Solar Cycle Variations of the First-Degree Spherical Harmonic Components of the Geomagnetic Field

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(Received October 25, 1978; Revised January 27, 1979)

Spherical harmonic analyses were made of the annual mean values of the geomagnetic components at 34 observatories for the period from 1940 to 1973. Annual values of the Gauss coefficients were separated into parts of external and internal origin. The external part of the first-degree component is parallel to the internal dipole and changes with solar activity. The maximum change from peak to trough amounts to 47 nanoteslas during this period. During one solar cycle, two peaks appear in the variation of this component. The first peak corresponds to the period of the maximum solar activity, the second at the declining stage of the cycle.

Examination of the time variations of the first-degree components was extended to an analysis for 1900 to 1973 based on annual means from 21 observatories. Although its amplitude is small, the internal part changes with a three-year time lag behind the external field change. After Fourier analyses of the time series of both the internal and external components, their ratio was computed for the 10-year-period band. The modulus of the internal/external ratio (i/e) was obtained to be 0.32 and the phase angle (internal-external) to be 105° .

1. Introduction

Variations in the geomagnetic field are strongly affected by solar phenomena. Various indices of the geomagnetic activity change with the solar cycle. However, the nature of the geomagnetic variation associated with the solar cycle is not yet fully understood. In this paper, year-to-year variation of the geomagnetic field caused by magnetic effects associated with solar activity is examined by expanding the annual mean values of the X , Y , and Z components at selected observatories into spherical harmonic series.

The simplest way of expressing the solar cycle variation is to assume a

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sinusoid with a period of 11 years (YUKUTAKE, 1965). However, this is too crude an approximation to represent the magnetic response to the solar activity. COURTILOT and LEMOUËL (1976) obtained a better representation of the geomagnetic changes by making use of the second and the third harmonics of the 11-year period (as well as the fundamental mode). Recently, HARWOOD and MALIN (1977) tried to correlate the geomagnetic variation with the change in relative sunspot number by making use of a linear combination of annual values of sunspot numbers and their annual rate of change. Although sunspot number is one of the best indices to measure solar activity, the phenomena taking place at the surface of the sun are not so simple as they can be described by a single index. From examination of magnetic indices, it has been recently inferred that two peaks exist in geomagnetic activity during one solar cycle (OL', 1971; SUTORIK and CRUICKSHANK, 1977; MEYERS and ALLEN, 1977). These are long-term variations in the frequency of rather short-term magnetic disturbances, whereas the present paper attempts to investigate the long-term change in the field itself. In this paper, the first-degree components of external origin are first extracted without referring to the sunspot number.

Investigation of the response of the Earth to the external field variations provides important information regarding the distribution of electrical conductivity within the Earth. Since the penetration becomes deeper as the period of the phenomena becomes longer, analysis of the long-term variation of the external field is particularly important to obtain an estimate of the conductivity in the deeper part of the mantle. Many efforts have been made to separate the solar cycle variations of the geomagnetic field into external and internal parts (YUKUTAKE, 1965; COURTILOT and LEMOUËL, 1976; ALLDREDGE, 1976; HARWOOD and MALIN, 1977). Accurate extraction of the internal components is more difficult than that of the external parts, because of the small amplitude of the former. Accumulation of accurate data over a long period is required to make a reliable separation. Because of the limited number of observatories which can supply reliable data over several solar cycles, the separation of the external and the internal parts is not yet conclusive. Nevertheless the internal field variation related to the solar activity was extracted by a spherical harmonic analysis of annual mean values of the observatory data.

2. The Process of the Analysis

Two approaches can be used in extracting solar cycle variations from the annual mean values of the observatory data. One is to first make a spectrum analysis of the observed secular changes at individual observatories to extract solar cycle variations with specific periods, and then make a spherical har-

monic analysis of these extracted variations to determine their global nature (YUKUTAKE, 1965; COURTILLOT and LEMOUËL, 1976; ALLDREDGE, 1976; HARWOOD and MALIN, 1977). This approach is a kind of frequency domain analysis. The other approach is to first employ a spherical harmonic analysis of the annual mean values and obtain the spherical harmonic coefficients for the successive years. The annual values of the Gauss coefficients thus obtained are then subjected to a spectral analysis (YUKUTAKE, 1973; ALLDREDGE *et al.*, 1979).

If an anomalously large solar cycle variation exists at a specific observatory, the first method may detect the anomaly much more easily. However, the year-to-year variation of the global feature is hard to examine by this method, since detailed features of the variation have already been decomposed into a frequency domain at the first step. On the other hand, the second approach can reveal the year-to-year variation of the dipole field caused by the solar activity. The second approach is adopted in this paper.

3. The Data

For detecting small fluctuations in the Gauss coefficients it is desirable to have the data as uniformly distributed in space and as continuous in time as possible. As will be shown in later sections, alterations of the low-order terms by aliasing from neglected higher order coefficients can be large compared to the variations being sought. Thus, it is best to avoid changes in baselines, sites of the stations, or number of stations.

Since there are large areas of the earth not represented by any magnetic observatory, any spherical harmonic analysis of observatory data alone can contribute only to a global knowledge of the lowest order coefficients. By so limiting the analysis it is possible to obtain meaningful results from a small subset of the total data available. The data from 34 observatories covering the period 1940–1973 were selected as being sufficient to estimate the changes in the first few degrees and orders of harmonic terms. The geographic positions of these stations are listed in Table 1 and are illustrated in Fig. 1 by the circles. As can be seen here the largest gaps between stations are of the order of 90° , corresponding to $n=4$ in the harmonic series. Although there are many excellent observatories elsewhere, especially in Europe, their addition would not close these gaps and could tend to bias the results to the areas of greater observatory density. Such a biased result could be less representative of the global variation than that determined from fewer but more evenly distributed locations.

The discontinuities caused by movement of the site or change in standards were corrected using the constants provided by the individual observatories.

Table 1. List of observatories.

Code	Name		Geographic coordinates		
			Latitude	Longitude	Altitude (km)
0	GDH	Godhavn	69.240	−53.522	.01
0	SOD	Sodankylä	67.368	26.630	.18
0	YAK	Yakutsk	62.017	129.717	.10
1	VOY	Voyeykovo	59.950	30.705	.07
1	SIT	Sitka	57.058	−135.325	.02
1	VDA	Vysokaya-Dubrava	56.733	61.067	.29
0	MNK	Meanook	54.617	−113.333	.68
0	RSV	Rude Skov	55.843	12.457	.05
0	ESK	Eskdalemuir	55.317	−3.200	.24
1	PTY	Patrony	52.167	104.450	.54
0	FUR	Fürstentfeldbruck	48.165	11.277	.57
1	CLF	Chambon-la-Forêt	48.023	2.260	.15
0	YSS	Yuzhno Sakhalinsk	46.950	142.717	.07
2	AGN	Agincourt	43.783	−79.267	.06
1	DUS	Dusheti	42.092	44.705	.98
0	TKT	Tashkent	41.333	69.617	.81
0	COI	Coimbra	40.222	−8.422	.10
0	FRD	Fredericksburg	38.205	−77.373	.07
1	SMG	San Miguel	37.773	−25.653	.18
1	KAK	Kakioka	36.230	140.190	.03
1	TUC	Tucson	32.247	−110.833	.77
1	MLT	Misallat	29.515	30.892	.12
1	HON	Honolulu	21.320	−158.002	.00
0	TED	Teoloyucan	19.747	−99.182	2.28
1	ABG	Alibag	18.638	72.872	.01
1	SJG	San Juan	18.117	−66.150	.10
2	KUY	Kuyper	−6.033	106.733	.00
2	MRI	Mauritius	−20.093	57.553	.05
0	HUA	Huancayo	−12.045	−75.340	3.31
1	API	Apia	−13.807	−171.775	.00
0	TAN	Tananarive	−18.917	47.500	1.38
1	VSS	Vassouras	−22.400	−43.650	.50
1	PLL	Pilar	−31.667	−63.833	.34
0	GNA	Gnangara	−31.783	115.950	.06
0	HER	Hermanus	−34.425	19.225	.03
1	TOO	Toolangi	−37.533	145.467	.46
1	AML	Amberley	−43.152	172.722	.04

Code=0 used only for 1940–1973 analysis

1 used for both analyses (1900–1973 and 1940–1973)

2 used only for 1900–1973 analysis

Fig. 1. Distribution of the observatories used for analysis.

The few missing data were carefully interpolated. The X , Y , and Z components thus compiled into a homogeneous and continuous data set are listed in YUKUTAKE *et al.* (1979). The observatories that changed their observation sites more than 3° in longitude were excluded. The second set of 21 observatories from which data were similarly treated by YUKUTAKE (1973) for the interval 1900–1965 are also indicated in Table 1 and in Fig. 1 by plus signs. These are given because the results of their earlier analysis will also be extended to 1973 and referenced later in this paper.

