z , the oceanic term is greater than the ionospheric term. This result supports the view of TARPLEY (1971) that most probable cause of $L(O_1)$ in Z is the electric currents induced in the ocean by O_1 tide. It is interesting to note that while the phase difference between oceanic and ionospheric parts in D and Z are greater than 90° , in H the difference is small.

When the oceanic part is separated, the decontaminated c_1 , i.e. c_I , is still less than c_2 in all the elements. This suggests that the absence of the prominence of c_1 over c_2 at Alibag is not due to the effect of oceanic dynamo contribution to c_1 . An obvious alternative interpretation is that even the yearly geomagnetic $L(O_1)$, derived here, may include an unavoidable contamination from the seasonal variations of the M_2 geomagnetic tide. There is reason to believe this because $L(M_2)$ at Alibag, all through the year, is not under the control of northern hemispheric current system associated with $L(M_2)$, as shown by RAJA RAO (1962) and ARORA (1977). Consequently the contribution of $L(M_2)$ to $L(O_1)$ may not average to zero even in the results for data that cover the whole year. The exception in H of c_1 being greater than c_2 could be due to the fact that oceanic and ionospheric parts are nearly in phase and add up in the resultant c_1 .

3.3 Seasonal variation of $L(O_1)$

Figure 1 shows the seasonal vectograms of the $L(O_1)$ variation in the horizontal plane. The vectograms show the greatest field change during daylight hours. The variation vector during winter and year describes a clockwise loop but during summer and equinox the variation vector rotates in an anticlockwise direction. It is interesting to note that the seasonal variation of the amplitude of $L(O_1)$ in H , D , Z and also the sense of rotation of $L(O_1)$ vector in horizontal field in different seasons are similar to the seasonal variations of $L(M_2)$. How-

Fig. 1. Vectograms showing the variation of $L(O_1)$ in the horizontal plane during the year and for each season. The curves refer to the epoch when $-2\nu - h = 0$. The numbers along the curve give local time. Day and night portions are separated by a short line across the curve.

ever, the sense of rotation of $L(O_1)$ variation vector during year is similar to that in winter; it is opposite to that for $L(M_2)$, suggesting that, unlike in $L(M_2)$, the yearly variation of $L(O_1)$ is dominated by the variation during winter. Similarity in seasonal variation of $L(O_1)$ and $L(M_2)$ may be only due to the contamination of $L(M_2)$ by $L(O_1)$ and *vice versa*. WINCH (1971) has given a method to remove the contribution of $L(M_2)$ to $L(O_1)$ and *vice versa*. As per the theory, the contamination can appear only in summer and winter and, therefore, his Eqs. 17, 18, 23 and 24 for a northern hemispheric station, are applied to the harmonics of $L(M_2)$ and $L(O_1)$ in the present analysis. $L(M_2)$ harmonics, for the same period, are taken from ARORA (1977). The decontaminated $L(M_2)$ and $L(O_1)$ after the removal of the $L(O_1)$ contribution from $L(M_2)$ and $L(M_2)$ contribution from $L(O_1)$ are given respectively in Tables 3 and 4 along with the observed harmonics for summer and winter. A striking feature of Tables 3 and 4 is that, though the amplitudes have improved in magnitude there is considerable decrease in the precision of decontaminated harmonics; amplitude to probable error ratio has appreciably decreased. This supports the

Table 3. Harmonics of $L(M_2)$ in H , D and Z prior to and after the removal of $L(O_1)$ contribution for summer and winter seasons (Amplitudes, probable errors in units of 0.01 nT and phases in degrees).

