The Hysteresis of Piezomagnetization

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(Received September 19, 1980)

The magnetization of rock acquired by applying one cycle of uniaxial compression comprising an increase of compression from zero to \( P \) and a decrease from \( P \) to zero in the presence of a magnetic field (\( H \)) is larger than its initial magnetization before applying the compression cycle. This phenomenon has been called the piezomagnetization effect.

In the course of the second cycle of the same compression in the same field \( H \), the magnetization is subjected to a hysteresis phenomenon, called the hysteresis of piezomagnetization. The hysteresis is reproducible in repeated processes of applying the same compression cycles.

The hysteresis of piezomagnetization can be theoretically interpreted as due to an effect of finite limits of magnetic domain volumes on the movement of 90° domain-walls.

1. Introduction

It appears that the key problems surrounding the effect of mechanical stresses upon the magnetization of natural rocks have already been reasonably well understood on the basis of both physics of ferromagnetism and mineralogical crystallography. A review of these problems (NAGATA, 1970) has summarized that the effects of mechanical stresses on the magnetization processes of magnetic mineral grains in natural rocks can be classified into two main categories, i.e. (a) the rotation of spontaneous magnetization of individual magnetic domains to minimize the sum of field, magnetocrystalline and magnetostrictive energies, and (b) the movements of 90° domain-walls to minimize the magnetostrictive energy in individual magnetic grains.

It is observed in these theories that the possible effect of mechanical stresses on the resistivity of domain-walls themselves is ignored as a minor secondary effect, but it is theoretically clear that the 180° domain-walls can not have any effect of applied elemental mechanical stresses so far as they are uniaxial.

The majority of experimental data of the mechanical stress effects on the natural rock magnetization demonstrated by KAPITSA (1955), KALASHNIKOV and KAPITSA (1952), DOMEN (1957), NAGATA and KINOSHITA (1965), NAGATA and CARLETON (1968, 1969a, b), OHNAKA and KINOSHITA (1968), and Pozzi (1977) have been satisfactorily interpreted on the basis of a combination of the rotation theory and the 90° domain-wall movement theory (e.g. NAGATA, 1970).

However, there still remain several unresolved problems in regard to the mechanical stress effects on rock magnetization—tentatively named the piezomagnetization of rocks.
One of these unresolved problems may be, as stated by Nagata (1966), the so-called irreversible hysteresis phenomenon of piezomagnetization. The observed hysteresis of piezomagnetization has been defined by Nagata (1966) as follows: Let us note the magnetization intensity of a rock sample in the presence of a magnetic field \(H\) by \(J(H_+)\). When a uniaxial compression \(P\) is applied on the sample along the direction of a vector \(
abla\), the resultant magnetization is denoted by \(J(H_+, P^+)\). In the presence of \(\nabla\), \(P\) is increased up to \(P_+\), and then \(P\) is decreased from \(P_+\) to zero. The magnetization in the course of decreasing \(P\) from \(P_+\) to zero is denoted by \(J(H_+, P_+, P^-)\). Then, the magnetization corresponding to \(P^- = 0\), denoted by \(J(H_+, P_+, P_0)\), is always larger than \(J(H_+)\), namely

\[
J(H_+, P_+, P_0) > J(H_+). \tag{1}
\]

The increased magnetization, \(J(H_+, P_+, P_0)\), obtained by applying \(P\) once in the presence of \(H\), has been well interpreted by Nagata and Carleton (1969a, b) as due to the irreversible movement of 90° domain-walls caused by the magnetostrictive effect owing to \(P\).

When \(P\) is again increased from zero to \(P_+\) in \(H\), the magnetization in the course is denoted by \(J(H_+, P_+, P_0P^+)\).

\[
J(H_+, P_+, P_0P^+) > J(H_+, P_+P^-) \quad \text{for} \quad 0 < P < P_+, \tag{2}
\]

and

\[
J(H_+, P_+, P_0P_+) = J(H_+, P_+) \quad \text{for} \quad P = P_+.
\]

When \(P\) is again decreased from \(P_+\), the observed value of \(J(H_+, P_+, P_0P_+P^-)\) can be represented by

\[
J(H_+, P_+, P_0P_+P^-) = J(H_+, P_+P^-) < J(H_+, P_+P_0P^+). \tag{3}
\]

Namely, an irreversible hysteresis of magnetization takes place with respect to \(P\), when \(P\) is repeatedly applied on a rock sample which has already acquired the increased magnetization, \(J(H_+, P_+, P_0)\). The hysteresis phenomenon has been named the hysteresis of piezomagnetization. The Nagata-Carleton theory cannot explain this hysteresis phenomenon, their theory suggesting that

\[
J(H_+, P_+, P_0P_+P^-) = J(H_+, P_+P_0P^+P_+). \tag{4}
\]

The hysteresis of piezomagnetization will be experimentally and theoretically re-examined in the present work.

