North-South Anisotropy and Radial Density Gradient of Galactic Cosmic Rays

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The cosmic ray north-south anisotropy arising from the heliocentric radial density gradient of cosmic rays and the interplanetary magnetic field has been studied using data for a wide range of rigidity (15–210 GV in median primary rigidity) during the period 1969–1973. Two components of the anisotropy; the north-south asymmetry and the associated sidereal diurnal variation have been analyzed to reveal a three-dimensional structure of the anisotropy in space. By means of the least-squares method and using improved coupling coefficients, the direction and certain other parameters of the anisotropy have been determined.

Definite evidence has been obtained for the existence of anisotropic flow perpendicular to the ecliptic plane. The magnitude of the flow is found to be $0.081 \pm 0.021\%$ and $0.072 \pm 0.018\%$ at 10 GV for the periods 1969–1970 and 1971–1973, respectively. It is also found that its rigidity spectrum is slightly rigidity dependent and has an upper limiting rigidity of 200 GV or more. Based on the derived parameters of the anisotropy, the heliocentric radial density gradient for high rigidity cosmic rays has been estimated as a function of rigidity for 1969–1973 as $(3.0 \pm 1.1) \times (P/10)^{-(0.7 \pm 0.2)}\%$/AU for $P \leq 200$ GV ($P$ is rigidity in GV). The diffusion coefficient is also derived, which seems to be consistent with those so far determined.

1. Introduction

During the last decade, a great many investigations have been made of the cosmic ray north-south anisotropy in connection with the direction sense (towards or away from the sun) of the sector structure of the interplanetary magnetic field (IMF) (Swinson, 1969, 1971; Bercovitch, 1970, 1971; Iucci and Storini 1972; Kondo et al., 1975; Mori et al., 1976). It has also been demonstrated that on the basis of the north-south anisotropy it is possible to determine the radial density gradient of cosmic ray particles in the vicinity of the earth.

In 1969, Swinson examined the relationship between the anisotropy and the IMF using data from underground meson telescopes, and found for the first time that the
sidereal diurnal variation might originate from the modulation effects due to the IMF polarity. He suggested that the field dependent sidereal diurnal variation might be caused by one of the components (perpendicular to the earth’s rotational axis) of a north-south anisotropy, and that the anisotropy could be due to the cosmic ray flow caused by a positive heliocentric radial density gradient \( (\mathbf{p}_n) \) of cosmic rays and the IMF \( (\mathbf{B}) \), as expressed by the vector product, \( \mathbf{B} \times \mathbf{p}_n \). Since the average direction of \( \mathbf{B} \) is known to lie in the ecliptic plane and \( \mathbf{p}_n \) also lies in the ecliptic plane, the direction of this vector product (or the north-south anisotropy) is expected to be perpendicular to the ecliptic plane and to reverse direction with reversal of the magnetic field direction.

Bercovitch (1970, 1971) studied the field dependent north-south asymmetry of cosmic ray intensity (hereafter abbreviated as N-S asymmetry), based on Gleeson’s formulations. Using intensity data from neutron monitors in the northern and the southern hemispheres, he suggested that the N-S asymmetry could be caused by another component (parallel to the earth’s rotational axis) of the north-south anisotropy. Based on the estimated value of N-S asymmetry, he derived the heliocentric radial density gradient of cosmic rays in free space. Uucci and Storini (1972) also performed a similar analysis using data from a pair of polar neutron monitors (Alert and McMurdo), and showed that the gradient has a positive solar activity dependence.

Kondo et al. (1975) pointed out a more distinct evidence of the N-S asymmetry, using intensity data from meson telescopes at Nagoya and Mt. Norikura. They analyzed an index called G-component which was defined earlier by Nagashima et al. (1972) and which represents an intensity difference between inclined telescopes at one station. And they found that it was a good index in representing the N-S asymmetry in cosmic ray intensity. Recently, Mori and Nagashima (1979) demonstrated a significant IMF polarity dependence of GG-component (described later in some detail) not only on the average but also on a daily basis. Further they showed a fairly good correlation (~75% in agreement) of the prevailing IMF polarity with this index for 1971–1973 and also 1974–1975 (Mori et al., 1980).

It seems, however, that some important problems still remain undetermined. For instance, the above analyses have only been concerned with a single component of the anisotropy either the N-S asymmetry or the sidereal diurnal variation. The information concerning the three-dimensional nature of the anisotropy has not yet been established. Another point is that little work has been performed to determine the spectral dependence of the north-south anisotropy. Earlier investigations on the rigidity spectrum of the anisotropy are inconclusive, since they were generally restricted to the nucleonic data representing relatively low primary rigidities (~15 GV).

