Electric Potential Difference between Conjugate Points in Middle Latitudes Caused by Asymmetric Dynamo in the Ionosphere

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A new method is proposed for calculation of electric charge distribution in the ionosphere associated with the dynamo action for any given air motion. If the dynamo action is asymmetric in the northern and southern hemispheres, the electric potential difference of the order of 1 kV will be produced (if the field-aligned current is prohibited) at the conjugate pair of stations in middle latitudes although the potential difference will be almost cancelled by the field-aligned currents in the magnetosphere flowing from the winter hemisphere to the summer hemisphere. It is also shown that the height-gradient of the ionospheric conductivity plays a very important role for charge separation in the ionosphere, the effect of which intensifies the eastward equatorial electrojet on the dayside.

1. Introduction

The dynamo action in the ionosphere is now believed to be the most important cause of the geomagnetic variation on quiet days, Sq. Figure 1 is a diagram showing the consequence of the dynamo action in the ionosphere, how the final three-dimensional electric current through the ionosphere and magnetosphere results, if the Sq-field is assumed to be due entirely to the dynamo action in the ionosphere. A number of papers have been published, which deal with the three-dimensional current in the magnetosphere-ionosphere (VAN SABBEN, 1966, 1969, 1970; MISHIN, 1968; MISHIN et al., 1971; MATVEEV, 1971; MÖHLMANN, 1974; MAEDA, 1974), with some simplified assumptions to avoid the mathematical difficulties involved (see also a review paper on the different models written by WAGNER, 1971). The present paper gives a new method of approach to the study of current distribution in the ionosphere and magnetosphere, i.e. through the estimation of electric potential distribution associated with the dynamo action in the ionosphere, especially when the dynamo action is asymmetric in the northern and southern hemispheres.

This paper also deals with the effect of the height-gradient of the electric conductivity in the ionosphere, and shows a possible consequence of the vertical charge separation associated with the dynamo action in the ionosphere.

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2. Electric Currents in the Ionosphere

The total electric current \( i \), in the ionosphere associated with the dynamo action \((v \times B)\) can be written as

\[
i = -\sigma_0 \nabla / S + \sigma_1 [(v \times B) - \nabla \perp S] + \sigma_4 (B/B) \times [(v \times B) - \nabla \perp S],
\]

(1)

where \( \sigma_0, \sigma_1 \) and \( \sigma_4 \) denote the longitudinal conductivity, Pedersen and Hall conductivities of the ionosphere, and \( S \) is the total electrostatic potential in the ionosphere due to the heterogeneous distribution of excess electric charge produced by the dynamo action; \( \nabla \parallel S \) and \( \nabla \perp S \) mean the gradient of \( S \) along and perpendicular to the local geomagnetic field line. We consider here that

\[
S = S_1 + S_2,
\]

(2)

where \( S_1 \) is the electrostatic potential produced when the electric conductivity perpendicular to the magnetic field in the ionosphere has only Pedersen conductivity \( (\sigma_1) \), whereas \( S_2 \) is the additional potential produced under the presence of Hall conductivity \( (\sigma_4) \). The total current \( i \), can be thought to consist of two parts, i.e.

\[
i = i_1 + i_2
\]

(3)

with

\[
i_1 = -\sigma_0 \nabla / S_1 + \sigma_1 [(v \times B) - \nabla \perp S_1],
\]

\[
i_2 = -\sigma_0 \nabla / S_2 + \sigma_4 (B/B) \times (-\nabla \perp S_2),
\]

(4)

and

\[-\sigma_1 \nabla \perp S_2 + \sigma_4 (B/B) \times [(v \times B) - \nabla \perp S_2] = 0.
\]

(5)

Equation (5) means that, in the electric current flowing perpendicularly to the magnetic field, the Pedersen current under the electric field of \(-\nabla \perp S_2\) is always cancelled out by the Hall current under the electric field of \((v \times B) - \nabla \perp S_2\).

Figure 2 shows the geomagnetic \( Sq \) current under an idealized dynamo theory which assumes that (i) the ionosphere is a thin shell with uniform Pedersen and Hall conductivities, (ii) the geographic and geomagnetic axes of the earth coincide, (iii) the geomagnetic field is vertical with its intensity proportional to the cosine of colatitude on the earth, (iv) the horizontal ionospheric wind is divergent from the subsolar point on the equator. In such a case it has been shown (FUKUSHIMA, 1968)
that the total $Sq$ current consists of two components with the same pattern, i.e. $\sigma_1(\mathbf{v} \times \mathbf{B} - \mathcal{V}_1 \mathbf{S}_1)$ and $\sigma_2(\mathbf{B}/\mathbf{B}) \times (-\mathcal{V}_1 \mathbf{S}_2)$, so that the resultant horizontal current is the Cowling current under the electric field of $(\mathbf{v} \times \mathbf{B}) - \mathcal{V}_1 \mathbf{S}_1$. The cancelling currents given by Eq. (5) are shown by broken arrows in Fig. 2.

