Linearity of Ferromagnetic Core Solenoids Used as Magnetic Sensors

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Linearity of a high- μ metal core solenoid as a magnetic sensor has been checked, comparing its output to that from an air-core loop antenna. Two induction magnetometer systems using these different types of sensors both parallel to the magnetic N-S direction, were run simultaneously in the same manner as for actual field observations. The outputs from these systems during a moderately intense Pc 3 event were compared with each other. No distortion of the magnetic signals due to the high- μ metal core sensor was detected. Theoretical consideration is presented also as to why high- μ metal core solenoids give rise to no signal distortion.

1. Introduction

For observation of magnetic pulsations, induction magnetometers are used most widely among many different types of magnetometers. As head sensors of induction magnetometers, two different types of coils are normally used, an air-core circular loop antenna and a high- μ metal core solenoid. There are a considerable number of disadvantages to a high- μ metal core solenoid compared to an air-core loop antenna (CAMPBELL, 1969). The most serious one is that a high- μ metal core might distort magnetic signals. The distortion is due to the following processes.

- 1) Non-linearity of the ' B - H ' curve might give rise to higher harmonics of sinusoidal signals and also cross modulation of two or more sinusoidal signals.
- 2) Energy loss caused by completing the B - H hysteresis loop.
- 3) Energy loss due to eddy currents caused in the core.

The purpose of this research is to see experimentally whether or not high- μ metal core solenoids actually distort magnetic signals. Two induction magnetometers, one having an air-core loop antenna and the other a high- μ metal core solenoid, were run simultaneously. The systems have identical amplifier/recording systems. The magnetometers were set up at a magnetic station of the University of British Columbia in the same manner as they are used for actual observations. Signals

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obtained by the two systems were compared and no signal distortion was detected. The setup of the magnetometer systems and the procedures of signal comparison will be described in the following two sections. In the last section, a theoretical consideration will be given as to why no signal distortion occurs using a high- μ metal core solenoid.

2. Experimental Setup

Three induction magnetometer systems were operated at the University of British Columbia Research Forest, Maple Ridge, B.C. for this experiment. One system used an air-core loop antenna as the head sensor and the other two used high- μ metal core solenoids. The air-core system and one of the high- μ metal core systems have identical amplifier/recording systems. The third system which uses a different type of amplifier was set up as a back up system. All three systems measured natural field changes of the magnetic N-S component.

The experimental setup is schematically shown in Fig. 1. System A uses an air-core coil and System B a permalloy core coil. Both systems use identical semicon-

Fig. 1. Schematic diagram of the experimental setup.

ductor parametric amplifiers, which can take as high an output impedance as the 200 k Ω of the sensor, with a noise figure less than 3 db. Direct comparison is possible between the outputs from these two systems. However, the parametric amplifiers were frequently found to be noisy. Therefore, another system, *viz.*, System C was set up for the purpose of checking. This system uses an FET chopper amplifier. The noise level of this amplifier which is about 0.1 μ V peak-to-peak, is much lower than that of the parametric amplifiers. However, this amplifier employs intensive highcut filtering so that direct comparison with the other systems can be carried out for frequencies lower than approximately 0.1 Hz. Outputs of all three systems, together with the output from a fluxgate magnetometer, clock pulses and a reference

FM carrier signal, were recorded on a 7 channel FM magnetic tape recorder. The sensitivity of this recorder was adjusted to be 2 V per 40% frequency deviation of the FM carrier signal. This FM recorder determined the dynamic range of the systems as 40 db in rms base.

The air-core coil for System A was designed by the late Dr. B. CANER (1970) of the Victoria Magnetic Observatory. The permalloy core coils for Systems B and C were designed and built at the Geophysics Research Laboratory, University of Tokyo. The air-core coil is 1.27 m in diameter and has 16,000 turns. Therefore, the total area is $2.03 \times 10^4 \text{ m}^2$ and the absolute sensitivity is 0.127 Caner.* The core of the sensor of System B is made of permalloy sheets, insulated and laminated. Its dimension is 1 m in length and 2 cm $\times$ 2 cm in cross section. The turn number is 20,000. The sensor of System C is similar to that of System B with an identical core. However, System C which has a larger turn number than System B, is about 30% more sensitive. Further details of the characteristics of the coils are given in a paper by the authors (UEDA and WATANABE, 1975b). The frequency response of each of the three systems, assumed to be linear, is given in Fig. 2.

Fig. 2. Frequency response of each of the three systems. Ordinates represent output voltages for time-derivatives of sinusoidal magnetic field variations having various frequencies. For Systems B and C, each high- μ metal core solenoid is assumed to be linear in response.