4. Spherical Harmonic Analysis of the Annual Mean Values of X , Y , Z Components

In order to separate the variations into external and internal parts, two different methods of spherical harmonic analysis have been employed. One is the classical way that analyzes the X and Y components together and Z separately to obtain two sets of Gauss coefficients (see e.g. CHAPMAN and BARTELS, 1940, p. 641). The internal-external separation is then made from the two sets of coefficients. The other is the analysis that treats all three components together and determines the external and the internal coefficients at once following the least squares program developed by CAIN *et al.* (1965, 1967). Since the variations from year to year are found to be essentially identical from each type of analysis only the results from the separation technique are used for interpretation. The combined analyses were made to establish confidence in the validity of the difference results and to explore the question of whether such a thin distribution of data could be used to determine absolute values of the internal and external coefficients.

Table 2a. Gauss coefficients (nT) ($g_n^m(XY)$, $h_n^m(XY)$) obtained by analyses of the X and Y components. σ represents standard deviation.

Period	g_1^0	σ	g_1^1	σ	h_1^1	σ
1940	-30403	221	-2569	241	5576	192
1941	-30385	221	-2569	242	5574	192
1942	-30392	223	-2571	244	5575	194
1943	-30378	225	-2563	246	5571	195
1944	-30378	225	-2561	246	5570	196
1945	-30371	226	-2558	248	5565	197
1946	-30345	227	-2555	249	5557	197
1947	-30342	228	-2546	251	5551	199
1948	-30339	230	-2538	252	5550	200
1949	-30329	232	-2531	254	5545	201
1950	-30323	232	-2524	255	5543	202
1951	-30389	234	-2517	257	5534	204
1952	-30300	236	-2511	259	5531	205
1953	-30290	238	-2510	261	5532	207
1954	-30284	239	-2508	262	5534	208
1955	-30272	240	-2503	262	5534	209
1956	-30247	240	-2496	263	5530	209
1957	-30232	241	-2486	263	5527	210
1958	-30221	242	-2481	265	5524	211
1959	-30202	243	-2478	266	5522	211
1960	-30178	243	-2471	267	5521	212
1961	-30181	244	-2469	268	5527	212
1962	-30174	245	-2463	269	5529	213
1963	-30154	245	-2455	269	5529	213
1964	-30141	246	-2445	270	5530	214
1965	-30123	247	-2436	270	5528	214
1966	-30095	247	-2425	271	5521	215
1967	-30064	247	-2410	271	5512	215
1968	-30041	248	-2401	271	5507	216
1969	-30023	248	-2389	272	5501	216
1970	-30001	249	-2379	272	5491	216
1971	-29984	249	-2369	272	5484	216
1972	-29959	249	-2359	273	5475	217
1973	-29935	250	-2346	274	5464	218

4.1 Treatment of the X , Y components separate from Z

Spherical harmonic analyses were made separately for the annual means from 1940 to 1973 of the X , Y components and the Z component of 34 observatories, assuming that all the fields are of internal origin. Thirty-four values of Z component for a particular year allow a determination of the series up to the maximum degree and order of four. As is the case with spherical harmonic analysis, the resulting coefficients are dependent on the maximum degree and

Table 2b. Gauss coefficients (nT) ($g_n^m(Z)$, $h_n^m(Z)$) obtained by analysis of the Z component. σ represents standard deviation.

Period	g_1^0	σ	g_1^1	σ	h_1^1	σ
1940	-31826	1347	-3342	871	5952	932
1941	-31816	1351	-3349	873	5940	935
1942	-31802	1358	-3355	878	5928	940
1943	-31794	1364	-3357	882	5918	944
1944	-31788	1370	-3363	886	5912	949
1945	-31786	1378	-3370	891	5906	954
1946	-31788	1384	-3376	895	5895	958
1947	-31788	1392	-3381	900	5882	964
1948	-31782	1401	-3381	906	5880	969
1949	-31777	1408	-3381	911	5871	975
1950	-31777	1415	-3377	915	5869	979
1951	-31774	1421	-3377	919	5864	983
1952	-31772	1426	-3377	922	5855	987
1953	-31766	1431	-3374	926	5846	991
1954	-31757	1438	-3375	930	5840	995
1955	-31746	1441	-3369	932	5834	997
1956	-31739	1444	-3364	934	5825	999
1957	-31731	1448	-3360	936	5820	1002
1958	-31720	1452	-3354	938	5813	1005
1959	-31782	1453	-3349	939	5809	1006
1960	-31688	1456	-3349	941	5894	1008
1961	-31665	1458	-3345	943	5801	1009
1962	-31638	1460	-3338	944	5798	1011
1963	-31620	1462	-3331	946	5795	1012
1964	-31599	1463	-3323	946	5788	1013
1965	-31575	1464	-3315	947	5786	1013
1966	-31563	1462	-3304	946	5786	1012
1967	-31549	1461	-3296	945	5779	1011
1968	-31538	1459	-3285	944	5775	1010
1969	-31526	1459	-3274	943	5771	1010
1970	-31520	1457	-3260	942	5770	1009
1971	-31504	1457	-3247	942	5766	1008
1972	-31491	1454	-3235	940	5758	1007
1973	-31478	1451	-3223	938	5753	1004

order at which the spherical harmonic series are terminated. The analyses were repeated, terminating the series at degrees from one to four. When only the dipole terms $n=1$ are used, long term variations in g_1^0 derived from the Z component are different from those obtained by terminating the series at higher degrees. The approximation is evidently poor if only the dipole terms are taken. However, the variations in the coefficients with periods shorter than 20 years are similar to each other and are stable for the change in the maximum degree

and order of the series except for the case[†] $n^*=4$. Stability of the short-period variation was examined by taking the time derivative of each Gauss coefficient and comparing the results for different models. If the errors induced by aliasing were caused by short-period noise, they would be amplified by taking derivatives. The time-differentiated coefficients obtained from the Z component with $n^*=4$ exhibited very different behavior with large amplitudes than did those determined from the Z component with n^* smaller than 4. The maximum degree and order of 4 seems too large for the 34 Z values to give stable results. Accordingly, the Gauss coefficients determined up to $n^*=3$ were adopted for the analysis of the Z component and were combined with those of $n^*=4$ for the (X, Y) analysis to separate the internal field from the external field. Gauss coefficients obtained separately from the X and Y analysis ($g_n^m(XY)$, $h_n^m(XY)$) and from the Z analysis ($g_n^m(Z)$, $h_n^m(Z)$) for the $n=1$ terms are shown in Table 2a and b.

The standard deviations given in Table 2a and b are very large. They range from 221 nT to 250 nT for $g_1^0(XY)$. Due to the large size of the standard deviation it might appear meaningless to discuss a year-to-year change in the dipole component that is less than 30 nT/yr for $g_1^0(XY)$. However, the variability of the year-to-year change in the magnetic components from one observatory to another is not so large as might be suspected from this large standard deviation for the main field. Taking the year-to-year variation of the observed field as ΔX , ΔY , and ΔZ , we repeated the spherical harmonic analysis for ΔX and ΔY separately from ΔZ . The Gauss coefficients thus obtained, e.g. $\Delta g_1^0(XY)$, were the same as those obtained by simply taking the annual differences of the coefficients in Table 2a and b. The standard deviations for $\Delta g_1^0(XY)$ were much reduced, ranging from 2.4 nT/yr to 3.6 nT/yr. This implies that the year-to-year variations in the Gauss coefficients in Table 2a and b are well above the range of scatter of the rate of change in the geomagnetic components at the individual observatories. Therefore, in spite of the large standard deviations of the Gauss coefficients for the main field, the relative yearly change in the coefficients is regarded as real and useful for examination of the time change in the dipole component.

Representation by a spherical harmonic series terminated at such low degrees as $n=3$ or 4 is only a crude approximation of the geomagnetic field. The number of the observatories used for the analysis is also very limited. If there are a few observatories whose fields deviate largely from the smooth field represented by the low-degree harmonics, the standard deviations assigned to each coefficient are expected to become large. At some of the observatories deviations from the smooth field indeed exceed 1,000 nT. Even though small fluctua-

[†] The convention used here, suggested by R. Hide, is to express the use of the whole series up to $n=m$ by the notation n^* .

tions such as the solar cycle variations are superposed, the major part of the deviations remains nearly constant during a period of several decades. Removal of this constant part of the deviation from the original data will greatly reduce the standard deviations of the Gauss coefficients for the main field.

