Non-Steady State of a Hydromagnetic $\alpha\omega$ -Dynamo and Its Application to the Geomagnetic Reversals

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The non-steady state of a simplified hydromagnetic $\alpha\omega$ -dynamo is investigated. The hydromagnetic α -effect is derived by taking into consideration Coriolis, Lorentz and viscous forces, based on a simple, nearly homogeneous and isotropic turbulence. The non-uniform rotation is also taken to be non-stationary and subjected to the above forces.

Numerical calculations of the initial value problem of the $\alpha\omega$ -dynamo equations have revealed that the dipole field undergoes relatively gradual variations and occasional polarity changes similar to the geomagnetic reversals. The reversal of the dipole field is caused essentially by the disruption of the non-uniform rotation. The time required for a complete decay of the dipole field is estimated to be about 3,000 years which is consistent with paleomagnetic observations.

By taking into account the electrical conductivity of the bottom part of the mantle, electromagnetic coupling of the mantle to the core has also been studied. It is found that the field reversal is accompanied by the acceleration of the rotational speed of the mantle, and a change in l.o.d. of 95 msec.

1. Introduction

The basic properties of the dynamo processes are now well understood owing to the rapid progress in the kinematic dynamo theory (e.g., GUBBINS, 1974), but the particular process actually operating as the geodynamo has not yet been identified. There still remains many possibilities, and research is proceeding on parallel tracks (GUBBINS, 1974; MOFFAT, 1976; LEVY, 1976; BUSSE, 1978; BRAGINSKIY, 1978). The correct theory of geodynamo may be distinguished in conjunction with the observational evidences of the earth's magnetic field.

Studies of paleomagnetism indicate that while the geomagnetic field has been mainly of a dipole type with an almost constant intensity (e.g., KOBAYASHI, 1968; SCHWARTZ and SYMONS, 1969; KONO, 1971), it has often changed its polarity very rapidly (within $\sim 10^3$ years) and at random intervals (10^5 – 10^6 years) (e.g., COX, 1968, 1969; HEIRTZLER, 1968). Both of the uniformity of the field intensity and the field reversals are of primary importance to the theory of geodynamo. In a previous paper (WATANABE, 1977) the paleomagnetic evidences of the uniform field intensity was important to infer the basic properties of geodynamo. Geomagnetic reversals may also offer another constraint on the

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theory of geodynamo. In this paper the geomagnetic reversals are analyzed on the same $\alpha\omega$ -dynamo model as in the previous paper.

Various models have been proposed to explain the geomagnetic reversals. They are divided into four groups; mechanical analogous models (e.g., RIKITAKE, 1958; LOWES and WILKINSON, 1967), statistical models (COX, 1968; KONO, 1972), kinematic models (e.g., TAKEUCHI, 1956; BRAGINSKIY, 1964; NAGATA, 1969; PARKER, 1969; LEVY, 1972) and hydromagnetic models (STEVENSON and WOLFSON, 1966; KROPACHEV, 1971a, b; GUBBINS, 1975).

Although the studies on the analogous dynamo models have revealed that the magnetic field performs complicated oscillations and occasional reversals, the application of these results to the actual geomagnetic reversals is limited because of the crudeness of the models. Statistical models have been proposed mainly to interpret the statistical properties of the geomagnetic field, and do not enter the physical processes of the field reversals. The most striking features of the reversals are that there is a marked reduction in intensity associated with reversals (e.g., OPDYKE *et al.*, 1973) and that the time required for a reversal is very short compared with the free decay time of the dipole field (e.g., YUKUTAKE, 1968). The intensity reduction during a reversal is naturally interpreted by almost all models. Concerning the time constant of the field reversal, RIKITAKE (1972) pointed out that the time required for completing the decay and growth of the dipole field would be much longer than the observed one, unless some degenerating mechanism was taken into consideration. In this point it is noted that only the model of PARKER (1969) and LEVY (1972) involves fluctuations in the distribution of cyclones which provide the negative feedback. The time constant of the Parker-Levy process, however, cannot be estimated, because they assumed arbitrary strong bursts of cyclones and stasis so as to simplify the calculations.