3. Data Analysis

There was a period of moderately intense Pc 3 pulsation activity on October 10, 1973 (Fig. 3). The data obtained by the three systems during this event were compared in two different ways. Firstly, the amplitude ratio of the permally core

* Absolute sensitivity of a coil in units of Caner is given by emf in microvolts when a uniform (in space) and sinusoidal (in time) magnetic field of an amplitude of 1 mT and a frequency of 1 Hz is applied parallel to the axis of the coil (UEDA and WATANABE, 1975a).

Fig. 3. A moderately intense Pc 3 event of October 10, 1973 recorded at the University of British Columbia Research Forest, Maple Ridge, B.C. All three systems were aligned to the magnetic N-S direction. The scale in units of GAMMA-Hz measures amplitudes in gammas of sinusoidal field changes at a frequency of 1 Hz. It differs from the scale in γ/sec by a factor of 2π .

coil signal to the air-core signal was examined and secondly, their power spectra were compared.

3.1 Amplitude ratio

An inter-channel comparison was made by taking the ratio of each permalloy core coil signal to the corresponding air-core coil signal. The data of about 15 min duration commencing 21h 42m 06s UT, October 10, 1973 were used. It is appropriate to use peak-to-peak readings since they are free of dc offset and phase shift. Further, they are less affected by noise because of relatively higher S/N ratios.

To obtain peak-to-peak readings, maximum and minimum values of the signal waveform were collected, and the difference between successive pairs of values was calculated. One reading, maximum or minimum, is used for calculating two successive peak-to-peak values. The two peak-to-peak values in the series are, therefore, opposite in sign.

Let y_1 and y_2 be the peak-to-peak readings in volt of System A and System B, respectively. If we denote the true peak-to-peak in System A with x , the actual reading y_1 is given by

$$y_1 = x + n_1, \quad (1.1)$$

where, n_1 , referred to as 'noise' hereafter, is the difference between the actual reading

and the true value. The difference includes the system noise and an error in peak-to-peak reading. The error in reading is likely from two sources. One is short-period (Pc 1) pulsations overlapping the Pc 3. The other is long-period fluctuations including dc drifts. Now, if System B is linear in response as System A, actual Pc 3 reading in System B should be given as follows,

$$y_2 = Sx + n_2, \quad (1.2)$$

where S , a constant, is the relative sensitivity of System B to System A. The two equations above show that the relative sensitivity is given by

$$S = \langle y_2 \rangle_{Av} / \langle y_1 \rangle_{Av}, \quad (2)$$

assuming, that n_1 and n_2 vanish on the average. With respect to each system, more than eighty peak-to-peak values were collected and the relative sensitivity was found to be 0.1438;

$$S(B/A) = 0.1438. \quad (3.1)$$

Similarly, it was found that the relative sensitivity of the sensor of System C to that of System A equals 0.1838,

$$S(C/A) = 0.1838. \quad (3.2)$$

If we assume, that n_1 and n_2 each follows a normal distribution function of a zero mean value, the quantity $(y_2 - Sy_1)$ should also follow a zero-mean normal distribution function. We see this from the following relation,

$$y_2 - Sy_1 = n_2 - Sn_1. \quad (4)$$

The theory of errors tells us that the difference between two normal random variables is also a normal random variable. We actually checked that $(y_2 - Sy_1)$ is distributed as a zero-mean normal distribution function. The standard deviation of difference between the readings by the two systems, A and B, turned out to be 85.4 mγ/sec. The readings in output voltage y_1 and y_2 were converted to the values in units of γ/sec which will be denoted with v_1 and v_2 . More specifically, v_1 and v_2 are related to y_1 and y_2 as follows,

$$v_1 = y_1 / S_A \quad (5.1)$$

and

$$v_2 = y_2 / (SS_A), \quad (5.2)$$

where S_A is the sensitivity of System A (3.59 V/mγ/sec). If we denote the true peak-to-peak reading in γ/sec with u , then we have

$$v_1 = u + N_1 \quad (6.1)$$

and

$$v_2 = u + N_2, \quad (6.2)$$

where

$$u = x/S_A.$$

Noise of each system is represented with N_1 and N_2 , respectively and the standard deviation of $(N_1 - N_2)$ is 85.4 mγ/sec as mentioned before.

From the Eqs. (6.1) and (6.2), we obtain ratio z of the readings by the two systems as follows,

$$z = \frac{v_2}{v_1} = \frac{u + N_2}{u + N_1}. \quad (7)$$

This ratio z is plotted in Fig. 4a as a function of v_1 , which is approximately equal to u , the true peak-to-peak value. The ratio between System C and System A was obtained in a similar manner and is plotted in Fig. 4b. The plots are scattered around the line $z=1$. Apparently, the smaller peak-to-peak values are more affected by noise; their plots are scattered more.

Fig. 4. Ratio of high- μ metal core system outputs to air-core system output. (a) System B to System A, (b) System C to System A. Peak-to-peak readings by System A are plotted on the abscissa. The inside pair of distribution limits correspond to one standard deviation (68.3%) and the outside pair to twice the standard deviation (95.5%).