The effect of the number of observatories was examined by reducing the number from 34 to 20. Observatories with relatively noisy Z variations were intentionally excluded. However, no improvement was observed by this procedure. Indeed, the scatter of the results had a tendency to be slightly greater.

Table 3. External and internal Gauss coefficients (nT) computed from $(g_n^m(XY), h_n^m(XY))$ and $(g_n^m(Z), h_n^m(Z))$.

Year	g_1^{0e}	g_1^{0i}	g_1^{1e}	h_1^{1e}	g_1^{1i}	h_1^{1i}
1940	949	-31352	515	-251	-3084	5827
1941	954	-31339	520	-244	-3089	5818
1942	940	-31332	523	-235	-3094	5810
1943	944	-31322	529	-231	-3092	5802
1944	940	-31318	535	-228	-3096	5798
1945	943	-31314	541	-227	-3099	5792
1946	962	-31307	547	-225	-3102	5782
1947	964	-31306	557	-221	-3103	5772
1948	962	-31301	562	-220	-3100	5770
1949	965	-31294	567	-217	-3098	5762
1950	969	-31292	569	-217	-3093	5760
1951	977	-31286	573	-220	-3090	5754
1952	981	-31281	577	-216	-3088	5747
1953	980	-31270	576	-209	-3086	5741
1954	982	-31266	578	-204	-3086	5738
1955	983	-31255	577	-200	-3080	5734
1956	995	-31242	579	-197	-3075	5727
1957	999	-31231	583	-195	-3069	5722
1958	999	-31220	582	-193	-3063	5717
1959	1000	-31202	581	-191	-3059	5713
1960	1007	-31185	585	-189	-3056	5710
1961	989	-31173	584	-183	-3053	5710
1962	976	-31159	583	-179	-3046	5708
1963	977	-31131	584	-177	-3039	5706
1964	972	-31113	585	-172	-3030	5702
1965	968	-31091	586	-172	-3022	5700
1966	979	-31074	586	-177	-3011	5698
1967	990	-31054	591	-178	-3001	5690
1968	998	-31039	589	-179	-2990	5686
1969	1002	-31025	590	-182	-2979	5681
1970	1013	-31014	587	-186	-2966	5677
1971	1013	-30997	585	-188	-2954	5672
1972	1021	-30980	584	-189	-2943	5664
1973	1029	-30964	585	-193	-2931	5657

If the observed variations originated entirely inside the earth (and there were no contamination from neglected higher order coefficients), then the terms $(g_n^m(XY), h_n^m(XY))$ and $(g_n^m(Z), h_n^m(Z))$ in Table 2a and b should show complete agreement with each other. The discrepancies between these terms, except for the part due to contamination, indicate the existence of geomagnetic variations of external origin. The separation into external (g_n^{me}, h_n^{me}) and internal (g_n^{mi}, h_n^{mi}) terms up to $n^*=3$ were made by the following formula:

$$g_n^{me} = \frac{(n+1)g_n^m(XY) - (n+1)g_n^m(Z)}{2n+1}, \quad g_n^{mi} = \frac{ng_n^m(XY) + (n+1)g_n^m(Z)}{2n+1},$$

$$h_n^{me} = \frac{(n+1)h_n^m(XY) - (n+1)h_n^m(Z)}{2n+1}, \quad h_n^{mi} = \frac{nh_n^m(XY) + (n+1)h_n^m(Z)}{2n+1}.$$

As seen in the above equations, the separation process depends on the difference between the horizontal and vertical solutions. Since there are twice as many points in the XY solution, the Z determination would contribute more noise even if it were as accurately observed as either X or Y . However, baseline maintenance is the hardest for Z , and errors most easily enter this component. It is not very certain whether the data available at present, particularly those for the Z component, are accurate enough to permit us to extend the separation up to the higher harmonics. Accordingly, only the results for the first-degree

Fig. 2. a. Time variations in the external and internal parts of the axial dipole component (g_1^0). b. Time variations in the external and internal parts of the equatorial dipole component (g_1^1, h_1^1). Unit in nT.

terms are shown in Table 3. If the contamination of the results is due to the neglected terms, it should be relatively constant since the changes in field are small compared to its magnitude. Thus, as will be shown later, one can rely on the changes revealed by this analysis, but not on the absolute values themselves. Figure 2 shows the time variations in the external and internal parts of the first-degree terms. In contrast to the smooth changes in the internal coefficients, zigzag features are noticeable in the external coefficients.

4.2 *Determination of the external coefficients simultaneously from three components*

Since the direct method of analysis using the classical component separation technique gave such large residuals and significant biases in the absolute results, some other techniques were applied to attempt to improve the analysis and to investigate the statistical validity of the variations discussed. These other numerical experiments will be reported on here, but are not used extensively in the interpretations, because they were only partially successful and the results were basically identical to those by the previous technique. However, as will be shown subsequently, they do indicate the difficulty in obtaining absolute values from the selected observatories and of using statistical evidence to either validate or refute the previous results.

The alternate method of computation is based on the technique developed by CAIN *et al.* (1965, 1967), by which a first-order Taylor's expansion is performed to linearize the least-squares solution for fitting any observed component of the magnetic field. Since the components chosen for this study were X , Y , and Z annual means, a non-linear solution was not actually required, but experience has shown that a second iteration very slightly improves the solution, and the same least-squares matrix must be constructed anyway to compute the standard errors and cross-correlation coefficients. The program takes into account the Earth's oblateness and the varying altitudes of the observatories. The oblateness correction was originally shown (CAIN *et al.*, 1965) to be important for obtaining a more coherent organization of the data. That is, even a remapping of the data back onto a spheroid by ignoring oblateness in both the analysis and re-computation of the field was shown to cause distortions in the components as great as 150 nT near the Earth's surface. Since the separation technique ignored oblateness, a numerical experiment was performed to test the effect on the external field computations. X , Y , and Z values of the Earth's field were first computed using the IGRF model at epoch 1965 up to $n^*=4$ on a 10° by 10° grid of latitude and longitude on the spheroidal earth. The resulting vectors were then analyzed by the field model program altered to set $F_\theta = -X$, $F_r = -Z$, and $a/r = 1$. That is, the geocentric components of field F expressed in spherical coordinates (F_θ and F_r) differ slightly in direction from the geodetically observed

X and Z , and the geocentric radius also varies with latitude from the standard value “ a ” normally taken in geomagnetic computations to be about 6,371 km (corresponding to the Earth’s mean radius). Using an $n^*=4$ solution plus the first external term, the following rms residuals were obtained:

Component	rms (nT)
X	52
Y	28
Z	75
All	55

False external coefficients of $g_1^0 = -82.2$ nT, $g_1^1 = 3.1$ nT, and $h_1^1 = -7.7$ nT were generated. The standard deviation of the coefficients was ± 1.8 nT and they had only small cross correlations with any of the internal terms.

It is thus clear that while a large spurious external component is created by neglecting oblateness, its size is still an order of magnitude smaller than that seen in the separation analysis. Since any absolute variations in dipole position over the epochs analyzed are very small, we could also assume its effect constant.

A second test was then performed to determine the extent to which spurious external components can be created by perturbations in the data, both on a uniform grid and at the positions of the selected observatories. In the latter case it was observed that adding gaussian noise whose variance was 10 nT to each component and fitting data generated from an $n^*=4$ internal model to a model that had the same internal complexity and had first-order externals, produced spurious external components no larger than 0.5 nT. These externals were statistically insignificant and equally distributed among the three coefficients.

The third experiment was to test for aliasing from neglected higher order terms by creating data on an equal grid from an internal model of terms $n^*=8$ but fitting the data only with $n^*=4$ internal and $n=1$ external coefficients. This procedure gave much larger false externals that were statistically insignificant. From this result it is clear that neglecting higher order terms is the basic cause for biases like those found in the results from the separation technique. These previous results were then duplicated (except that a consistent n^* was used) by analyzing the 1940–1973 selected data from the 34 observatories with a series limited to only $n^*=4$ internal and $n=1$ external terms. This computation gave rms residuals to the data from 750 to 813 nT, standard errors of the resulting coefficients from 50 to 200 nT, and external terms parallel to the equator (g_1^1 and h_1^1 external) as much as several hundred nanoteslas. The low-order terms were inspected and seen to exhibit year-to-year variations very similar to those produced by the separation analysis.

We then addressed the question of whether it would be possible to obtain accurate absolute values of the low-order field components using the data from 34 observatories.

The technique attempted was to use information other than that provided by the selected observatories to reduce the contamination. As mentioned earlier the spacing between the selected points (as much as 10,000 km) was so great that determinations of coefficients beyond $n^*=4$ would not be reliable. The

Table 4a. Average deviations of 1940–1973 annual means from the GSFC(12/66) field model (nT).