Although the Parker-Levy model may result in such rapid reversals as shown by the observations, the basic question is that why and how do fluctuations in the distribution of cyclones occur. It should be also noted that even if a kinematic mechanism can cause a field reversal, the field cannot grow again from a stray field when the steady state fluid motion is recovered. That is because the steady state fluid motion can only maintain the magnetic field present. It is therefore necessary to take into account the dynamics of the fluid motion that has a sufficient amount of kinetic energy that can be converted to magnetic energy. The best approach to the dynamical problem is to integrate the equations of a dynamo model forward in time. This is a substantially harder numerical problem than simply obtaining steady or exponentially growing solutions. There have already been several attempts at a numerical solution of the dynamo model, including rotation, and driven by a body force (STEVENSON and WOLFSON, 1966; KROPACHEV, 1971a, b). But none of them have been successful because of the severe restriction on the size of the time step imposed by the rotation.

Avoiding this difficulty by omitting the rotation, GUBBINS (1975) could get a numerical solution of this problem. In his model fluid motions were driven in a sphere by an applied body force. Calculating the time-evolution of the magnetic field, he found that when the dynamo was driven well above a critical limit the magnetic field underwent growing oscillations which might ultimately lead to a geomagnetic-type reversal. But the

time required to complete a reversal was very long (3×10^4 years). It is also unrealistic to omit the rotation that has a significant role in the dynamics of the earth's core. Considering these difficulties it seems very desirable to explore a hydromagnetic dynamo model which is simple to permit the numerical solution but includes appropriately both of the Coriolis and the Lorentz forces dominant in the earth's core. In the hydromagnetic $\alpha\omega$ -dynamo model studied in this paper, the effects of these forces are evaluated analytically. The numerical solution of the $\alpha\omega$ -dynamo would then be easier than the direct numerical analysis of the effects of the rotation and the magnetic field.

2. A Hydromagnetic $\alpha\omega$ -Dynamo Model

In the earth's core non-uniform rotation is supposed to prevail due to the momentum transfer of convective motions. The non-uniform rotation is effective in generating a toroidal field from a poloidal field. To complete the dynamo process a feedback mechanism from toroidal to poloidal fields is necessary. We take a standpoint that turbulent motions play an essential part in producing the poloidal field by generating the mean electromotive forces parallel to the mean magnetic field, which was called α -effect by STEENBECK and KRAUSE (1969). This so called $\alpha\omega$ -dynamo is considered as a whole process of geodynamo.

The induction equations of the mean magnetic field ($\mathbf{B}$) are

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl} (\mathbf{V}_\varphi \times \mathbf{B} + \alpha \mathbf{B}) + \eta \nabla^2 \mathbf{B} \quad (1)$$

where α and η are the α -effect of turbulence and the magnetic diffusivity. The α -effect of core turbulence under the influence of the Coriolis, Lorentz, and viscous forces is as follows (WATANABE, 1977),

$$\left. \begin{aligned} \alpha &= \alpha_\Omega + \alpha_B \\ \alpha_\Omega &= 8/3\pi\rho\lambda^2 \boldsymbol{\Omega} \cdot \nabla v^2 (B^2 + 12\pi\rho v\eta/\lambda^2)^{-1} \\ \alpha_B &= -1/(3\eta)(\lambda v)^2 \mathbf{B} \cdot \text{curl} \mathbf{B} \cdot (B^2 + 12\pi\rho v\eta/\lambda^2)^{-1} \end{aligned} \right\} \quad (2)$$

Here ρ and ν are the density and the kinematic viscosity of core fluid. $\boldsymbol{\Omega}$ is the rotation rate of the earth. λ and v are the characteristic length scale and intensity of the core turbulence.