In the present paper, the three-dimensional direction as well as the spectral dependence of the anisotropy will be evaluated using data in a wide range of rigidities (Yasue et al., 1977) from neutron monitors, meson telescopes at the surface and underground. In the analysis, improved coupling coefficients will be utilized for
meson components (Fujimoto et al., 1977), of which calculations have been made based on precise response functions given by Murakami et al. (1976). On the basis of the magnitude and the spectral dependence of the north-south anisotropy, the heliocentric radial density gradient in free space will be estimated as a function of rigidity. Also the diffusion coefficient for high rigidity cosmic rays will be evaluated.

2. Formulation of the Anisotropy

In the present work, the analytical methods given by Nagashima (1971) and Nagashima et al. (1972) have been utilized. According to their formulation, the variational intensity distribution of the anisotropy in free space is expressed by the following axis-symmetric distribution,

\[ \frac{\delta J(P, \zeta)}{J(P)} = \eta_1 g(P) \cos \zeta \]  

(1)

where \( P \) is the particle rigidity in GV and \( \zeta \) denotes the angle of the incident cosmic ray direction with respect to the direction of anisotropy. \( \eta_1 \) represents the magnitude in free space and \( g(P) \) the differential rigidity spectrum of the anisotropy. The direction of anisotropy is specified by the co-declination, \( \theta_R \), and the right ascension, \( \alpha_R \), in the equatorial coordinate system. It is noted that no restriction is made on the direction of anisotropy.

A vector \( R \) is then introduced (see Fig. 1), the components of which are connected by the following expression with the parameters \( \eta_1, \theta_R, \) and \( \alpha_R \) which define the anisotropy,

\[
\begin{align*}
R_x &= \eta_1 \sin \theta_R \cos \alpha_R \\
R_y &= \eta_1 \sin \theta_R \sin \alpha_R \\
R_z &= \eta_1 \cos \theta_R
\end{align*}
\]  

(2)

These components of \( R \) are called space harmonic components. In our case, Eq. (2) indicates that \( R_z \) represents the component parallel to the earth’s rotational axis, and \( R_x \) and \( R_y \) are the components perpendicular to that axis. Thus the vector \( R \) can be considered to represent the anisotropy vector. Hereafter the north-south anisotropy is defined as one half of the difference between anisotropies for toward and away polarities, since the anisotropy reverses direction with reversal of the IMF polarity.

The anisotropy produces the following daily variation \( D(t) \) observed on the earth,

\[ D(t) = D_z \cos 2\pi t/24 + D_y \sin 2\pi t/24 + D_x \]  

(3)

where \( t \) is chosen to be sidereal time in hours, since the constant direction in the equatorial coordinate, e.g., the direction of the ecliptic north pole is observed to be stationary only when sidereal time is employed. \( D_z \) and \( D_y \) therefore represent the component of the sidereal diurnal variation and \( D_z \) indicates the N-S asymmetry. Figure 1
shows the relation between the anisotropy vector $\mathbf{R}$ and the three components $D_x$, $D_y$, and $D_z$ of the daily variation vector $\mathbf{D}$. The components $D_x$, $D_y$, and $D_z$ are related to $R_x$, $R_y$, and $R_z$, by the following expression,

$$
\begin{pmatrix}
D_x(i) \\
D_y(i) \\
D_z(i)
\end{pmatrix} =
\begin{pmatrix}
c_1(i) & s_1(i) & 0 \\
-s_1(i) & c_1(i) & 0 \\
0 & 0 & c_1(i)
\end{pmatrix}
\begin{pmatrix}
R_x \\
R_y \\
R_z
\end{pmatrix}
$$

(4)

where $c_1(i)$, $c_1(i)$, and $s_1(i)$ are called coupling coefficients for $i$-th station or $i$-th component. The coupling coefficients are functions of the rigidity spectrum $g(P)$, the cosmic ray asymptotic orbit, and the response function in the atmosphere. Therefore, the coupling coefficients depend on the location of a station and on the directional component of the telescope. Recently, numerical calculations of the coupling coefficients have been made precisely for meson telescopes above ground and underground (Fujimoto et al., 1977). In their calculations, improved response functions computed by Murakami et al. (1976), and also the observational conditions, such as the geometry of the telescope, the shape of the ground above the station, and the density of the overhead material are taken into account. The coupling coefficients for neutron monitors have been taken from the computations by Yasue et al. (personal communication).