3. A New Approach to Calculate Electric Charge Distribution in the Ionosphere

Equation (5) holds only for the electric currents perpendicular to the magnetic field. However, if the equipotentiality of magnetic field lines can be assumed, i.e.

$$\mathcal{V}_/ \mathbf{S}_2 = 0,$$

the following three-dimensional equation holds:

$$-\sigma_1 \mathcal{V}_2 + \sigma_2(\mathbf{B}/\mathbf{B}) \times [(\mathbf{v} \times \mathbf{B}) - \mathcal{V}_1 \mathbf{S}_1] = 0.$$  \hspace{1cm} (7)

Once we are allowed to assume the validity of Eq. (7), we can proceed to the following calculation for the electric charge density in the ionosphere associated with the dynamo action. Equation (7) can be written as

$$\mathcal{V}_2/ \mathbf{S}_2 = \{\sigma_2/(\sigma_1 \mathbf{B})\} \mathbf{B} \times [\{\mathbf{v} \times \mathbf{B}\} - \mathcal{V}_1 \mathbf{S}_1].$$  \hspace{1cm} (8)

Here the quantity $\sigma_2/(\sigma_1 \mathbf{B})$ depends on height, but it can be thought to be independent of latitude and longitude over the earth, with much higher accuracy than $\sigma_2/\sigma_1$. Taking the divergence of Eq. (8), the density $\rho_2$ of electric charge responsible for the electric potential $\mathcal{V}_2$ can be obtained as

$$\rho_2 = \varepsilon (-\Delta \mathcal{V}_2) = \varepsilon \frac{\sigma_2}{\sigma_1} \mathbf{B} \cdot \text{curl} (\mathbf{v} \times \mathbf{B}) - \varepsilon \mathcal{V}_2 \frac{\sigma_2}{\sigma_1} \mathbf{B} \cdot [\mathbf{B} \times (\mathbf{v} \times \mathbf{B} - \mathcal{V}_1 \mathbf{S}_1)]$$

$$= \varepsilon \frac{\sigma_2}{\sigma_1} \mathbf{B} \cdot \text{curl} (\mathbf{v} \times \mathbf{B}) - \varepsilon \mathcal{V}_2 \frac{\sigma_2}{\sigma_1} \mathbf{B} \cdot \mathcal{V}_2 \mathbf{S}_2,$$  \hspace{1cm} (9)

where $\varepsilon$ is the dielectric constant of the medium concerned.
Here we must consider how the assumption of $\mathcal{F}_1 S = 0$ influences the calculation for the electric charge in the ionosphere. $\mathcal{F}_1 S = 0$ means the equipotentiality of geomagnetic field lines, but it also means $e_0 \mathcal{F} = 0$, i.e. no field-aligned currents in the magnetosphere and ionosphere. Hence Eq. (9) gives the electric charge distribution in the ionosphere when it is not at all modified by the field-aligned currents. In other words, Eq. (9) gives the charge distribution in the ionosphere before the field-aligned currents begin to modify it. The real charge distribution in the ionosphere under the allowance of field-aligned currents must be obtained after the process shown in Fig. 1 is studied in detail.

It is seen in Eq. (9) that $\rho_s$ responsible for $S_R$-potential has contributions from two sources, both of which originate from the dynamo action in the ionosphere. It is worthwhile noting that the second term disappears when the vertical geomagnetic field alone is taken into consideration in the dynamo action, because $\mathcal{F} e_{\alpha 2} (\alpha, B)$ is vertical and $B \times (\nu \times B - R S)$ or $R S$ is horizontal in such a case. Hence the second term is essentially important when we discuss the dynamo theory without neglecting the horizontal component of the geomagnetic field.

4. Estimation of Electric Charge Distribution in the Ionosphere along the Noon Meridian

The contributions from both the terms on the right-hand side of Eq. (9) are examined and compared for the case with a simple horizontal wind motion in the ionosphere.

4.1 The contribution from the first term for the ionospheric dynamo

The calculation is made for a simple case when the wind can be derived from the velocity potential $\alpha$ in the form of

$$\phi = r \kappa \alpha^2 [\sin \delta \cos \theta \cos \delta \sin \cos (\phi - \phi_0)],$$

where $r$ is the geocentric distance of the ionosphere. The horizontal wind $v(\nu, \nu)$ in the dynamo region is then given by