	<i>X</i>	<i>Y</i>	<i>Z</i>
Godhavn	265	−245	480
Sodankylä	−210	−66	−553
Yakutsk	0	−1231	192
Voyeykovo	44	73	−203
Sitka	6	9	−90
Vysokaya-Dubrava	−305	−124	−414
Meanook	88	−2	−140
Rude Skov	22	26	−2
Eskdalemuir	−18	−20	−40
Patrony	9	64	6
Fürstenfeldbruck	−16	17	15
Chambon-la-Forêt	−61	−2	133
Yuzhno Sakhalinsk	8	−166	−131
Dusheti	−236	45	−49
Tashkent	−271	37	−93
Coimbra	33	−15	−75
Fredericksburg	−3	−83	67
San Miguel	675	431	1667
Kakioka	−6	−7	−95
Tucson	−142	−61	158
Misallat	−39	65	100
Honolulu	−142	80	−416
Teoloyucan	−82	25	19
Alibag	−119	464	580
San Juan	−183	178	252
Huancayo	119	116	117
Apia	−123	224	−938
Tananarive	291	11	−385
Vassouras	54	−55	−22
Pilar	83	−3	−24
Gnangara	−78	−85	135
Hermanus	−37	9	38
Toolangi	−10	−82	84
Amberley	16	−30	5

data were corrected for the higher order terms of the field by use of the GSFC(12/66) field model (CAIN *et al.*, 1967). Since the intent was only to test the technique, no extensive comparisons were made with results from other models. Of course the GSFC model could only correct for field complexity up to its limit of $n^*=10$, while it is known (e.g. REGAN *et al.*, 1975) that there are significant components of much higher degree and order extending to the very small scale local anomalies. However, if we assume that most of this upper part of the spatial spectrum is crustal and thus constant, a further noise reduction is possible by computing the average difference over the time span of the data between the field model and each observatory component. Table 4a gives the average deviation between the model field and each component for each observatory; these deviations were used to correct each component prior to analysis. Thus the analysis was performed on the data after the values in Table 4a were subtracted. Only the $n=1$ external and $n^*=4$ internal terms were allowed to vary; whereas the $n=5$ to 10 terms of the model were used to further correct the data. The results of this analysis for the $n=1$ external terms are

Fig. 3. First-degree external coefficients obtained by one step analysis to $n^*=4$ of annual means precorrected for higher degree internal terms.

illustrated by the solid lines in Fig. 3, and a tabulation for both internal and external terms for $n=1$ is given in Table 4b. The limitation to $n=1$ in the external components is based on past experience (CAIN and DAVIS, 1973) that the higher order components are relatively small.

Table 4b. External and internal coefficients (nT) resulting from analysis of 34 station annual means after correction for GSFC(12/66) field model terms above $n=4$. Adjustments made to coefficients up to $n=4$ correcting data for first and second coefficients of GSFC(12/66) model from $n=5$ through $n=10$.

Year	External terms			Internal terms		
	g_1^0	g_1^1	h_1^1	g_1^0	g_1^1	h_1^1
1940	8.2	-8.7	.6	-30343.6	-2155.6	5751.9
1941	15.1	-7.2	-2.0	-30351.1	-2157.5	5758.8
1942	1.9	-10.0	-1.7	-30361.3	-2157.0	5765.5
1943	3.7	-9.6	-.8	-30367.8	-2155.6	5767.1
1944	-3.6	-8.5	-1.4	-30378.0	-2156.9	5772.7
1945	-3.9	-7.0	-5.3	-30387.2	-2158.3	5777.0
1946	11.5	-4.3	-7.9	-30392.1	-2159.9	5777.0
1947	7.2	-.6	-8.4	-30402.9	-2159.9	5777.6
1948	2.0	2.4	-12.7	-30410.4	-2157.9	5782.9
1949	.6	5.1	-13.4	-30415.8	-2157.6	5782.5
1950	-1.1	6.1	-20.0	-30423.7	-2156.1	5789.1
1951	.6	9.9	-22.3	-30429.1	-2158.6	5787.7
1952	-.4	10.7	-23.4	-30435.9	-2159.0	5789.4
1953	-5.4	7.7	-17.2	-30437.8	-2159.7	5788.5
1954	-9.3	7.8	-14.7	-30443.4	-2161.9	5791.4
1955	-7.2	8.6	-11.2	-30443.3	-2162.3	5791.8
1956	7.0	9.5	-10.3	-30446.6	-2162.2	5789.5
1957	10.9	9.1	-10.1	-30448.2	-2160.7	5790.7
1958	9.2	7.5	-7.3	-30449.5	-2162.4	5788.7
1959	12.0	5.8	-5.8	-30446.7	-2163.3	5788.9
1960	17.4	6.7	-4.7	-30442.0	-2166.5	5790.1
1961	2.1	4.2	-.1	-30441.9	-2170.0	5793.4
1962	-8.5	3.1	1.8	-30436.1	-2169.9	5796.6
1963	-8.2	2.0	5.2	-30429.3	-2171.0	5796.5
1964	-14.1	2.4	11.3	-30422.7	-2171.3	5793.9
1965	-18.5	2.5	12.7	-30412.0	-2170.2	5792.5
1966	-12.9	1.8	11.1	-30403.2	-2170.4	5789.7
1967	-3.3	3.1	13.8	-30394.2	-2171.7	5783.3
1968	-1.0	1.3	15.6	-30386.7	-2172.9	5778.5
1969	-4.7	-2.2	17.8	-30379.1	-2172.1	5772.2
1970	-2.4	-6.1	17.5	-30372.6	-2172.0	5765.7
1971	-4.9	-8.5	19.0	-30364.7	-2170.2	5758.7
1972	.5	-12.4	22.3	-30356.0	-2169.8	5748.2
1973	5.6	-14.6	24.9	-30347.9	-2170.0	5736.7

The validity of these results can be examined in the light of the previously mentioned tests on the synthetic data and the statistics from the analysis. Table 4c presents the residuals (averaged to 5-year intervals), standard errors, σ_p , and $t=p/\sigma_p$ (Student's "t" test); p is the magnitude of the total external field vector (CAIN *et al.*, 1965). These averages were taken from the results for each year simply for convenience in reducing the size of the table, not because there were any large deviations for a given year. Although the standard errors are taken from those determined for magnitude of the external field vector, they are comparable to those for all of the components. Values of 1.96 and 0.67 for t refer to confidence levels of 95% and 50% respectively. The cross-correlation between the parameters was also examined and found to be very low.

The second technique used to investigate the validity of these results was to take successive differences ("secular variation") of the annual means, and to separately analyze these changes. This approximation was considered appropriate since the non-linear secular change terms are insignificant over a time span as short as a year.

In making the analysis of secular change the same computer code was employed, but the $\dot{X}$, $\dot{Y}$, $\dot{Z}$ data replaced X , Y , Z , and the GSFC(12/66) coefficients $\dot{g}$, $\dot{h}$, $2\ddot{g}$, $2\ddot{h}$, 0, 0 replaced g , h , $\dot{g}$, $\dot{h}$, $\ddot{g}$, $\ddot{h}$ respectively. The solution was then for the potential $\dot{V}$ in place of V , and only the $\dot{g}$ and $\dot{h}$ terms were adjusted.

The results of this difference analysis are seen to be approximately equivalent to those from the combined analyses. This analysis was done with adjustments in the linear secular change terms up to $n^*=4$ and in the externals, (now also representing changes of field) allowing both linear and quadratic secular terms of the GSFC(12/66) model to continue in effect (no equivalent "anomaly" correction) from $n=5$ to 10. The coefficients of secular change from 1941 through 1973 are given in Table 4d. Here of course, since the previous dates referred to the middle of the year (e.g. 1940.5), the first differences are more correctly assigned to the epoch at the end (e.g. 1941.0).

Table 4c. Statistics from analyses of annual means averaged over five year intervals.

Years	Residuals (nT)				External fields	
	X	Y	Z	All	Standard error (nT)	t
1940-1944	37	43	77	55	12	1.0
1945-1949	36	36	59	45	9	1.4
1950-1954	39	23	44	37	7	2.9
1955-1959	33	16	28	28	6	2.7
1960-1964	24	21	32	26	5	2.3
1965-1969	37	36	59	46	9	2.0
1970-1973	82	59	100	82	14	1.4

The residuals of fit to the various components over the interval are similar in pattern to those of Table 4c, but the absolute values are smaller. They average from 7 to 9 nT/yr from 1941 through 1966 and rise to 12 nT/yr by 1973, the increase being due to the errors in the higher order correction terms since the field model was constructed with data only to 1965. The $\dot{Y}$ component contributes very little since its residuals are almost always below those for $\dot{X}$ and $\dot{Z}$. The residuals in $\dot{Z}$ were not commonly higher than those in $\dot{X}$, implying

Table 4d. External and internal coefficients (nT) of annual change based on annual mean differences from 34 observatories.