On the basis of above relations and by means of the observed values $D_x^{obs}(i)$, $D_y^{obs}(i)$, and $D_z^{obs}(i)$, we can determine the anisotropy vector $\mathbf{R}$ ($R_x$, $R_y$, $R_z$) and its spectral dependence by the following procedures. In the first place, we assume the
differential rigidity spectrum \( g(P) \) to be expressed by the power-law type spectrum as,

\[
g(P) = \begin{cases} (P/10)^r & \text{for } P \leq P_u \\ 0 & \text{for } P > P_u \end{cases}
\]

where \( P_u \) denotes the upper limiting rigidity beyond which the anisotropy vanishes. When a parameter set \((r, P_u)\) is given, the values of coupling coefficients \( c^i(i) \), \( s^i(i) \), and \( D^i(i) \) are obtained from the numerical tables cited earlier. Also, the components \( D_1(i) \), \( D_2(i) \), and \( D_3(i) \) are given by the expressions including \( R_x, R_y, \) and \( R_z \) (see Eq. (4)). Finally, by means of the least-squares method that minimizes the following \( \chi^2 \), the best-fitted parameters of \( r, P_u, R_x, R_y, \) and \( R_z \) are determined,

\[
\chi^2 = \sum_i w(i)\{(D_{xbs}^0(i) - D_x(i))^2 + (D_{ybs}^0(i) - D_y(i))^2 + (D_{zbs}^0(i) - D_z(i))^2\}
\]

where \( w(i) \) denotes the weighting factor on the \( i \)-th component of data (the counting rate in the present analysis).

3. Observed Data

In Table 1 are listed the observation stations from which the N-S asymmetry and the sidereal diurnal variation are derived. As shown in the table, the present data cover a wide range of rigidity, i.e., they include the data from various cosmic ray detectors; neutron monitors (Thule, McMurdo, and Deep River) and meson telescopes at the surface (Mt. Norikura and Nagoya) and an underground telescope (Takeyama). Neutron monitors respond to relatively low primary rigidities of around 15 GV.

<table>
<thead>
<tr>
<th>Detector</th>
<th>Station</th>
<th>Geographic Lat.</th>
<th>Long.</th>
<th>Counting rate ( \times 10^3 ) h</th>
<th>Median primary rigidity (GV)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Neutron monitor</td>
<td>Thule</td>
<td>76.60°N</td>
<td>68.80°W</td>
<td>43.1</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>McMurdo</td>
<td>77.90°S</td>
<td>160.60°E</td>
<td>90.7</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Deep River</td>
<td>46.10°N</td>
<td>77.50°W</td>
<td>197.6</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Meson telescope</td>
<td>Mt. Norikura</td>
<td>36.11°N</td>
<td>137.55°E</td>
<td>620*</td>
<td>62*</td>
<td>no absorber</td>
</tr>
<tr>
<td></td>
<td>Nagoya</td>
<td>35.15°N</td>
<td>136.97°E</td>
<td>276*</td>
<td>5 cm P&lt;sub&gt;n&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>Underground</td>
<td>Takeyama</td>
<td>35.22°N</td>
<td>139.62°E</td>
<td>3.9*</td>
<td>214*</td>
<td>54 m.w.e. in depth</td>
</tr>
</tbody>
</table>

* for vertical telescope.
Meson telescopes at the surface respond to a median primary rigidity of around 60 GV and the underground telescope (54 m.w.e. in depth) to about 210 GV. It is of great importance to use data of wide rigidity range in determining the spectral dependence of the anisotropy. In Table 2 are also listed the details of the telescopes whose data are used in the present analysis. The data cover the 5-year period 1969–1973, which is divided into two periods; 1969–1970 and 1971–1973 (hereafter designated as (A) and (B), respectively). Period (A) corresponds to the phase of moderate solar activity (average sunspot number ~ 100) in the sunspot cycle and period (B) to the declining phase (average sunspot number ~ 50). In period (B), the data from Nagoya are used instead of Mt. Norikura which is used in period (A), since observations at Nagoya started from the end of 1970 (SEKIDO et al., 1975).

The N-S asymmetry for the nucleonic component (Th-Mc in Table 2) is determined by taking the difference between daily mean intensities from two neutron monitors at the polar regions (Thule and McMurdo). It is reasonable to expect that any common intensity variations will be eliminated by taking the difference, and that the N-S asymmetry can be extracted effectively. The N-S asymmetry for Mt. Norikura and Nagoya meson telescopes (G and GG in Table 2) are derived according to the following definition (NAGASHIMA et al., 1972); G = (30°N – 30°S) + (30°N – 30°E) and GG = (49°N – 49°S) + (49°N – 49°E), where 30°N etc. represent the daily mean intensities of each directional telescope having its central direction of viewing toward the north (N), the south (S), and the east (E) direction with zenith angle of 30° and 49°. For the Takeyama underground telescope (MURAKAMI et al., 1971), the N-S asymmetries (N-S, V-S, W-S, and E-S in Table 2) are determined in a similar manner to the above, by taking the difference of intensities between the north and the south-pointing tele-

Table 2. Details of the telescopes whose data are used in the present analysis. The legend of each symbol is mentioned in the text.