According to GEARY (1930), distribution of z approximately follows the probability density function $Q(z)$;

$$Q(z) = \frac{1}{\sqrt{2\pi}} \cdot \frac{(\sigma_2^2 + \sigma_1^2 z)u}{\sigma_1 \sigma_2 (\sigma_1^2 z^2 + \sigma_2^2)} \cdot e^{-\frac{1}{2} \cdot \frac{(z-1)^2 u^2}{\sigma_1^2 z^2 + \sigma_2^2}}, \quad (8)$$

where σ_1 and σ_2 are the standard deviations of N_1 and N_2 , respectively. That the approximation is valid is readily seen from the fact that the exact value of the prob-

ability is greater than $Q(z)$ by not more than 0.0027 even if the standard deviation σ_1 is as large as one third of the value u ; $\sigma_1=1/3u$. According to Geary, we also know that the probability function is simplified as follows,

$$Q(z)dz = \frac{1}{\sqrt{2\pi}} e^{-\frac{t^2}{2}} dt, \quad (9)$$

transforming the variable from z to t by

$$z = \frac{1 + t\sqrt{(\sigma_1/u)^2 + (\sigma_2/u)^2 - t^2(\sigma_1/u)^2(\sigma_2/u)^2}}{1 - t(\sigma_1/u)^2}. \quad (10)$$

Let z_{-1} and z_1 be the values of z corresponding to $t=-1$ and $+1$. Then, $\int_{z_{-1}}^{z_1} Q(z)dz = 68.3\%$, gives the probability that z falls in the range $z_{-1} \leq z \leq z_1$. In Figs. 4a and b, the inside pair of curves represents z_{-1} and z_1 for various values of u . To draw these curves, it is unnecessary to know σ_1 and σ_2 separately. For $|t|=1$ and sufficiently large values of u as in the present case, the relation is well approximated by

$$z = 1 + \frac{t}{u} \sqrt{\sigma_1^2 + \sigma_2^2}. \quad (11)$$

In this relation, $\sqrt{\sigma_1^2 + \sigma_2^2}$ is equal to the standard deviation of $(N_2 - N_1)$ as proved in the theory of errors, and was also determined experimentally. The outside pair of curves correspond to $t=\pm 2$, specifying the range of 95.5% occurrence of probability. In each of the Figs. 4a and b, approximately 66% of the points fall within the inside pair of curves and 94% within the outside pair.

3.2. Comparison of power spectra

For another inter-channel comparison, the power spectra of the records from the three systems were calculated for 282 sec record length beginning at 21h 42m 06s UT, October 10, 1973. During this period, the data are free of sporadic noise inherent to the parametric amplifiers. The magnetic tape containing the data was played back at a speed 15 times greater than the recording speed, and the data on each channel was digitized at a rate of 29 points per second after passing through a low pass filter with cutoff frequency and attenuation slope 8 Hz and 24 db/oct respectively.

To the data thus digitized, the Blackman-Tukey method and the maximum likelihood method were applied for estimating the power spectrum of each. Firstly, a power spectrum due to the maximum likelihood method was obtained. This method is excellent for reproducing spectral estimators for relatively small number of data points (LACOSS, 1971). The spectrum obtained in this way was used as a reference for the Blackman-Tukey method. For more details of this method, ref-

erence is made to JENKINS and WATTS (1968). A lag window of the Tukey type (see Table 6.5 of J-W) is used. For various maximum lags of the Tukey window, smoothed spectral estimators were computed using the formula (6.3.28) or (6.3.30) and the one which was nearest to the spectrum by the maximum likelihood method was chosen. Then, a confidence interval was estimated following the procedure described in pp. 254–255 of Jenkins and Watts.

The results of the spectral analysis are presented in Figs. 5a and b. In each, spectral information is given of the data by the two systems; one is the air-core system (System A) and the other a high- μ metal core system (System B or C). The spectra of both the systems given in the figures are those which were obtained by the

Fig. 5. Difference in the power spectra between the high- μ metal core system and the air-core system. (a) System B and System A, (b) System C and System A. The horizontal cross line shows the bandwidth, and the vertical one is the 96% confidence interval. Also, the center point of the cross corresponds to zero difference of the power spectra by the scale on the right-hand side. Difference of the power spectra was taken after each spectrum was normalized to its peak.

Blackman-Tukey method. The spectrum for System A, normalized to the peak value, is given on a logarithmic scale. An estimate of the 96% confidence interval is also given on the same scale. The ratio of the two spectra which is given as a difference on a logarithmic scale, is also shown (on an expanded scale). As seen in the figures, the ratio is constant as a function flat of frequency; there is no significant deviation from unity (0 db). The deviation in Fig. 5b around frequency of 0.14 Hz is due to the difference in frequency response between System A and System B (refer to Fig. 2).