$$v = -k_1^2 \alpha [\sin \delta \sin \cos \delta \cos \cos (\phi - \phi_0)],$$

$$v = k_1^2 \cos \delta \sin (\phi - \phi_0).$$

This air motion shows a horizontal diverging wind in the ionosphere (such as the one shown in Fig. 2) from the point of $\theta = \pi/2 - \delta$, $\phi = \phi_0$, which may be assumed to be the subsolar point. The geomagnetic field is assumed to be a centred dipole with an inclined geomagnetic axis. By means of the Gauss coefficients for the spherical harmonic expansion of the geomagnetic field, $B(B_r, B_\theta, B_\phi)$ can be expressed by

$$B_r = 2(g_1^2 \cos \theta - g_1^2 \sin \theta \cos \phi - h_1^2 \sin \theta \sin \phi),$$

$$B_\theta = g_1^2 \sin \theta - g_1^2 \cos \theta \cos \phi - h_1^2 \cos \theta \sin \phi,$$

$$B_\phi = g_1^2 \sin \phi - h_1^2 \cos \phi.$$
Figure 3 shows the numerical value of \( (B/B) \cdot \text{curl} (v \times B) \) along the noon geomagnetic meridian in the unit of \( k[s/r_E] \), which is proportional to electric charge density in the dynamo region when the field-aligned current is prohibited. The diagrams are shown for the solar declination \( 0^\circ, \pm 10^\circ \) and \( \pm 20^\circ \); (a) is for the 21°E or 159°W

Fig. 3. The value of \( (B/B) \cdot \text{curl} (v \times B) \) in the unit of \( k[s/r_E] \) in the ionosphere along the noon meridian, which is proportional to the excess electric charge produced in the ionosphere associated with the idealized horizontal wind from the subsolar point, for different solar declinations. The mark \( \circ \) denotes the latitude of subsolar point.
meridian where the geomagnetic and geographic equators coincide, (b) and (c) are for 111°E or 69°W meridian with the greatest discrepancy between the geomagnetic and geographic equators.

The height-integrated electric charge density \( Q \) (coulomb/m\(^2\)) over the ionosphere is given by

\[
Q(\theta, \phi) = \int \rho(r, \theta, \phi) \, dr = \int \varepsilon \frac{\sigma_2}{\sigma_1} \frac{B}{B} \, \text{curl} (\mathbf{v} \times \mathbf{B}) \, dr
\]  

(13)

where the integration is to be taken over the height range of the dynamo region.

4.2 The contribution from the second term with height-gradient of ionospheric conductivity

The second term on the right-hand side of Eq. (9) disappears in high latitudes (because \( \mathcal{L}[\sigma_2/(\sigma_1 B)] \) has only the radial component and \( \mathcal{L} S_2 \) or \( B \times (\mathbf{v} \times B - \mathcal{L} S_1) \) is horizontal there), and near the focus of the ionospheric \( S_q \) current where \( \mathcal{L} S_2 = 0 \). A schematic distribution of positive and negative electric charge due to the second term of Eq. (9) is shown in Fig. 4 for the meridian of \( S_q \)-current focus on the daytime, although the radial charge separation must be greatly reduced through the diffusion of excess electric charge along the inclined magnetic field lines, except for the equatorial region where the magnetic field is horizontal. Near the equator the charge separation has an effect of intensifying the eastward electrojet along the geomagnetic equator in the daytime.

4.3 Comparison of charge quantity due to the first and second terms

Here we estimate which one of the above two contributions plays a more important role in the production of excess electric charge in the ionosphere associated with the dynamo action. The second term on the right-hand side of Eq. (9) is written as

![Fig. 4. Schematic illustration of vertical electric charge separation within the ionosphere on the dayside associated with the dynamo action.](image-url)
because of $BF(1/B) \ll \sigma_i/\sigma_2$. In the above equation $I$ is the dip angle of the geomagnetic field. Therefore, the ratio $R$ of the second to the first term of Eq. (9) is given by

$$R = -\left(\frac{\sigma_1}{\sigma_2}\right)^2 \frac{\nabla \cdot \mathbf{S}_2}{\nabla \cdot \mathbf{S}_1} \cdot \cos \left(\mathbf{\nabla} \cdot \mathbf{B}\right) \cdot \mathbf{v} \times \mathbf{B}^{-1}.$$

The denominator of $R$ in Eq. (15) is shown numerically in Fig. 3 for the noon meridian in the unit of kg/r, the order of which being about $100 \text{ ms}^{-1} \times 3 \times 10^{-5}$ Tesla/6500 km = $5 \times 10^{-10}$ V/m. In the numerator, $\nabla \cdot \mathbf{S}_2$ can be assumed to be about the order of $10 \text{ kV}/5000$ km = $2 \times 10^{-3}$ V/m. Since $\nabla (\sigma_2/\sigma_1)$ is the order of $10^{-4}$ m$^{-1}$, it is safely concluded that $R > 1$ at least in low latitudes, i.e. the contribution from the second term of Eq. (9) is usually greater in comparison with the first term, unless the charge separation is neutralized along the magnetic field lines within the dynamo region.