<table>
<thead>
<tr>
<th>(A) 1969–1970</th>
<th>(B) 1971–1973</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>N-S Asym.</strong></td>
<td><strong>Sidereal D. V.</strong></td>
</tr>
<tr>
<td>Th-Mc Mt. Norikura</td>
<td>G</td>
</tr>
<tr>
<td>GG</td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>S</td>
</tr>
<tr>
<td></td>
<td>E</td>
</tr>
<tr>
<td></td>
<td>W</td>
</tr>
<tr>
<td>Takeyama N-S</td>
<td>V</td>
</tr>
<tr>
<td>V-S</td>
<td>N</td>
</tr>
<tr>
<td>W-S</td>
<td>S</td>
</tr>
<tr>
<td>E-S</td>
<td>E</td>
</tr>
</tbody>
</table>

Note: \( G = (30°N – 30°S) + (30°N – 30°E) \), \( GG = (49°N – 49°S) + (49°N – 49°E) \).
The sidereal diurnal variations are derived by using data from the middle or high latitude stations, since the observations at these stations show a more significant diurnal variation than the polar or equatorial stations. As shown in Table 2, data from Deep River (in Canada) have been selected as representative of neutron monitors. For the meson data the five directional components of the multi-directional telescopes are utilized for the vertical (V in Table 2), the north (N), the south (S), the east (E), and the west (W) components.

In the present work we are only concerned with the field dependent parts of the variations, and these are derived by the following procedure. The N-S asymmetry and the sidereal diurnal variation have been computed on a daily basis for the data listed in Table 2, and divided into two groups according to the direction sense of the IMF (toward or away). The average values in both groups and their difference are then evaluated on the solar rotation basis. And finally the average values for period (A) and period (B) are calculated. Field dependent variations are defined as a half of the difference, that is, the field dependent variation = \{(average for toward polarity) - (average for away polarity)\}/2. Thus G, GG, N-S, and Th-Mc etc. in Table 2 are re-defined such that G = \{(G for toward polarity) - (G for away polarity)\}/2 and so on. The sidereal diurnal variation is also re-defined in the same way as above; V = \{(V for toward polarity) - (V for away polarity)\}/2 and so forth. In what follows, both the N-S asymmetry and the sidereal diurnal variation exhibit these field dependent variations. It is emphasized that these re-defined N-S asymmetries and the sidereal diurnal variations are almost free from atmospheric effects as well as other field independent variations (e.g., Forbush decrease and solar diurnal variation), because the difference tends to cancel out such effects that are common to both polarities. The IMF polarity is taken from the atlas prepared by SVALGAARD (1975), which is inferred from data for the polar geomagnetic field variations (Thule and Vostok).

Figures 2 and 3 show the obtained results for the N-S asymmetry and the sidereal diurnal variation for each station and for each period. In the right hand side of Figs. 2 and 3, are plotted the observed N-S asymmetry as a function of the expected value \( c_i \), which is the coupling coefficient described in the previous section. \( c_i \) is estimated under the assumption that the source in space is rigidity independent \((r=0)\) for all rigidity ranges. In the left hand side of the figures the sidereal diurnal vectors are also plotted on sidereal harmonic dials. For both variations statistical errors are derived from counting rates. As is shown in these figures, the coupling coefficients \( c_i \) for meson telescopes above ground and underground are not so small compared with those for neutron monitors. This feature indicates one advantage of using meson telescopes in studying the N-S asymmetry, and the significance is emphasized by the fact that the daily averages of GG-component bear a good correlation with the IMF polarity (Mori and Nagashima, 1979). In the present analysis the nucleonic and meson data are combined together for this reason and also to cover a wide range of
rigidity.

4. Results

For each parameter set of \( (\gamma, P_u) \), the best-fitted three components \( R_x, R_y, \) and \( R_z \) are computed, and the corresponding \( \chi^2 \)-values are also obtained. Based on the derived \( \chi^2 \)-values the contour maps of iso-\( \chi^2 \) lines are plotted in the plane \( (\gamma, P_u) \) in Figs. 4 (a) and 4 (b) for each of the periods (A) and (B), respectively. It can be seen in these figures that both cases have a single minimum value of \( \chi^2 \). In Fig. 4 (a) for period (A), a minimal value of \( \chi^2 (\chi^2 = 216) \) is found at a point where \( \gamma = 0.2 \) and \( P_u = 200 \text{ GV} \); and Fig. 4 (b) for period (B) shows a minimal value of \( \chi^2 (\chi^2 = 414) \) at a point where \( \gamma = 0.4 \) and \( P_u = 200 \text{ GV} \). These values of \( \chi^2 \) for minimal points are somewhat greater than those expected from the present number of degrees of freedom. In the present case, the standard errors used in the evaluations of \( \chi^2 \) are estimated from the counting rates. If, however, in the calculation of \( \chi^2 \), we assume standard errors several times larger than those from the counting rates, the above cases would still be
statistically significant. This assumption of larger errors (a factor of several) may be more appropriate for most cases of analyzing cosmic ray intensity variations. In the present case, the errors might be due mostly to some disturbances of isotropic intensity variations in the N-S asymmetry, and solar modulation of the sidereal diurnal