The non-uniform rotation is expressed by differential rigid rotations of spherical shells (outer core) and a sphere (inner core),

$$\mathbf{V}_\varphi = -\mathbf{r} \times \boldsymbol{\omega} \quad (3)$$

They are subjected to the electromagnetic couples of time-varying magnetic fields (Γ^B), the angular momentum transfer due to the convective mixing of fluids in the rotating core (Γ^Ω), and the viscous forces (Γ^ν). The equation of motion for the i -th shell or sphere is then,

$$I_i \frac{\partial \omega_i}{\partial t} = \Gamma_i^B + \Gamma_i^\Omega + \Gamma_i^\nu \quad (4)$$

where I_i is the moment of inertia of the i -th shell or sphere and ω_i is its angular velocity.

The former two couples were given by WATANABE and YUKUTAKE (1975) for the shellular model of the earth. The electromagnetic couple acting on the i -th shell is

$$\Gamma_i^B = 1/(4\pi) \left[r^3 \int \int B_r B_\phi \sin^2 \theta \, d\theta \, d\phi \right]_{r_i}^{r_{i+1}} \quad (5)$$

where B_r and B_ϕ are the radial and azimuthal components of the magnetic field, and r_i and r_{i+1} are the inner and outer radii of the i -th spherical shell. The magnetic permeability is taken to be free space value throughout the core. The effective couple due to the angular momentum transfer within the core is

$$\Gamma_i^\Omega = A[r^3 v_r(r)]_{r_i}^{r_{i+1}} \quad (6)$$

where A is a parameter indicating the efficiency of the angular momentum transfer due to mixing of fluids, and v_r is the normalized mean radial velocity ($\max(v_r) = 1$ cm/sec) of the up- and down-going flow.

It is well known that the effect of viscosity is small on the large scale motion in the earth's core, compared with those of other forces such as the Coriolis and Lorents forces. But it will be safe to take into account the viscosity, since it may be useful to suppress instabilities in the numerical solution of the hydromagnetic $\alpha\omega$ -dynamo equations. Furthermore it is noted that the large scale motion feels the total effect of two viscosities, the molecular kinematic viscosity (ν) and the turbulent viscosity (ν_t), and that the turbulent viscosity is usually much larger than the molecular one. The turbulent viscosity is given, in order of magnitude, as

$$\nu_t \simeq \lambda \nu \quad (7)$$

from the analogy between turbulence and molecular chaos. The viscous couple acting on the i -th shell is reduced to

$$\Gamma_i^\nu = 8/3\pi\rho\nu_t \left[r^4 \frac{d}{dr} \omega(r) \right]_{r_i}^{r_{i+1}} \quad (8)$$

The governing Eqs. (1)–(8) are numerically solved in the next section.

3. Numerical Solutions of the Hydromagnetic $\alpha\omega$ -Dynamo

Evolution of the mean field $\mathbf{B}$ and the angular velocities ω_i of the differential rotations are calculated by numerical integrations of the simultaneous equations of the hydromagnetic $\alpha\omega$ -dynamo, Eqs. (1)–(8). To solve Eq. (1) numerically, we adopt the spherical harmonic expansion technique (BULLARD and GELLMAN, 1954) which is a powerful tool for numerical analysis of the kinematic dynamo theory. In the present case, however, the procedure is not so simple even with the Bullard and Gellman formalism, because the α -effect depends also on $\mathbf{B}$. For simplicity we deal only with the axisymmetric fields, and further neglect the latitude dependence of B^2 , replacing it by the latitude-averaged value. The turbulent intensity is also assumed to depend only on the radial coordinate, $\nu(r)$.

We take the radius of the core $b=3,480$ km as the unit of length, b^2/η as the unit of time and measure the velocity in units of η/b . $\mathbf{B}$ and $\mathbf{V}_\phi$ in Eq. (1) are expanded in the forms,

$$\left. \begin{aligned} \mathbf{B} &= \text{curl} (T\hat{\mathbf{r}}) + \text{curl}^2(S\hat{\mathbf{r}}) \\ \mathbf{V}_\phi &= \text{curl} (t\hat{\mathbf{r}}) \end{aligned} \right\} \quad (9)$$

where $\hat{\mathbf{r}}$ is the unit radial vector. The functions T , S , t , and α may be divided into spherical harmonic components as