5. Electric Potential Difference between Summer and Winter Hemispheres and Associated Field-Aligned Currents in the Magnetosphere

The non-uniform distribution of excess electric charge in the ionosphere associated with the dynamo action can be obtained by Eq. (9) or (13), before it is modified by the field-aligned currents through the magnetosphere. If $Q$ in Eq. (13) can be given as

$$Q(\theta, \phi) = \sum_n \sum_m q_n^m S_n^m(\theta, \phi)$$

by means of the surface spherical harmonics $S_n^m$ with the numerical coefficients $q_n^m$, the electric potential $V$ at the ionospheric level is given by

$$V(\theta, \phi) = 4\pi r_E^2 \sum_n \sum_m \frac{q_n^m}{2n+1} S_n^m(\theta, \phi).$$

Although the electric potential distribution associated with the dynamo action is not shown in this paper, the curves in Fig. 3 show approximately the noon-meridian value of electric potential at the ionospheric level (the unit may be assumed to be about 10 kV, referring to the ordinary estimation of electric potential associated with the geomagnetic $Sq$ current in the ionosphere), insofar as the field-aligned currents are prohibited. In such an idealized state, the electric potential in middle latitudes is generally a few kV lower in the summer hemisphere than in the winter hemisphere for a pair of conjugate points connected with the geomagnetic field lines.

The potential difference in the northern and southern hemispheres will trigger the field-aligned currents in the ionosphere flowing from the winter side to the summer side, which tends to cancel out the electric potential difference between the both hemispheres at the level just above the ionosphere. On the other hand, the dynamo action in the ionosphere will continuously supply the electric charges. In
the final stationary state a balance must be held between the charge neutralization along the geomagnetic field lines and the production of excess charges due to the dynamo action in the ionosphere. This balance will result in a certain amount of stationary field-aligned current in the magnetosphere from the winter hemisphere to the summer hemisphere.

6. Conclusion and Discussion

In most of the previous papers on the middle-latitude field-aligned currents associated with the ionospheric dynamo (referred to in the introduction) the field-aligned currents are calculated from the divergence of horizontal currents in the ionosphere or the currents perpendicular to the geomagnetic field. The present paper adopts a different method of approach; the starting point is to calculate the distribution of excess charge or electric potential at the ionospheric level when the field-aligned currents are prohibited. Under such a condition the potential difference between the north-south conjugate pair of stations will trigger the field-aligned currents which flow from the winter side to the summer side in the daytime.

Once the field-aligned currents start to flow, the electric potential in the ionosphere must be redistributed so as to show little (or almost no) potential difference between the north-south conjugate pair of stations. The resultant final three-dimensional current distribution in the ionosphere and magnetosphere must be worked out by taking account of the balance between the production of excess charge by the ionospheric dynamo and the charge neutralization along the geomagnetic field-lines. The charge neutralization current along the geomagnetic field-lines in the magnetosphere must constitute a closed divergence-free current in the ionosphere. Hence the intensity of field-aligned current is controlled by the Pedersen conductivity of the ionosphere between the north-south conjugate points, which is much smaller than the conductivity along the geomagnetic field-lines in the magnetosphere. Because of this the field-aligned currents on the night-side must be much weaker in comparison with the dayside currents, even if the electric potential difference between the north-south conjugate points is of the same magnitude in the sunlit and dark hemispheres.

In the polar regions where the geomagnetic field-lines are open to the magnetospheric tail, the potential difference between the northern and southern hemispheres may possibly remain unequalized. In such a case, a poleward electric field will be produced near the boundary of open and closed magnetic field-lines in the summer polar region, and this will produce an eastward circumpolar zonal current. In the winter polar region, on the other hand, an equatorward electric field will be produced, but the associated westward zonal current will be weak because of a poor Hall conductivity in the winter polar regions. Although this mechanism seems to be favourable for interpreting a remarkable annual variation in the Z-component of the geomagnetic field near the geomagnetic pole, the quantitative examination must be made in the future.
It is also shown in this paper that the height-gradient of the ionospheric conductivity plays a very important role in the production of excess charge in the ionosphere. The neutralization of excess electric charges of opposite signs above and below the level of maximum $a_2/a_1$ value will constitute closed electric circuits in the vertical cross-section of the ionosphere. These currents will produce a toroidal magnetic field within the ionosphere, which does not leak out of the ionosphere. At the geomagnetic equator the vertical polarization electric field must exist because of no charge neutralization along the horizontal magnetic field lines, and this vertical electric field intensifies the eastward electrojet flowing in the dayside ionosphere. The importance of the height-gradient of the ionospheric conductivity (Kato, 1977) must be taken into consideration in the future study of the ionospheric dynamo, because this effect is automatically eliminated if we ignore the horizontal component of geomagnetic field in the discussion of the dynamo action in the ionosphere.

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