Fig. 3. Observed N-S asymmetries (right) and diurnal variations (left) averaged over period (B) 1971-1973. Remarks are the same as Fig. 2 except that the data from Nagoya meson telescope are used instead of those from Mt. Norikura.

Fig. 4. (a) Contour map of iso-$\chi^2$ lines for period (A) 1969-1970. The figure of each contour represents one-tenth of the $\chi^2$-value. A single minimum value ($\chi^2=216$) is found at a point marked with a cross (×) at $\tau=0.2$ and $P_u=200$ GV. (b) Contour map of iso-$\chi^2$ lines for period (B) 1971-1973. A single minimum value ($\chi^2=414$) is also found at $\tau=0.4$ and $P_u=200$ GV.
We can here construct the three-dimensional anisotropy $R$ in free space, using the best-fitted values of the three components $R_x$, $R_y$, and $R_z$. Figure 5 shows the anisotropy $R_A$ for period (A) and $R_B$ for period (B) constructed in the equatorial coordinate system. From the figure we find that the direction of the anisotropy is very close to the direction of the ecliptic north pole. The angle of the directions of anisotropy with respect to the ecliptic north pole are obtained respectively as $8^\circ \pm 12^\circ$ for period (A) and $14^\circ \pm 13^\circ$ for period (B). Thus it can be concluded that the cosmic ray north-south anisotropy is really perpendicular to the ecliptic plane within statistical uncertainty. The present result shows definite evidence for the existence of the persistent north-south anisotropy perpendicular to the ecliptic plane. The average amplitude of the anisotropy at 10 GV which corresponds to $\eta_1$ in Eq. (1) is determined to be $0.081 \pm 0.021\%$ for period (A) and $0.072 \pm 0.018\%$ for period (B), respectively. Some characteristics of the north-south anisotropy derived in the present analysis are summarized in Table 3, in which the errors for various parameters are due to those

Table 3. Some characteristics of derived north-south anisotropy for each period.

<table>
<thead>
<tr>
<th>Period</th>
<th>$\eta_1$ (%)</th>
<th>Angle from ecliptic north pole (Deg.)</th>
<th>Rigidity spectrum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\gamma$</td>
</tr>
<tr>
<td>(A) 1969–1970</td>
<td>0.081±0.021</td>
<td>8±12</td>
<td>0.2±0.1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B) 1971–1973</td>
<td>0.072±0.018</td>
<td>14±13</td>
<td>0.4±0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
from the best-fit procedures. As is seen in Table 3, the best-fitted variational spectra have a power index $\gamma$ of a small positive value of $0.2 \pm 0.1$ for period (A) and $0.4 \pm 0.2$ for period (B), and the upper limiting rigidity $P_u$ is around 200 GV or more for both periods. When no upper limiting rigidity is imposed, i.e., if $P_u$ is assumed to be infinity, $\gamma$ is found to be 0.1 for both periods, and the amplitude $\eta_1$ of the anisotropy at 10 GV are found to be 0.065% for period (A), and 0.074% for period (B). Above results indicate that the variational spectrum is slightly rigidity dependent. The present analysis, however, does not rule out the possibility that the spectrum is rigidity independent as has so far been assumed.

In addition, the least-squares method has also been applied to each single component (N-S asymmetry and sidereal diurnal variation) separately. Almost the same contour maps of iso-$\chi^2$ lines as above are obtained for both of the separate cases, and the minimal $\chi^2$-values are found at the points having very close values of $(\gamma, P_u)$; for the N-S asymmetry $(\gamma, P_u)$ is $(0.2, 100 \text{ GV})$, and for the sidereal diurnal variation $(\gamma, P_u)$ is $(0.5, 200 \text{ GV})$ for period (A). These results would definitely support the idea that the N-S asymmetry and the sidereal diurnal variation are both due to a single origin in interplanetary space.