$$\left. \begin{aligned} T &= \sum_n T_n(r)P_n(\theta), & S &= \sum_n S_n(r)P_n(\theta) \\ t &= \sum_n t_n(r)P_n(\theta) = t_1(r)P_1(\theta) \\ \alpha_\Omega &= a_1(r)P_1(\theta), & \alpha_B &= a_B(r)\mathbf{B} \cdot \text{curl} \mathbf{B} \end{aligned} \right\} \quad (10)$$

where

$$\left. \begin{aligned} t_1(r) &= b^2/\eta r^2 \omega(r) \\ a_1(r) &= 8\pi/(3\eta)\rho\lambda^2\Omega[B^2(r) + 12\pi\rho v\eta/\lambda^2]^{-1} \frac{d}{dr} v^2(r) \\ a_B(r) &= -1/3(\lambda v(r)/\eta)^2[B^2(r) + 12\pi\rho v\eta/\lambda^2]^{-1} \end{aligned} \right\} \quad (11)$$

and $P_n(\theta)$ denotes the Legendre polynomial of order n . Using Eqs. (9)–(11), equation (1) is reduced to a series of simultaneous ordinary differential equations,

$$\left. \begin{aligned} \frac{\partial S_\gamma}{\partial t} &= \nabla_\gamma^2 S_\gamma + \sum_\beta (a_1 T_\beta S_\gamma) + \sum_{ijk} (S_i T_j T_k S_\gamma) \\ \frac{\partial T_\gamma}{\partial t} &= \nabla_\gamma^2 T_\gamma + \sum_\beta (t_1 S_\beta T_\gamma) + \sum_\beta (a_1 S_\beta T_\gamma) + \sum_{ijk} (S_i T_j S_k T_\gamma) \end{aligned} \right\} \quad (12)$$

where

$$\nabla_\gamma^2 = \frac{d^2}{dr^2} - \frac{\gamma(\gamma+1)}{r^2}.$$

Here $(t_1 S_\beta T_\gamma)$, $(a_1 T_\beta S_\gamma)$, and $(a_1 S_\beta T_\gamma)$ are the well known coupling terms that have been already evaluated (e.g., BULLARD and GELLMAN, 1954; ROBERTS, 1972). New terms $(S_i T_j T_k S_\gamma)$ and $(S_i T_j S_k T_\gamma)$, arising from α_B in Eq. (10), have also been calculated (WATANABE, 1977) as follows,

$$\left. \begin{aligned} N_\gamma(S_i T_j T_k S_\gamma) &= \frac{a_B}{r^2} \left[L_{ijk\gamma} \frac{i(i+1)j(j+1)}{r^2} S_i T_j T_k + M_{ijk\gamma} (\dot{S}_i \dot{T}_j - T_i \nabla_j^2 S_j) T_k \right] \\ N_\gamma(S_i T_j S_k T_\gamma) &= K_{ijk\gamma} \frac{a_B}{r^6} i(i+1)j(j+1)k(k+1)\gamma(\gamma+1) S_i T_j S_k \end{aligned} \right\}$$

$$\left. \begin{aligned} & + L_{k\gamma ij} \frac{a_B}{r^4} k(k+1)\gamma(\gamma+1)(\dot{S}_i \dot{T}_j - T_i \nabla_j^2 S_j) S_k \\ & - L_{ijk\gamma} \frac{d}{dr} \left[\frac{a_B}{r^4} i(i+1)j(j+1)S_i T_j \dot{S}_k \right] \\ & - M_{ijk\gamma} \frac{d}{dr} \left[\frac{a_B}{r^2} (\dot{S}_i \dot{T}_j - T_i \nabla_j^2 S_j) \dot{S}_k \right] \end{aligned} \right\} \quad (13)$$

where

$$\left. \begin{aligned} N_\gamma &= \frac{4\pi\gamma(\gamma+1)}{2\gamma+1} \\ K_{ijk\gamma} &= \iint Y_i Y_j Y_k Y_\gamma \sin \theta \, d\theta \, d\varphi \\ L_{ijk\gamma} &= \iint \frac{\partial Y_i}{\partial \theta} \frac{\partial Y_j}{\partial \theta} Y_k Y_\gamma \sin \theta \, d\theta \, d\varphi \\ M_{ijk\gamma} &= \iint \frac{\partial Y_i}{\partial \theta} \frac{\partial Y_j}{\partial \theta} \frac{\partial Y_k}{\partial \theta} \frac{\partial Y_\gamma}{\partial \theta} \sin \theta \, d\theta \, d\varphi . \end{aligned} \right\} \quad (14)$$