Year	External coefficients			Internal coefficients			
	$\dot{g}_1^0$	$\dot{g}_1^1$	$\dot{h}_1^1$	$\dot{g}_1^0$	$\dot{g}_1^1$	$\dot{h}_1^1$	$\dot{g}_2^0$
1941	8.4	1.9	-1.8	7.2	4.9	2.1	-20.2
1942	-11.7	-3.1	.9	4.4	8.0	1.8	-21.5
1943	3.2	.2	1.6	7.9	8.9	-3.0	-23.3
1944	-5.9	1.1	.1	4.3	6.3	.9	-21.8
1945	.9	1.8	-3.6	5.2	6.6	.0	-19.9
1946	17.2	2.6	-2.3	9.3	6.2	-4.6	-21.1
1947	-3.4	3.5	.1	3.6	7.8	-3.8	-18.9
1948	-4.3	2.1	-2.8	7.3	10.3	.2	-24.2
1949	.1	2.5	-.5	8.6	8.3	-4.7	-20.7
1950	-.3	.8	-6.3	6.5	9.7	2.7	-18.9
1951	2.0	3.2	-2.1	8.9	5.2	-5.6	-21.1
1952	-.3	.8	-.8	7.5	7.8	-2.5	-21.1
1953	-4.1	-3.1	6.4	12.4	7.7	-4.8	-21.9
1954	-3.3	.2	3.0	8.8	6.1	-1.6	-20.1
1955	2.5	.7	3.7	12.3	8.1	-3.6	-20.9
1956	14.6	1.2	1.7	12.8	7.8	-7.1	-19.6
1957	4.2	-.5	.3	12.5	10.0	-2.6	-20.4
1958	-1.7	-2.1	2.9	12.2	7.3	-6.0	-22.3
1959	2.9	-1.5	1.4	16.8	7.9	-3.6	-19.1
1960	5.5	1.1	1.1	18.7	5.7	-2.4	-22.9
1961	-15.9	-2.4	4.6	14.5	5.3	-.2	-22.4
1962	-10.7	-1.2	1.7	19.8	8.9	-.5	-21.3
1963	-.1	-1.1	3.1	20.7	7.9	-3.8	-21.7
1964	-6.1	.5	6.0	20.6	8.7	-6.1	-19.8
1965	-5.2	.5	1.7	25.2	10.0	-4.9	-23.2
1966	5.0	-1.0	-1.7	22.8	9.7	-6.6	-17.9
1967	8.4	1.6	1.6	23.2	7.6	-8.7	-19.7
1968	1.6	-1.9	1.3	21.3	8.1	-8.1	-19.1
1969	-4.3	-2.6	3.5	22.2	10.4	-10.9	-20.5
1970	1.4	-4.3	-.8	20.3	9.5	-9.6	-19.4
1971	-3.3	-1.6	2.3	22.3	11.6	-10.9	-19.2
1972	4.3	-4.3	2.5	22.4	10.1	-13.3	-19.0
1973	3.8	-.5	3.0	21.6	9.2	-15.6	-19.5

Adjustments made to coefficients up to $n=4$ correcting data for first and second order coefficients of GSFC(12/66) model from $n=5$ through $n=10$.

that there was no evidence for a higher observational error in this component. The estimated standard errors of the resulting external coefficients range from 1.9 to 1.4 nT/yr from 1941 to 1965 and rise to 2.7 nT/yr by 1973.

In reviewing these results to evaluate the amount of confidence we may have in either the coefficients of the absolute determination or the differences, we are struck by the fact that the statistics themselves are insufficient. That is, even though the standard errors of the absolute determinations using the combined method on the reduced data are satisfyingly low, the resulting internal coefficients differ significantly from those determined from more complete data sets. For example, a comparison of the low-order terms for 1965 from Table 4b with those of two other models from 1975 is as follows:

Internal Coefficient	Model (data in nT)		
	Table 4b	GSFC(12/66)	IGS75
g_1^0	— 30412	— 30333	— 30337
g_1^1	— 2170	— 2117	— 2117
h_1^1	5792	5759	5759

Since the GSFC and IGS75 (BARRACLOUGH *et al.*, 1975) models were derived without regard to any possible external field, one might ask whether they might be in error instead of the values from Table 4b. A numerical experiment was performed to determine the effect of a known external field on such determinations. The IGRF model (CAIN and CAIN, 1971) was used to create a uniform grid of data to which was then added an external field parallel to the dipole of 50 nT. The resulting vectors were then analyzed up to $n^*=4$ without allowing the external coefficients to vary. The largest change was to g_1^0 and was only 7.6 nT. The other coefficients were altered by 1 nT or less. It is concluded that although the statistics indicate that the coefficients from the combined analysis of corrected data (both internal and external) are in error only of the order of 10 nT (Table 4c) it is likely that they could be biased as much as 70 nT from the true values.

A similar comparison is tabulated for the secular change results as follows (for epoch 1970 in nT/yr):

Internal Coefficient	Model			
	Table 4d	GSFC(12/66)	AWC75 [†]	IGS75
g_1^0	20	13	25	23
g_1^1	10	11	10	10
h_1^1	— 10	— 0	— 10	— 10
g_2^0	— 19	— 26	— 25	— 24

[†] PEDDIE and FABIANO (1976).

This table appears to indicate that the 2–3 nT/yr error estimates on the secular change coefficients are not unreasonable when the variability of the values in Table 4d is taken into account. That is, one cannot argue that these biases are large compared to those from other models, as was the case for the absolute determinations. It would appear that standard errors of the order of 2 nT/yr in the year-to-year changes are not unreasonable.

5. Double Peaks in g_1^{0e} Variations during a Solar Cycle

From the time variations in the dipole components shown in Fig. 2a, general trends, approximated by a parabolic equation for g_1^{0e} and a third degree polynomial for g_1^{0i} , were subtracted, and the results are shown in Fig. 4. In the g_1^{0e} variation a peak appears in 1960, and then g_1^{0e} decreases rapidly, reaching a minimum in 1965. This peak-to-trough variation amounts to about 47 nT. Besides this large amplitude variation, small fluctuations with peaks at intervals of several years are noticeable. Regarding the plot of g_1^{0i} , a phenomenon having a period of about 20 years is so predominant that any changes with shorter periods are hard to see in this diagram. Then a high-pass filter was used to extract shorter period variations, and the results are illustrated in Fig. 5.

There is seen in Fig. 4 a relatively large variation in the internal parts of the equatorial dipole component. The amplitudes of g_1^{1i} and h_1^{1i} are nearly the same as those of g_1^{1e} and h_1^{1e} . From induction theory such a large variation in the internal terms is unlikely to be induced by the change in the corresponding external component. The maximum amount theoretically expected for the equatorial dipole on a homogeneous perfect conductor model is 50 percent of the inducing field (see, e.g. CHAPMAN and BARTELS, 1940, pp. 732–749). If the fluctuation of the equatorial dipole shown in Fig. 4 is real, and if this is due to induction, we must consider such mechanisms as lateral inhomogeneities within the earth. Another possible interpretation is to regard this relatively large variation as a phenomenon originating from inside the core. As will be discussed later, variations of internal origin of the axial dipole for periods less than 10 years can be regarded mostly as effects of the field induced by the external variation. Therefore, at the present stage it seems rather difficult to ascribe this internal equatorial dipole variation to a core origin.

Residuals after removing the general trend were passed through a numerical high-pass filter with its cut off period around 10 years. The characteristics of this 11 point-filter were such that periods over 20 years were reduced to less than 2% and those of 15 years 24%, whereas periods under 10 years remained between 92 and 110% of the original amplitude. Variations symmetric to the end points were added before and after the data window. The high-passed

Fig. 4. Time variations in the dipole terms after trend removal.

results for g_1^{0e} and g_1^{0i} are shown in Fig. 5 in comparison with time variations in some other phenomena. From the top down, the figure shows the relative sunspot numbers, the annual number of days when A_p indices exceeded 60 (MEYERS and ALLEN, 1977), and the annual Dst values (SUGIURA and POROS, 1977).