5. Radial Density Gradient

The formulation for the cosmic ray anisotropy $\xi_a$ due to the heliocentric radial density gradient and the IMF is given by (Forman and Gleeson, 1975; Yoshida et al., 1973)

$$\xi_a = \frac{(\omega \tau)^2}{1 + (\omega \tau)^2} \rho G \times B / B \quad (7)$$

where $\omega$, $\tau$, and $\rho$ are the gyrofrequency, the mean collision time and the gyroradius, respectively. $B$ denotes the magnetic induction and $G$ is the density gradient vector. In Eq. (7), it is readily understood that the direction of the vector $\xi_a$ is perpendicular to the ecliptic plane, since both vectors $G$ and $B$ lie, on the average, in the ecliptic plane. We have already shown in the previous section that the direction of the north-south anisotropy is almost perpendicular to the ecliptic plane (see Fig. 5). Hence, by combining Eqs. (1), (5), and (7), we can obtain the relation,

$$\eta_1 (P/10)^{\gamma} = \rho G \times (1/\sqrt{2}) \quad (8)$$

under the assumption that $(\omega \tau)^2 \gg 1$, and that the average angle of $B$ with respect to the sun-earth line is $-45^\circ$ (or $135^\circ$). $\rho$ is given by $P/45B$, where $B$ is measured in gammas, and $P$ is in GV. Based on this equation, we can estimate the magnitude of the radial density gradient when the values of $\eta_1$, $\gamma$, and $B$ are known. Adopting those values determined in the previous section for period (A), namely $\eta_1 = 0.081 \pm 0.021\%$, and $\gamma = 0.2 \pm 0.1$ and also the average values of the IMF, $B = 6.2$ gammas (Wada,
1977), the radial density gradient $G$ for 1969–1970 is represented by

$$G = (3.20 \pm 0.85) \times (P/10)^{-0.8 \pm 0.1} \%/\text{AU} \text{ for } P \leq 200 \text{ GV}.$$  \hspace{1cm} (9)

Similarly for period (B) (1971–1973), by adopting $\eta_1 = 0.072 \pm 0.018 \%$, $\gamma = 0.4 \pm 0.2$, and $B = 6.1$ gammas, the radial gradient is given by

$$G = (2.80 \pm 0.70) \times (P/10)^{-0.6 \pm 0.2} \%/\text{AU} \text{ for } P \leq 200 \text{ GV}.$$  \hspace{1cm} (10)

The average radial gradient for the 5-year period (A+B) (1969–1973) is then represented by

$$G = (3.01 \pm 1.11) \times (P/10)^{-0.7 \pm 0.2} \%/\text{AU} \text{ for } P \leq 200 \text{ GV}.$$  \hspace{1cm} (11)

In Eqs. (9), (10), and (11), errors are due to those for the anisotropy determined in the previous section. It is noted that in contrast to previous work (e.g., BERCOWITCH, 1970; IUCCI and STORINI, 1972; KUDO and WADA, 1977) the present radial gradients are derived without any assumption on the spectral dependence of the north-south anisotropy. In Fig. 6 the above radial gradient for the whole period (A+B) is shown with a heavy line. In the figure the hatched region indicates the error band. The upper limiting rigidity $P_u$ of this gradient is about 200 GV. The dashed line in the same figure shows the radial gradient with the upper limiting rigidity of infinity, which is estimated as $G = 2.72 \times (P/10)^{-0.9}$. This line lies fully in the hatched region.

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**Fig. 6.** The radial gradient derived from the cosmic ray north-south anisotropy (the heavy line) for period (A+B); 1969–1973. The hatched region indicates the error band. The dashed line represents the radial gradient without upper limiting rigidity.
Therefore, the present result would not be inconsistent with the idea that the rigidity dependence of the radial gradient is inversely proportional to the rigidity $P$, which has often been used as an assumption up to now (Iucci and Storini, 1972).

Figure 7 shows a comparison of the present radial gradient with some of those obtained by other authors for a different period of time. In the figure, the area surrounded by the dotted line gives the present result for 1969–1973, and the solid lines, solid circles, a triangle, and squares indicate respectively the gradients by Bercovitch (1971), Iucci and Storini (1972), Duggal and Pomerantz (1977), and Kudo and Wada (1977). Bercovitch estimated the radial gradient based on the IMF fluctuations by assuming the relationship between the so-called force-field gradient and the diffusion coefficients. In the other three cases, the gradients were evaluated by means of the cosmic ray N-S asymmetry. In Fig. 7, it is shown that all radial gradients lie in or very close to the error band of the present gradient, except the result of Iucci and Storini for the solar activity minimum (1965–1966). It is noted that these gradients for high rigidity regions connect smoothly to the gradients in lower rigidity regions ($\leq 1$ GV), where the data were obtained from direct measurements in space (McKibben et al., 1977; Nagashima, 1977). The present result is given as a function of rigidity $P$, and is found to fall off slightly slower than $P^{-1}$, as is shown in Fig. 7. This tendency is similar to that given by Kudo and Wada (1977). On the other hand,
in higher rigidity regions (10 ~ 100 GV) the slope of the functional form derived by Bercovitch (1971) based on IMF fluctuations falls off significantly faster than $P^{-1}$, being closer to $P^{-2}$. This is markedly different from the dependence determined from cosmic ray data.