Y denotes any surface harmonic, and is simply a Legendre polynomial P here, and the dot is differentiation with respect to r . S_γ and T_γ must also satisfy the boundary conditions,

$$S_\gamma = T_\gamma = 0, \quad \text{at } r=0 \quad (15)$$

$$\frac{dS_\gamma}{dr} + \frac{\gamma}{r} S_\gamma = T_\gamma = 0, \quad \text{at } r=1. \quad (16)$$

For the axisymmetric poloidal field, the radial function $S_\gamma(r)$ is related to the Gauss coefficients of the geomagnetic potential in the following way,

$$g_\gamma^0 = \gamma \left(\frac{b}{R} \right)^{\gamma+2} S_\gamma(1) \quad (17)$$

where R is the radius of the earth. Numerical calculations were carried out by truncating the spherical harmonic series at degree 3.

The above equations of the hydromagnetic $\alpha\omega$ -dynamo contain physical constants and parameters to be postulated in the earth's core; the properties of core turbulence (the length scale λ and the averaged intensity v_0), the efficiency of angular momentum transfer (A) due to mixing of fluid and the configuration of radial mixing velocity (v_r) in the core, the physical properties of the core fluid (ρ , ν , η), and the rate of rotation of the earth (Ω). Among physical properties, the viscosity is most uncertain. But fortunately it was shown (WATANABE, 1977) that the viscous effect is almost negligible compared with those of the Coriolis or magnetic forces, unless such a very high value as $\nu \sim 10^8$ stokes is taken. The values of the physical properties of the core used in this paper are summarized in Table 1.

Table 1. Physical properties in the earth's core.

Rate of rotation of the earth	$\Omega = 7.27 \times 10^{-5} \text{ sec}^{-1}$
Radii (core-mantle)	$b = 3,480 \text{ km}$
(inner-outer)	$c = 1,250 \text{ km}$
Density	$\rho = 10 \text{ g/cm}^3$
Viscosity	$\nu = 10^{-2} \text{ stokes}$
Magnetic diffusivity	$\eta = 2.7 \times 10^4 \text{ cm}^2/\text{sec}$

It is known from the hydromagnetic considerations of the steady state $\alpha\omega$ -dynamo (WATANABE, 1977) that the microscale magnetic Reynolds number ($R_\lambda = \lambda v_0 / \eta$) is limited to the range, $0.2 < R_\lambda \leq 0(1)$, in the present model so as to maintain the magnetic field of 30 G in the core. By considerations for energy balance of geodynamo another constraint has been imposed independently on λ and v_0 . It follows that the scale (λ) must be larger than 10 km and the mean turbulent intensity (v_0) must be smaller than $5 \times 10^{-3} \text{ cm/sec}$. In view of these results the length scale (λ) is taken as 50 km in the present problem. For the turbulent intensity we taken $v_0 = 3 \times 10^{-3} \text{ cm/sec}$ and the form,

$$v(r) = 12 v_0 r_* (1 - r_*)^2 \quad (18)$$

where r_* is the reduced radius as $r_* = (br - c)/(b - c)$. Here c is the radius of the inner-outer core boundary. The same configuration is assumed for the mean radial mixing velocity (v_r) of core flow,

$$v_r(r) = 12 r_* (1 - r_*)^2 \quad (19)$$

The coefficient (A) of the angular momentum transfer (Eq. (6)) due to the radial mixing is adjusted so that the averaged angular velocity of the shallower parts of the core, where the velocity is westward relative to the mantle, may represent the observed westward drift, $1.7 \times 10^{-10} \text{ rad/sec}$, of the geomagnetic secular variation.