One of the remarkable features revealed by a straightforward comparison

Fig. 5. Comparison of variations in the axial dipole field with those of other geomagnetic indices. From the top down are shown the annual values of the relative sunspot number, number of days when A_p index exceeded 60, annual Dst values, and the axial dipole components of external and the internal origin after being passed through a high-pass filter.

of g_1^{0e} with the relative sunspot number in Fig. 5 is the rather poor correspondence of their year-to-year variations, although minima of the sunspot number coincide with those of g_1^{0e} . A peak in g_1^{0e} appears in 1946, preceeding the maximum of the sunspot number. The next peak in g_1^{0e} corresponds to a decreasing period of the sunspot number. In a general sense, g_1^{0e} changes with the same cycle as the sunspot numbers, but two peaks appear in the change of g_1^{0e} during each solar cycle.

This phenomenon is also seen in other geomagnetic parameters. Ap indices, shown in Fig. 5, have two peaks in a solar cycle. Dst values show the same tendency. Among these geomagnetic quantities almost complete correspondence exists from year to year. Correlation coefficients are very high between these quantities, as will be discussed in the following section. These results are almost identical with those obtained by ALLDREDGE *et al.* (1978) for recent observatory data.

Another aspect we should note here is a small variation in g_1^{0i} . No significant variation is seen during the period from 1947 to 1964.

6. The External Field along the Geomagnetic Axis

Variations of the external coefficients g_1^{0e} , g_1^{1e} , and h_1^{1e} (Fig. 4) were passed through a numerical band pass filter that has a frequency characteristic with its peak around a 10-year period. The results are shown in Fig. 6 by open circles. It can be seen that changes in the equatorial components g_1^{1e} and h_1^{1e} closely correspond to those of the axial component g_1^{0e} . The quantity g_1^{1e} changes in phase with g_1^{0e} , whereas h_1^{1e} is completely out of phase. If the ratios of the equatorial coefficients (g_1^{1e} and h_1^{1e}) to the axial one (g_1^{0e}) are the same as the ratios of the main field that specify the inclined angle of the geomagnetic axis, the direction of the variation exactly parallels the inclined axis. This relationship is partly satisfied in the present case, i.e. regarding the phase. When g_1^{0e} changes in a negative direction, g_1^{1e} changes in the same direction but h_1^{1e} changes in a positive direction. On the other hand, the main field coefficients g_1^0 and g_1^1 are both negative, while h_1^1 is positive. Therefore the variations represented by the first degree terms in Fig. 4 must be approximately aligned in a direction parallel to the inclined geomagnetic axis.

The coordinate system was thus rotated from the geographic coordinates to those in which one of the axis coincides with that of the internal dipole. Assuming a boreal dipole position of 79°N and 69°W , the coefficients may be transformed by the expressions

$$G_1^0 = g_1^0 \cos 11^\circ + Q \sin 11^\circ \quad \text{where} \quad Q = g_1^1 \cos 69^\circ - h_1^1 \sin 69^\circ,$$

Fig. 6. The values g_1^0 , g_1^1 , h_1^1 are passed through a numerical band-pass filter with a peak around a 10-year period. Open circles are the filtered results. + signs show variations for the new coordinates, one of whose axis is taken along the geomagnetic axis. Reduction of the amplitude by the rotation of the coordinates is seen for h_1^1 and g_1^1 .

$$G_1^1 = Q \cos 11^\circ - g_1^0 \sin 11^\circ, \quad \text{and}$$

$$H_1^1 = h_1^1 \cos 69^\circ + g_1^1 \sin 69^\circ.$$

Since the secular changes are all linear, the above expressions may also be applied to the $\dot{g}$ and $\dot{h}$ terms.

The variations obtained by this transformation are shown by the dotted curves in Fig. 6. The variation along the geomagnetic axis is the largest and preserves the main feature of g_1^{0e} . Reduction of the amplitudes of the orthogonal components is obvious particularly in h_1^{1e} . Our previous qualitative conclusion that the solar cycle variation is approximately aligned along the geomagnetic axis rather than along the geographic axis is thus further strengthened.

The results obtained by the combined analyses in Tables 4b and 4d were also transformed into the magnetic coordinates by the above rotation expressions and are illustrated by the dotted curves in Fig. 3. Then, the relations of the rotated and non rotated components to other geophysical indices were examined. The first-order externals from Fig. 6 were correlated with the number of days per year whose A_p index exceeded a specified level (MEYERS and ALLEN, 1977), with the Wolf sunspot number, and with annual average Dst values. These results are illustrated in Table 4e. It is clear from this table that variation of

Table 4e. Statistical comparisons between external field results and geophysical parameters (SPSS package used, NIE *et al*, 1975).

		Pearson correlation coefficients								
		Dst	$Ap > 40$	Sunspots	g_1^0	G_1^0	g_1^1	G_1^1	h_1^1	H_1^1
Percent probability of chance correlation (0.1% is upper bound)	Dst	—	-.82	-.75	-.95	-.92	-.13	-.23	.41	.22
	$Ap > 40$	.1%	—	.63	.93	.94	.29	.42	-.58	-.12
	Sunspots	.1%	.3%	—	.75	.75	.40	.44	-.55	.10
	g_1^0	.1%	.1%	.1%	—	.98	.17	.31	-.50	-.24
	G_1^0	.1%	.1%	.1%	.1%	—	.35	.49	-.66	-.10
	g_1^1	31. %	13. %	6. %	25. %	8. %	—	.94	-.86	-.83
	G_1^1	18. %	5. %	4. %	11. %	2. %	.1%	—	-.98	.60
	h_1^1	5. %	.7%	1. %	2. %	.2%	.1%	.1%	—	-.44
	H_1^1	20. %	32. %	34. %	18. %	36. %	.1%	.5%	4. %	—

the external values obtained are the results of magnetic activity as manifested by the changes in average Dst*. It is equally clear that the rotation of coordinates to the dipole completely eliminates the weak correlation of the h_1^1 coefficient with Dst, while neither g_1^1 nor G_1^1 are significantly correlated. A similar correlation was done between the secular change coefficients and the successive differences of Dst and gave the following results:

Correlation coefficient with Dst		Probability of chance correlation (.1% is upper bound)
$\dot{g}_1^0$	— .98	.1%
$\dot{G}_1^0$	— .90	.1%
$\dot{g}_1^1$	— .11	35%
$\dot{G}_1^1$	— .07	40%
$\dot{h}_1^1$	.67	.2%
$\dot{H}_1^1$	.15	28%

Although the correlation with the tilted dipole is not quite as high as for the “absolute” analysis, both of the terms perpendicular to the dipole are satisfyingly uncorrelated.

Both of these results would confirm our previous conclusions that the year-to-year external variations are indeed real. However, the biases noted earlier for the internal absolute values would indicate the possibility also of similar biases (as large as a few tens of nT) in the external values as well.

The mean G_1^0 for 1957–1973 is -3 nT, whereas that for $Dst = -14$ nT, implying that G_1^0 is -17 nT when $Dst = 0^{**}$. The results given by CAIN (1966) on

* For background information on the Pearson correlation coefficients see NIE *et al.* (1975, p. 280).

** Note that G_1^0 is positive when the external field is downward on the boreal (north) pole whereas Dst is positive when the field is northward.

the average external field for the interval 1945–1964 can be rotated to give a value for G_1^0 of -21 nT, implying that the quiet day value is fairly small. However, M. Sugiura (private communication, 1977) believes on the basis of analysis of satellite data that $Dst=0$ when $G_1^0=+40$ nT. Also, if the all-day curves of FORBUSH (1964) are correct, then the absolute G_1^0 determined herein would be some 40 nT too large. It would appear that further work is indicated before these discrepancies can be resolved.

7. Double Solar Cycle

It is still a point of argument whether or not the “22-year” variation exists in the geomagnetic field in association with the polarity change in the sunspot

Fig. 7. Variations in the dipole (g_1^{0e} , g_1^{0i}) when they are subjected to a numerical band pass filter with a peak around 20 years.

field. CURRIE (1973) reported a 22-year periodicity in the observatory data, but COURTILLOT and LEMOUËL (1976) cast a serious doubt about its significance, and ALLDREDGE (1977) considered that variations having a period of about 20 years were of internal origin.

To the axial dipole components g_1^{0e} and g_1^{0i} in Fig. 4, we applied a numerical band pass filter with its peak at about a 20-year period. The results are shown in Fig. 7. In the external variation g_1^{0e} , the interval from one peak to another is 18 years. This is not the period of the pure double solar cycle. A large variation amounting to 33 nT from peak to trough is seen between 1957 and 1965. This trend is obviously recognizable in the unfiltered variation in Figs. 3 and 4. The effects of the solar cycle on the geomagnetic field are not identical from one cycle to another. This could be one cause of long-term variations in the external field in Fig. 7, though it is uncertain if this is the only cause or not.