The radial density gradient might be expected to show a solar activity dependence, since the cosmic ray intensity variation observed on the earth exhibits the well known 11-year modulation (Urch and Gleeson, 1972b). In Fig. 7, it can be seen that the gradient derived by Iucci and Storini shows a marked solar cycle dependence. On the other hand, Bercovitch (1971), using IMF fluctuation data, did not obtain such a large difference for the same period, as shown in the figure. The present result gives the ratio of the gradient as 3.20/2.80 = 1.14 ± 0.42 at 10 GV between period (A) and (B). Because of the large statistical uncertainty, we can not derive any positive conclusion on the solar cycle variation of the gradient, but rather suggest a weak activity dependence of the gradient for high rigidity regions.

6. Diffusion Coefficient

It is well known that the radial gradient in the high rigidity region ($\geq 1$ GV) is represented by the so-called force-field gradient (Jokipii and Coleman, 1968; Urch and Gleeson, 1972a),

$$G = CV/\kappa$$

(12)

where $C$, $V$, and $\kappa$ denote the Compton-Getting factor, the solar wind velocity, and the diffusion coefficient, respectively. Thus, when the values of $G$, $C$, and $V$ are known we can estimate the diffusion coefficient $\kappa$ at high rigidities based on this equation. Adopting the values $C=1.5$, $V=430$ km/sec and the present gradient $G$ for 1969–1973 (Eq. (11)), the coefficient $\kappa$ is given by

$$\kappa = (3.2 \pm 1.2) \times 10^{22} \times (P/10)^{0.7 \pm 0.2} \text{ cm}^2/\text{sec}.$$  (13)

Figure 8 shows the diffusion coefficient $\kappa/\beta$, divided by the particle velocity $\beta$. The present result is shown by the heavy line and hatched region. The dotted line indicates the result of Jokipii and Coleman (1968) and the dashed lines the result of Bercovitch (1971). Jokipii and Coleman derived their result from data from the IMF fluctuations during 32 days of Mariner 4 for 1964–1965, while Bercovitch obtained his result from data from Explorer 28, 33, and 35 for 1967–1968. The present diffusion coefficient is close to that of Bercovitch up to 10 GV, although the method of derivation and the period are different. The difference between them, however, becomes gradually larger with increasing rigidities. Since the diffusion coefficient is given by the reciprocal of the gradient (see Eq. (12)), this difference could be due to the difference between the gradients described in the previous section.
7. Discussion and Conclusion

In the present paper, the average direction of the field dependent north-south anisotropy has been examined in three-dimensional space, without making any assumption regarding its direction. By utilizing the least-squares method and using data in a wide range of rigidity, together with improved coupling coefficients, it is found that the direction of this anisotropy coincides with that of the ecliptic north pole within statistical error. Therefore, it can be concluded that definite evidence is obtained for the existence of a field dependent anisotropy in the stationary condition, whose direction is perpendicular to the ecliptic plane. The variational rigidity spectrum was also examined, and it is found that the best-fitted rigidity spectrum is characterized by a power-law spectrum with an exponent of small positive value (∼0.3) and an upper limiting rigidity of 200 GV or more.

In the present analysis, the data from Takeyama (54 m.w.e. underground) are used; the median primary rigidity from the Takeyama vertical telescope is as high as 214 GV. It will be noticed here that the value is very close to the present best-fitted upper limiting rigidity. In general, it might be difficult to determine a precise upper limiting rigidity when the genuine upper limit is out of the range of effective primary rigidities of the used data. For this reason, data from much deeper underground might be necessary. Recent observations at Sakashita (80 m.w.e. underground) (UENO

Fig. 8. Diffusion coefficient $\kappa/\beta$ (divided by particle velocity $\beta$) in the vicinity of the earth. The present result is shown with the heavy line and hatched region. The dotted line indicates the result by Jokipii and Coleman (1968), and the dashed lines of Bercovitch (1971). Data period is also shown.
et al., 1979) and at Poatina (365 m.w.e. underground) (Fenton et al., 1977), whose median primary rigidities are $\sim 340$ GV and $\sim 1300$ GV respectively, suggested some indications of IMF polarity dependence of the sidereal and/or the solar diurnal variation even at these underground depths. This would suggest that the upper limiting rigidity of the north-south anisotropy might be higher than 200 GV. In this respect, new underground telescopes under construction at Grand Canyon Cavern (Swinson, 1979) at 111 m.w.e. in depth, and at Matsushiro (Yasue et al., 1979) at 250 m.w.e. might give valuable data in the near future.