With these physical constants and parameters the differential Eqs. (12) with the boundary conditions (15, 16) for the radial functions and the Eqs. (4) for the non-uniform rotation, are simultaneously integrated forward in time using the finite difference method. The range from $r=0$ to $r=1$ is divided into M equal parts. A grid point width, Δr , is equal to $1/M$. A time step width is denoted by Δt . Nonlinear interaction between the magnetic field and the fluid motion (here, the non-uniform rotation) gives rise to the time step restriction:

$$\Delta t \leq \frac{\eta}{b} \frac{\Delta r}{\beta} \quad (20)$$

in a nondimensional form, where β is a typical value of the Alfvén velocity ($\beta = B/\sqrt{4\pi\rho}$). This has given the most severe restriction on computation time.

First, calculations are carried out in the case of $M=20$ (21 radial grid points), where we have taken $\Delta t = 10^{-6}$ as a nondimensional time step. Figure 1 shows the evolution of the dipole (g_1^0) and the quadrupole (g_2^0) fields. It is clearly seen that the dipole field undergoes relatively gradual variations and occasional reversals. A typical behavior of the dipole field is a gradual increase in field intensity and its rapid decay after reaching a

Fig. 1. Evolution of the dipole (g_1^0) and quadrupole (g_2^0) fields with time, for $M=20$.

Fig. 2. Distribution of the non-uniform rotation ω (left side) and the toroidal field T_2^0 (right side) within the core, for the time sequence numbers 1 to 4.

maximum state. It is noted that associated with a reversal of the dipole, the quadrupole field also changes its sign. This indicates that the dipole family (of odd degrees) indeed couples with the quadrupole family (of even degrees) through nonlinearities of the α -effect (α_B). Behaviors of the hexapole (g_3^0) is very similar to those of the dipole. These results suggest that when the dipole reverses, the polarity of the toroidal field from which poloidal fields (such as g_1^0 , g_2^0 , and g_3^0) are produced by the α -effect, also reverses. Indeed this can be seen in Fig. 2 where the magnitude of the toroidal field along the radial distance is shown. Variations in the amplitude of a toroidal field (T_2^0) are described on the

Fig. 3. Distribution of the non-uniform rotation ω (left side) and the toroidal field T_2^0 (right side) within the core, for the time sequence numbers 5 to 8.

Fig. 4. Distribution of the dipole field (S_1^0) in the core, for the time sequence numbers 1 to 8.

right side of the figure. On the left side variations in the non-uniform rotation within the core are shown. The numbers in the figure (1, 2, 3, and 4) correspond to those in Fig. 1, and denote the order of those events. It is seen in the figure that angular velocities of the central parts of the core decrease with time. At number 4 the sense of the non-uniform rotation is reversed, outer parts being eastward and some inner parts being westward relative to the mantle. Corresponding to these variations the toroidal field (T_2^0) also

decreases at the moderate depth in the core, but not becoming the opposite sense yet. So far the dipole field has not been much reduced in its intensity. The following sequence of variations (the numbers 5, 6, 7, and 8) in the non-uniform rotation and the toroidal field is shown in Fig. 3. At number 6 the non-uniform rotation has been recovered again. At this moment the toroidal field is weakened and has an opposite component at the central parts of the core. The dipole field is also weakened, but yet having the same sign as before. At number 8 the toroidal field is of quite the opposite sense and the dipole field has reversed its polarity.

From these figures a picture of the field reversal can be described as follows. First, fluctuations in the non-uniform rotation of the core are caused by the nonlinear interactions between the magnetic field and the fluid motion. The non-uniform rotation happens to become the opposite sense and produce the negative flux of toroidal field from the dipole field present. This results in the reversed toroidal field, and from which the negative flux of the dipole field is created by the α -effect. In these ways the reversal of the dipole field is completed. Evolution of the dipole amplitude (S_1^0) in the core is shown in Fig. 4.