In the internal variation g_1^{0i} , a more periodic variation amounting to 21 nT from peak to trough was obtained. The period is 19 years. The ratio of the internal to the external coefficient becomes larger than 0.6. The cause of such a high ratio is unknown.

Although the present analysis does not succeed in settling debate on the double solar cycle variation, the possibility cannot be ignored that considerable contribution to this period range comes from inside the core. A further analysis will be needed to derive conclusive results.

8. Solar Cycle Variation in the First-Degree Components since 1900

The axial dipole component in the analysis of data from 21 observatories that cover the period from 1900 to 1973 (YUKUTAKE, 1973) was examined to extract the internal parts of the solar cycle variation. A few of these observatories were not included in the previous set of 34 observatories as shown in Fig. 1.

The quality of data is poorer in the older records. Perhaps because of the difficulty in accurate maintenance of base-line values, the amplitude of the solar cycle variation in the dipole coefficient obtained from the Z component is unreasonably large for the first few decades of this century, compared with that from the X and Y components, though the phases of the variation look real. Therefore, a weighting function to reduce the large amplitude was adopted.

The dipole components obtained separately from (X, Y) and Z were first passed through a numerical high-pass filter that has its cut off frequency around a 10-year period. The variations obtained from the Z components were unreasonably large for the earlier part of the data set, whereas those from (X, Y) were nearly stationary for the whole period. Since it appeared that the early

Z variation increase was merely due to the higher inaccuracies of measurement prior to 1940, a weighting function was constructed to minimize its effect. Then the internal field variations were separated from the external changes. The weighting function adopted was of the form $w_i = 1 - at_i'^2$, where $t_i' = t_i - 1973$, since the "noise" appeared to increase parabolically to 1900. The a was tentatively determined to minimize the sum square of either (1) the external variation, or (2) the internal variation. In both cases, however, use of the weighting function gave such a ridiculous result that w_i became negative at the beginning of the data series (though the results were nearly the same except for the epochs when the weight was negative). Since such a negative weight is physically meaningless, a was simply determined so as to make w_i zero at 1900. The results did not differ very much by these three different treatments, except for the internal variations before 1910.

Fig. 8. Solar cycle variations in the axial dipole component from 1900 to 1973. From the top down are shown relative sunspot numbers, the external variations (g_1^{0e}) obtained from 34 observatory data, the external variations (g_1^{0e}) obtained from 21 observatory data, and the internal variations (g_1^{0i}) in the axial dipole component.

Table 5. External and internal coefficients (nT) of the first order terms for the period from 1900 to 1973 after being passed through a high pass filter.

Year	External	Internal	Year	External	Internal
1900	11.6	5.8	1937	6.4	-7.8
1901	-2.8	-1.2	1938	7.0	-1.0
1902	-5.1	-2.3	1939	5.8	-2.7
1903	-4.0	-3.6	1940	8.4	.9
1904	-1.4	-4.0	1941	10.0	3.8
1905	-.2	-2.9	1942	-11.0	-2.3
1906	-2.4	-5.3	1943	-7.3	3.4
1907	-2.0	.3	1944	-9.5	1.7
1908	-.7	1.1	1945	-7.8	-.4
1909	2.2	3.5	1946	11.7	-2.0
1910	6.4	3.9	1947	2.2	-2.3
1911	2.3	3.9	1948	3.7	-3.0
1912	-6.8	.9	1949	-.1	-.5
1913	-9.6	2.3	1950	-.9	2.2
1914	-5.8	-.5	1951	1.3	7.1
1915	6.5	-2.9	1952	1.2	5.8
1916	9.8	-1.6	1953	-4.2	3.8
1917	7.2	-.4	1954	-9.7	.6
1918	6.1	2.3	1955	-6.4	-2.2
1919	7.7	2.8	1956	.4	-1.3
1920	-8.4	6.1	1957	5.4	-1.5
1921	-4.6	2.1	1958	4.0	-.6
1922	-.3	-.8	1959	7.0	1.2
1923	-5.0	-3.1	1960	14.7	2.4
1924	-7.2	-2.4	1961	.5	-.3
1925	-.4	-2.1	1962	-5.7	2.0
1926	4.1	2.4	1963	-3.7	1.0
1927	-2.5	.4	1964	-4.8	-.2
1928	1.9	-2.2	1965	-8.6	-.4
1929	3.3	1.5	1966	-1.4	-.4
1930	10.0	-1.2	1967	8.4	-.8
1931	4.5	-7.6	1968	1.6	-.8
1932	2.0	1.9	1969	.4	-.7
1933	-4.0	4.2	1970	-6.7	2.7
1934	-10.4	2.9	1971	-5.9	.9
1935	-11.2	1.6	1972	-1.1	-2.1
1936	-6.4	-6.2	1973	11.6	-6.8

The external (g_1^{0e}) and the internal (g_1^{0i}) components thus obtained are plotted in Fig. 8 and listed in Table 5. The external coefficients obtained for the latest data set, i.e. 34 observatory data from 1940 to 1973, are also shown for comparison. Except for peaks in 1946 and 1967 for the 21-observatory data set, the variations in g_1^{0e} are almost identical between the two data sets.

It is evident that the variation in g_1^{0e} is not parallel with that of the relative sunspot number. Almost without exception, double peaks appear in g_1^{0e} during one solar cycle. Mostly, the first peak appears either at the same time as the maximum of the sunspot number or one year before it. The second peak is at the declining stage of solar activity. This relation can be seen more clearly in Fig. 9, where g_1^{0e} , g_1^{0i} , and the annual values of the relative sunspot number were picked up at an interval of 10 years for 50-year period and were averaged. It is clear from this figure that, on an average, epochs of minimum g_1^{0e} correspond to those of the sunspot number.

The variation in the internal coefficient g_1^{0i} looks very noisy in Fig. 8. However, when the annual values are superposed as in Fig. 9, at the interval

Fig. 9. Variations in the relative sunspot numbers g_1^{0e} and g_1^{0i} averaged at 10-year intervals over 50 years. For example, the points for 1905–1955 are averages of 1905, 1915, 1925, 1935, 1945, and 1955.

Fig. 10. Variations in g_1^{0e} and g_1^{0i} . The top two curves are the variations in g_1^{0e} and g_1^{0i} , which includes short period phenomena. The bottom two curves show the same variations after the band pass filter has been applied.

of 10 years, the shape of g_1^{0i} becomes similar to that of g_1^{0e} with a time lag of about 3 years. In order to see the relationship between the variations in g_1^{0i} and g_1^{0e} they were passed through a band pass filter whose frequency characteristic has its peak around a 10-year period. The ends of the data interval were treated generally as described for the case of high-pass filtering. The variations before and after the application of the band pass filter are shown in Fig. 10. High-frequency phenomena have been well smoothed out in the bottom two curves. If the peak-to-peak or trough-to-trough correspondence between the internal and external curves is examined, a good correlation may be noticed with a slight time lag. The variation in g_1^{0i} is, on an average, about 3 years behind g_1^{0e} . This is equivalent to the result obtained from Fig. 9. The time lag is so constant during the whole period that, in spite of its small amplitude, the variation in g_1^{0i} may be regarded as a real change which is probably induced by the g_1^{0e} variation.

9. Fourier Analysis of the Variation in the First-Order Components

In order to see the quantitative relationship between external and internal parts of the solar cycle variations, a Fourier analysis was made of the data from 1900 to 1973 shown in the two top curves of Fig. 10, which had been passed

through the high-pass filter with a cut off period around 10 years. The cosine and sine terms were computed respectively by the following equations.

$$A(\omega) = \frac{2}{73} \sum_{n=0}^{73} g(t) \cos(n\omega\Delta t),$$

and

$$B(\omega) = \frac{2}{73} \sum_{n=0}^{73} g(t) \sin(n\omega\Delta t),$$

where $g(t)$ is either g_1^{0i} or g_1^{0e} . The ω is the angular frequency, and $\Delta t=1$ year in the present case.

The amplitude $C(\omega)$ was computed for $A(\omega)$ and $B(\omega)$, and is shown in Fig. 11. A peak for the 10-year period is remarkable in the external field g_1^{0e} . The amplitude amounts to 5.5 nT. Relating to the solar cycle variation, a peak around the 5.1-year period is also conspicuous. The internal field variation also has its peak at 10.5 years, but the 5-year period phenomenon is hardly recognizable.

It should be noted that a significant amount of energy is contained in a 7-year period band of the external field variation. However, the cause of this phenomenon is completely unknown.

In order to avoid the effect of finite values at the beginning and the end

Fig. 11. The results of Fourier analysis for the high passed data. Solid circles are the results for g_1^{0e} , and the open circles the results for g_1^{0i} . Periods in years are given for the conspicuous peaks.