If we do not impose any upper limit on the spectrum, the exponent $\gamma$, which has a best-fitted value of 0.3 at $P_u = 200$ GV, will take a smaller value of 0.1 for both periods (A) and (B). If that is the case, the resultant amplitude of the anisotropy at 10 GV are found to be 0.065% for period (A) and 0.074% for period (B), and the corresponding radial gradient falls off as $P^{-0.9}$, which is almost inversely proportional to rigidity $P$.

The radial diffusion mean free path $\lambda$ is evaluated by the equation $\lambda = 3\kappa/\beta c$ ($c$ is the velocity of light). Thus the present diffusion coefficient $\kappa$ yields the following mean free path $\lambda$,

$$\lambda = 0.2 \times (P/10)^{0.7} \text{ AU}.$$ (14)

The present mean free path is consistent up to about 10 GV with the results of Iucci and Storini (Moraal, 1976) and Jokipii and Coleman (1968). Also the value of $\lambda$ of 0.2 AU at 10 GV is not inconsistent with the mean free path given by Webber et al. (1977), which was derived from helium and oxygen data from Pioneer 10 and 11. The mean collision time $\tau = 3\kappa/(\beta c)^2$ is estimated to be $\sim 100$ seconds at 10 GV, and indicates that the previous assumption that $(\omega \tau)^2 \gg 1$ is well satisfied, since $(\omega \tau)^2 \sim 25$ at 10 GV.

Recently some models of the heliosphere configuration, such as the 'two hemisphere model', has been proposed in which the heliosphere is magnetically divided into two (toward and away) hemispheres by a neutral current sheet (Saito, 1975; Svalgaard and Wilcox, 1978, and references therein). The current sheet is considered to have a variable extent in heliographic latitude across the sector structure. In the present work, it is implicitly assumed that if the average polarity of the IMF for a given day is toward (or away from) the sun, the area in the vicinity of the earth, having a characteristic length comparable to the gyroradius of the cosmic ray particle, is also dominated by the same polarity. This assumption would be reasonable so long as the gyroradius of the particle is relatively small. On the other hand, however, the above model might lead us to a speculation that the north-south anisotropy may vanish at the higher rigidities, since our assumption does not hold for those particles whose gyroradii are sufficiently large compared to the sector structure. Thus the upper limiting rigidity of the north-south anisotropy may give informations on the latitudinal extent of the neutral current sheet. The present upper limit of $\sim 200$ GV
would give a lower limit of \( \sim 0.7 \) AU of the extent of the sector structure in the direction perpendicular to the ecliptic plane. Detailed examinations of the cosmic ray north-south anisotropy and its relation to the IMF can be expected to give further information about the heliosphere models so far presented.

The present paper examined the north-south anisotropy during the period 1969–1973. YASUE et al. (1976) have pointed out in a similar analysis that the anisotropic flow perpendicular to the ecliptic plane also exists during the succeeding period of 1974–1975. The analysis for the period from 1976 onward is now under way, and will be reported elsewhere.

In conclusion the present analysis can be summarized as follows:

1) A three-dimensional nature of the field dependent north-south anisotropy of cosmic rays has been revealed and examined using data in a wide range of rigidity (15–210 GV) for 1969–1973. The direction and certain other parameters of the anisotropy have been determined based on the information deduced from both the N-S asymmetry and the sidereal diurnal variation.

2) Definite evidence has been obtained of the existence of an anisotropic flow perpendicular to the ecliptic plane responsible for the north-south anisotropy under stationary conditions. The magnitude of the anisotropy at 10 GV is \( 0.081 \pm 0.021 \% \) and \( 0.072 \pm 0.018 \% \) for the periods 1969–1970 and 1971–1973, respectively. The rigidity spectrum is found to be slightly rigidity dependent and to have an upper limiting rigidity of 200 GV or more. The present analysis, however, does not rule out the possibility that the spectrum is rigidity independent (\( \gamma = 0 \)).

3) The heliocentric radial density gradient and the diffusion coefficient for the high rigidity range of cosmic ray particles up to 200 GV have been estimated, based on the derived parameters of the north-south anisotropy. The radial density gradient is given as a function of rigidity as \( (3.0 \pm 1.1) \times (P/10)^{(-0.7 \pm 0.2)} \) \%/AU for 1969–1973. The diffusion coefficient is estimated as \( (3.2 \pm 1.2) \times 10^{22} \times (P/10)^{0.7 \pm 0.2} \) cm\(^2\)/sec.

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