Similar calculations were also carried out with $M=40$. As the time step we took a value of $\Delta t = 5 \times 10^{-7}$ to avoid numerical instabilities. In order to take into account the electrical conductivity of the bottom part of the mantle, the outermost part between the grid points 41 and 43 was taken to be conducting ($\sigma_m = 10^{-8}$ emu). This corresponds to the conducting layer of 124 km thickness. The boundary condition of the magnetic field at the core-mantle interface (Eq. (16)) is then modified to

$$\frac{\partial S_\gamma}{\partial r} + \frac{\gamma}{r} S_\gamma = T_\gamma = 0, \quad \text{at } r = 1 + \delta r \quad (21)$$

where δr is the non-dimensional thickness of the conducting layer of the mantle and now equal to 2/40.

In this case the time-variation of the magnetic field is smoother than in the case of $M=20$, but nevertheless the field reversals in fact take place. In a long series of numerical calculation, only a part where the dipole changes its polarity is shown in Fig. 5. From that figure the time required for a complete decay of the dipole field is estimated to be about 2,880 years. This is quite the same order of magnitude as the time constant suggested from paleomagnetic observations.

In the present earth model the bottom part of the mantle has a low but finite electrical conductivity ($\sigma_m = 10^{-8}$ emu). A small amount of leakage of toroidal fields from the core into the conducting mantle results in the electromagnetic coupling of the mantle to the outer core. Then associated with variations of the magnetic field in the core, the electromagnetic couples (Eq. (5)) acting on the mantle and the core will also change and lead to fluctuations in the rotational speed of the mantle as well as the outer part of the core. In Fig. 5 variations in the rotation rate of the mantle (ω_m) and those in the angular velocity of the surface layer of the core (ω_w) are also shown. The angular velocity of the surface layer was obtained by averaging the velocities of the upper eight layers, the total thickness of which being about 700 km.

It is clearly seen in the figure that the disruption of the westward drift of the surface

Fig. 5. Evolution of the dipole field (g_1^0) and the angular velocities of the mantle (ω_m) and the outer part of the core (ω_w), for $M=40$.

layer precedes the decay of the dipole field by about 5,000 years. About 1,440 years after the disruption of the surface westward motion, the mantle is accelerated by the electromagnetic coupling. The increase in the angular velocity amounts to about 8×10^{-11} rad/sec in a period of 8,640 years or so. Consequently the length of a day decreases about 95 msec during the period.

4. Discussion and Conclusion

In this paper the non-steady state of the hydromagnetic $\alpha\omega$ -dynamo model has been investigated. The non-uniform rotation has been also taken to be non-stationary and subjected to the various forces relevant in the earth's core; the electromagnetic torques, the effective torques due to fluid mixing in the rotating system and the viscous torques. The governing equations of the $\alpha\omega$ -dynamo have been numerically integrated forward in time, using the spherical harmonic expansion and finite difference methods. Calculating the time-evolution of the magnetic field it has been found that the dipole field performs relatively gradual variations and occasional reversals as seen in Fig. 1. From Figs. 2 and 3 it has also been found that the reversal is preceded by variations in the toroidal field and further by those in the non-uniform rotation. This suggests that the reversal of the dipole field is caused essentially by the disruption of the non-uniform rotation which is resulted in through the electromagnetic interactions with the time-varying magnetic field.

The time required for a complete decay of the dipole field is about 3,000 years which

is consistent with paleomagnetic observations. Polarity intervals has been short (tens of thousand years), however, compared with those suggested by paleomagnetic observations (about once in 0.2 m.y.). For the mean motion we have considered only the non-uniform rotation and neglected other components. The meridional circulation is known to stabilize the kinematic $\alpha\omega$ -dynamo (ROBERTS, 1972). Therefore if we consider some meridional circulation as well as the non-uniform rotation, even the hydromagnetic $\alpha\omega$ -dynamo may become stable so that the polarity intervals can be reconciled with the observed ones.

By taking into account the electrical conductivity of the bottom part of the mantle, the electromagnetic coupling of the mantle to the core has also been studied. Associated with the field reversals the rotational speed of the mantle is accelerated by 8×10^{-11} rad/sec in a period of about 8,600 years.

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