Element	Season	Har- monic ' n '	Contaminated $L(M_2)$ tide					Decontaminated $L(M_2)$ tide					
			Phase		Partial		pe	Phase		Partial		pe	
			l_n	λ_n	l_n'	λ_n'		l_n	λ_n	l_n'	λ_n'		
H	j	0	245	191			205	577	120			1177	
		1	48	351	56	139	15	133	2	107	85	65	
		2	36	174	18	278	12	76	216	71	211	49	
		3	14	13	14	61	11	31	69	30	359	43	
		4	14	286	17	132	10	25	297	37	89	40	
	d	0	363	40			230	431	126			870	
		1	129	340	55	250	14	208	40	142	184	59	
		2	139	171	49	156	09	240	211	130	148	36	
		3	57	2	12	226	05	104	46	37	206	22	
		4	15	177	08	269	04	23	211	31	230	19	
	D	j	0	161	11			68	266	335			359
			1	65	95	34	349	08	114	123	62	330	33
			2	126	285	42	59	05	233	317	111	55	22
			3	81	105	21	160	03	150	139	63	126	11
			4	02	159	05	59	03	14	298	12	319	11
d		0	97	226			93	233	267			357	
		1	66	221	12	244	05	110	247	16	211	30	
		2	148	54	21	306	04	233	101	103	308	19	
		3	64	220	20	349	03	86	250	38	314	15	
		4	24	8	06	75	02	41	53	32	59	09	
Z	j	0	238	337			56	275	292			412	
		1	56	184	33	02	05	65	223	93	316	22	
		2	73	334	18	51	04	138	11	53	32	17	
		3	72	172	06	358	03	142	213	27	43	13	
		4	02	277	09	93	02	09	287	18	45	09	
	d	0	79	302			82	318	272			332	
		1	30	213	22	268	05	61	190	47	229	24	
		2	106	99	23	241	04	148	141	48	225	19	
		3	88	259	20	349	03	123	296	71	314	13	
		4	31	71	09	355	02	52	112	26	10	09	

view of WINCH (1971) that any dynamo theory devised for $L(M_2)$ and $L(O_1)$ should be able to deal with the contaminated results.

From the consideration of $L(M_2)$ and $L(O_1)$ results at Toolangi, for data averaged over all seasons, WINCH (1970) states that $L(M_2)$ results for summer months would be enhanced and for winter months diminished by the presence

Table 4. Harmonics of $L(O_1)$ in H , D , and Z prior to and after the removal of $L(M_2)$ contribution for summer and winter seasons (Amplitudes, probable errors in units of 0.01 nT and phases in degrees).

Element	Season	Harmonic 'n'	Contaminated $L(O_1)$ tide					Decontaminated $L(O_1)$ tide				
			Phase		Partial		pe	Phase		Partial		pe
			c_n	λ_n	c_n'	λ_n'		c_n	λ_n	c_n'	λ_n'	
H	j	0	298	337			344	655	20			1280
		1	14	55	54	69	15	103	293	104	129	65
		2	29	240	26	237	10	66	166	79	272	47
		3	14	77	15	355	08	31	21	31	46	41
		4	08	19	14	68	08	14	234	33	139	39
	d	0	451	305			148	651	273			808
		1	134	236	65	11	13	219	183	156	56	58
		2	115	67	14	259	07	193	350	98	38	34
		3	49	255	05	7	05	89	184	31	91	22
		4	12	79	07	67	05	16	354	30	114	20
	j	0	128	142			100	198	225			383
		1	46	175	21	264	07	77	77	38	24	32
		2	91	1	11	329	05	170	270	82	127	22
		3	60	180	14	118	02	113	92	56	187	11
		4	07	236	08	347	02	19	228	17	14	11
	d	0	76	105			64	205	34			336
		1	47	124	10	337	09	70	23	10	59	33
		2	134	312	12	208	05	203	245	98	204	20
		3	52	128	15	96	04	52	41	30	181	16
		4	21	263	05	291	02	36	193	31	311	09
Z	j	0	226	100			132	239	159			467
		1	51	272	29	313	05	50	206	89	12	22
		2	56	47	07	5	04	109	324	43	98	17
		3	58	242	07	165	03	119	166	28	129	13
		4	01	237	08	24	02	09	215	16	91	09
	d	0	58	321			68	303	7			322
		1	18	167	17	22	06	43	275	40	99	24
		2	94	2	12	339	05	117	292	31	106	20
		3	75	163	15	142	03	90	86	67	197	13
		4	26	328	03	16	02	42	252	21	273	09

of the $L(O_1)$ contribution. At Alibag, when the effect of $L(O_1)$ is removed from $L(M_2)$, $L(M_2)$ during winter as well as in summer is found to increase. The winter change is of the type expected by theory but it may be recalled that during this season $L(M_2)$ is primarily determined by the southern hemispheric current system (RAJA RAO, 1962; ARORA, 1977). This may necessitate the

inclusion of additional correction in the numerical procedure for decontamination. Hence, the observed enhancement of decontaminated $L(M_2)$ harmonics during winter may not be the true result of decontamination procedure. The increase of $L(M_2)$ during summer is contrary to the expectation by theory. The sense of rotation of variation vector in the vectograms of $L(O_1)$ and $L(M_2)$ in the horizontal plane during summer and winter, even after the removal of the effect of one from the other, remains unaltered and is opposite between winter and summer. This suggests that opposite trends in the rotation of the variation vector during summer and winter are not the effects of contamination.

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