Table 6. Ten year components of the external (g_1^{0e}) and the internal field variation (g_1^{0i}) obtained by a Fourier transformation.

Time (years)	Unweighted				Sine-weighted				i/e	$\iota-\varepsilon$
	g_1^{0e}		g_1^{0i}		g_1^{0e}		g_1^{0i}			
	C	Theta	C	Theta	C	Theta	C	Theta		
10.53	4.63	−118.6	2.22	−24.0	3.75	−117.0	1.37	−16.8	0.367	100.3
10.00	5.41	−53.1	1.85	31.7	3.84	−48.7	1.23	52.7	0.317	101.4
9.52	4.43	16.6	0.73	107.0	3.21	23.6	0.84	138.1	0.263	114.5
Mean									0.316	105.4

C is the amplitude (nT), and Theta is the phase angle in degrees.

point of the time series, a sine function was adopted as a weighting factor to make both ends zero. Then Fourier analyses were repeated with the weighted data. The amplitudes and the phases of the 10-year period bands are listed in Table 6 along with the results for the unweighted data. The weighting process reduces the original amplitude, but yields more stable results, particularly for the phase angles. Accordingly, for calculating the ratio of the internal to the external field variation, we finally adopted the results obtained from the sine-weighted data. The amplitude ratios and phase differences were averaged over the 10-year band, the mean period being 10.5 years, and the modulus (i/e) was found to be 0.32, with a phase difference ($\epsilon - \varepsilon$) of 105° . The phase difference corresponds exactly to a 3-year lag of g_1^{0i} behind g_1^{0e} , which was pointed out in the previous section.

10. Discussion and Conclusion

10.1 The nature of the geomagnetic variation associated with the solar activity

One of the interesting features revealed by this investigation is the existence of double peaks during one solar cycle in the time variation of g_1^{0e} . One peak nearly coincides with the maximum of sunspot number, and the other appears at the declining stage of the solar activity. Recently, a few papers have appeared reporting the double peaks in the geomagnetic activity. As was discussed in a previous section, MEYERS and ALLEN (1977) provided a frequency distribution of magnetically disturbed days for the period from 1930 to 1975 in terms of Ap index that clearly indicated two peaks in a solar cycle. Examining the Kp index, OL' (1971) noticed that two maxima existed in the magnetic disturbances. He related the first maximum to the frequency of nonrecurrent magnetic storms and the second, which appeared a few years preceding the sunspot minimum, to recurrent disturbances. SUTORIK and CRUICKSHANK (1977) examined the Ap

index for the period from 1868 to 1976 and confirmed that the recurrent geomagnetic activity was seen at the declining stage of the solar activity. From examination of the solar magnetic field S. Matsushita (private communication, 1977) suggests the existence of the double peak phenomenon in statistics of the number of days when the away-from-the-sun sector structure was observed on the earth. Certain features of geomagnetic disturbances are likely to be closely related to solar-wind streams from the coronal holes of the sun and to interplanetary fields (SHEELEY *et al.*, 1976; SVALGAARD, 1977). Investigation of the physical processes that produce magnetic disturbance is beyond the scope of the present work. One thing, however, we should note here is that the double peak is not a phenomenon that can only be seen in the statistics of the magnetic disturbances, but that it actually exists in the variation in the axial dipole component.

Annual mean values of the geomagnetic components on which our analyses are based are computed from daily means of each component. Magnetic variations with periods shorter than a year contained in the annual mean values are suspected to contaminate the results. There seem to be two possibilities of contamination. One is the effect of disturbances on the mean values. For the investigation of the long-term variation in the geomagnetic field, one of the greatest concerns is whether shorter period disturbances superposed on it may conceal the long-term trend. The second problem is the effect of diurnal variations. The amplitude of Sq changes with solar activity (MATSUSHITA and MAEDA, 1965; MALIN *et al.*, 1975). If the midnight values are assumed to represent the undisturbed state of the geomagnetic field, the diurnal variation will lower the daily means from the nighttime level, and the amplitude variation in Sq due to solar activity can be a cause of solar cycle variation in the annual mean values.

Regarding the first question, no detailed investigation has been made as yet. One way of solving this question is to compare the variations obtained on the quiet-day data with those on the all-day data. A preliminary check of the annual mean values of the three components at Tucson (YUKUTAKE, 1965) suggested little difference between them. Investigation now under way for several observatory data indicates that, though there exists significant difference, the solar cycle variation in the quiet-day annual mean is still large.

The change in the Sq amplitude has been discussed by MALIN *et al.* (1975), and HARWOOD and MALIN (1977), in which the necessity of correction of the amplitude variation in Sq was pointed out. Even if the correction is made, a major part of the solar cycle variation is likely to be preserved in the long period variations of the adjusted values.

Differences between the annual means of the nighttime values and those

of the all-day hourly values were examined on the data obtained at the Kakioka Magnetic Observatory. It has been revealed that the differences range from 1 to 3 nT around the mean of 1.4 nT with a standard deviation of 0.63 nT for the horizontal component and from 4 to 8 nT around the mean of 5.3 nT with a standard deviation 1.07 nT for the vertical component during the period from January 1960 to December 1976, which covers the largest peak-to-trough variation in the external axial component g_1^{0e} . The difference in the behavior of the all-day value and the nighttime value is considered to be contained in the standard deviations rather than in the mean value of the difference itself. The standard deviations estimated above are small compared with the peak-to-trough variation in g_1^{0e} computed on the all-day data, about 50 nT, which yields about 40 nT variation of the horizontal component at Kakioka. Therefore, it may be said that the nighttime value is also subjected to the solar cycle variations. Although further investigation is needed on the problem of the contamination of short period phenomena into the annual mean values, most of the solar cycle variations in the annual mean values may be expected to represent the long-term change in the geomagnetic field.

10.2 *The relationship between the external and the internal terms*

The ratio of the internal to the external coefficients provides important information regarding the distribution of the electrical conductivity in the Earth, provided that the internal term is entirely due to the field induced by the external field variations. Phenomena with periods as long as 10 years supply particularly important information about the deeper part of the mantle. The ratios of g_1^{0i}/g_1^{0e} obtained by other analyses for the solar cycle variation of "11 years" are listed in Table 7 together with the results of the present work. COURTILLOT and LEMOUËL (1976) gave only the 11-year harmonics of the X and Z components. The ratio i/e given in the table was calculated from these values. The results seem to be quite consistent with each other. The ratios range from 0.26 to 0.34, except for those of YUKUTAKE (1965) and HARWOOD and MALIN (1977) who give the low value as 0.16 and 0.12 respectively. The phase angle does not

Table 7. The ratios of the internal to external parts of the "11-year" period variation.

	Modulus (i/e)	Phase (i/e)
YUKUTAKE (1965)	0.16 ± 0.08	96°
COURTILLOT and LEMOUËL (1976)	0.26	100°
ALLDREDGE (1976)	0.34 ± 0.13	30°
HARWOOD and MALIN (1977)	0.12 ± 0.07	$8 \pm 30^\circ$
ISIKARA (1977)	0.28	77°
The present work	0.32	105°

scatter very much either; it ranges from 77° to 105° if ALLDREDGE's (1976) and HARWOOD and MALIN's (1977) figures are discarded.

It is necessary to consider again the effect of Sq . If the effect of Sq on the all-day mean is not ignorable, the ratio would be affected. HARWOOD and MALIN (1977) made an adjustment of the all-day means to the midnight values. The adjustment reduced the amplitude of the external and the internal coefficients by about 20%. However, the ratio of the internal to the external by this adjustment was changed only from 0.13 to 0.12 in their case. Since the specific process of their adjustment is not very clear to us, we made no such adjustment.

In the present work, the analyses for the recent data set (1940–1973), which is considered more accurate than the older one, gave variations in the internal coefficient that were too small to permit us to derive a reliable ratio between the internal and external fields. It is not known whether this shortcoming is due to the errors that might be incidently included in the data or to some other cause. In spite of the fair convergence of the various analyses in Table 7, they cannot yet be said to be conclusive. Only the accumulation of accurate data will be able to clarify the ambiguity.

We are grateful to M. Sugiura for providing his numerical filters. We have benefited by discussion with L.R. Alldredge, J.H. Allen, and S. Matsushita. Discussion with K. Svendsen on the observatory data was useful. We would like to express our thanks to C. Kisslinger for his interest and support. This work would not have been possible without the assistance of the National Geophysical and Solar-Terrestrial Data Center of the National Oceanographic and Atmospheric Administration, Boulder, Colorado, and the assistance of the Office of Applications of the National Aeronautics and Space Administration under Grant NAS-W-2129.

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