4. Discussions and Conclusions

The inter-channel comparisons described show that no signal distortion occurs using a high- μ metal core solenoid. The processes mentioned in Section 1, which possibly give rise to signal distortion, probably do not occur in practice, however, even if they do they would not be significant.

Of the three possible distortion processes, non-linearity and hysteresis loop loss are presumably irrelevant, since the effective permeability of a high- μ metal core solenoid is most likely determined by the core's geometry rather than by the intrinsic permeability of the core material. The intrinsic permeability μ for an infinitesimal magnetic field ΔH is equal to the gradient of the B - H curve. The change in B is then given by

$$\Delta B = \mu(H) \Delta H. \quad (12)$$

The permeability μ depends on the location on the B - H curve. Its value under the presence of a strong bias field, as in the case of N-S component measurements, is generally different from the value for the non-biased case. Now, the change in magnetic flux density in the core is not given by the above Eq. (12), since a demagnetizing field is induced within the core. The change in the flux density is given as follows,

$$\Delta B_i = \frac{\mu}{1 + N_d(\mu - 1)} \Delta H \quad (13)$$

where N_d is the demagnetization coefficient, a non-dimensional quantity determined by the geometry of the core. If μ is much greater than 1 and also than $1/N_d$, the above-given equation approximates very well to

$$\Delta B_i = \frac{1}{N_d} \Delta H. \quad (14)$$

The electromotive force E generated in the coil is proportional to the time derivative of B_i ,

$$E \propto \frac{dB_i}{dt} = \frac{1}{N_d} \frac{dH}{dt}. \quad (15)$$

This equation shows that the effective permeability μ_{eff} relevant to a high- μ metal core solenoid is the inverse of the demagnetizing coefficient;

$$\mu_{\text{eff}} = 1/N_d. \quad (16)$$

The relation (13) has been shown to hold precisely if the core is an ellipsoid in shape (STRATTON, 1941; OSBORN, 1945; BOZORTH, 1951). The flux density ΔB_i is uniform in the core in that case. The relation holds true in good approximation for long cylindrical rods also. The experimental values of demagnetizing coefficients for long cylindrical rods agree well with the theoretical values for prolate spheroids (BOZORTH, 1951).

It is well known that intrinsic permeability of a ferromagnetic material changes due to thermal and/or mechanical stress. This would imply instability in sensitivity of a high- μ metal core solenoid. However, the instability of this nature would not be the case since the intrinsic permeability μ remains much larger than the inverse of the demagnetizing coefficient. Under the same condition, presence of a bias magnetic field would cause no change in sensitivity. Obviously, no bias field should be as great as to cause the B - H curve to approach saturation level. In this case, the intrinsic permeability μ defined by the Eq. (12) would not be large enough to satisfy the condition that $\mu \gg 1/N_d$.

Eddy current loss, the third possible process for signal distortion, needs consideration different from that for the first and second processes discussed above. Insulation/lamination of high- μ metal strips seems to reduce eddy current loss substantially. One of the present authors (H.U.) carried out measurements of self-inductances of two high- μ metal core solenoids, with similar dimensions (core length ~ 5 to 6 feet); one an insulated-laminated high- μ metal core and the other a solid cylindrical rod. It was found that the self-inductance of the rod core coil begins to decrease for a frequency of about 10 Hz whereas self-inductance of the other does not change up to about 1 kHz.

The Pc 3 event in this investigation is a moderately strong one. The largest peak-to-peak amplitude is 3γ approximately. Magnetic variations much larger in size take place in nature, however. For example, Pc 5 often surpasses 100γ at high latitudes. The experiment reported in this paper should be extended for such large amplitude magnetic variations, since non-linearities due to high- μ metal cores are more likely to occur for larger-size magnetic changes. However, the experimental work of this paper was limited to mid-latitudes, and magnetic variations more than $\sim 100\gamma$ seldom occur there. R.D. Russell (private communication, 1977) suggests an active experiment to overcome this difficulty. The idea of the experiment is to apply sinusoidal magnetic fields with two different frequencies to a magnetic sensor and to examine if there is any cross modulation between the two exciting fields. If the sensor is non-linear, the output should contain a component with a frequency equal to the sum of the two exciting frequencies and also to their difference. A cross

modulation technique has actually been attempted in measurements of electric conductivities of rocks. As demonstrated by MITCHELL (1978), a student of Russell, different kinds of rocks yield different degrees of cross modulations. Natural magnetic field variations are bound to give rise to 'noise' in this mode of approach. However, we can apply exciting fields as long as necessary to ensure that, the S/N ratio in spectral analysis is increased sufficiently to reveal any cross modulation that may occur.

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