2. Experimental Results

The magnetometer system to measure the magnetization of a sample under the effects of both a magnetic field and a uniaxial compression is the same in principle as that used by Nagata and Kinoshita (1965), namely, a ballistic magnetometer, in which a pick-up system comprising two inversely connected search coils is fixed to a long solenoid to supply a magnetic field (Nagata, 1961). The sample to be examined is fixed between two pieces of non-magnetic anvil and the moving search-coil method is adopted for measurements.
Since it has been experimentally confirmed that all characteristics of the piezomagnetization of natural igneous rocks, dependent on various combination of application and removal of a magnetic field \( (H) \) and a uniaxial compression \( (P) \) (NAGATA and CARLETON, 1968, 1969a, b), are exactly the same as those of assemblages of ferromagnetic mineral grains extracted from natural rocks and also as those of assemblages of synthesized magnetite (or titanomagnetite) grains (NAGATA and KINOSHITA, 1965), magnetite.
samples are examined in the present work in order to achieve a higher sensitivity with respect to the piezomagnetic phenomena.

Figures 1 and 2 show two examples of these experimental results. As mentioned in the introduction, \( J(H, P, P_{\downarrow}) \) becomes \( J(H, P_{+}, P_{0}) \) which is considerably larger than \( J(H, \) before applying one cycle of a uniaxial compression up to \( P_{+} \). When the magnetic field \( (H) \) is removed, the remanent magnetization \( J_{R}(H, P_{+}, P_{0}H_{0}) \) is defined as the piezoremanent magnetization which has a considerably larger value than the ordinary isothermal remanent magnetization, \( J_{R}(H_{+}, H_{0}) \), acquired in the same magnetic field. The Nagata-Carleton theory indicates that

\[
\begin{align*}
\Delta J &= J(H, P_{+}, P_{0}) - J(H_{+}) > 0, \\
\Delta J_{R} &= J_{R}(H, P_{+}, P_{0}H_{0}) - J_{R}(H_{+}, H_{0}) > 0,
\end{align*}
\]

and

\[\Delta J > \Delta J_{R}.\]

As clearly seen in these two figures, the curve of \( J(H_{+}, P_{+}, P_{0}P'_{\downarrow}) \) versus \( P \) is considerably higher than the \( J(H_{+}, P_{+}, P_{\downarrow}) \) versus \( P \) curve, but the \( J(H_{+}, P_{+}, P_{0}P', P_{\downarrow}) \) versus \( P \) curve is almost identical to the \( J(H_{+}, P_{+}, P_{\downarrow}) \) versus \( P \) curve within experimental errors. To verify the hysteresis of piezomagnetization, experiments of a further application of \( P \) on \( J(H_{+}, P_{+}, P_{0}P_{+}, P_{0}) \) were repeated. The experimental results have indicated, within experimental errors, that

\[
\begin{align*}
J(H_{+}, P_{+}, P_{0}P_{+}, P_{0}P'_{\downarrow}) &\approx J(H_{+}, P_{+}, P_{0}P'_{\downarrow}), \\
J(H_{+}, P_{+}, P_{0}P_{+}, P_{0}P'_{\downarrow}) &\approx J(H_{+}, P_{+}, P_{0}P_{+}, P_{\downarrow}) \approx J(H_{+}, P_{+}, P_{\downarrow}).
\end{align*}
\]

These experimental results may demonstrate a reproducible hysteresis phenomenon of piezomagnetization after the piezoremanent magnetization is acquired by the initial one cycle application of \( P \) in the presence of \( H \).

Thus, the hysteresis of piezomagnetization with respect to \( P \) change in a range of \( 0 \leq P \leq P_{+} \) can be represented by

\[
\begin{align*}
\Delta J &= J(H_{+}, P_{+}, P_{0}P'_{\downarrow}) - J(H_{+}, P_{+}, P_{\downarrow}) > 0 \quad &\text{for} \quad P_{0} < P < P_{+}, \\
&= 0 \quad &\text{for} \quad P = 0 \quad \text{and} \quad P = P_{+}.
\end{align*}
\]

3. Theoretical Interpretation

As the magnetization \( (J) \) in a magnetic field \( (H) \) is due partially to the rotation of spontaneous magnetization and partially to the movements of 90° and 180° domain-walls, \( J \) can be expressed as

\[J = \alpha J_{\text{rot}} + \beta J_{180} + \gamma J_{90}\]
with $\alpha + \beta + \gamma = 1$. It has been generally believed that the contribution of $J_{\text{rot}}$ is about a half of $J$, whereas the ratio $\beta/\gamma$ varies in different cases, sensitively depending on the grain size of ferromagnetic minerals in the case of natural rocks.

In (7), both $J_{\text{rot}}$ and $J_{90}$ are dependent on the uniaxial stress ($P$) whereas $J_{180}$ is independent of $P$. The dependence of $J_{\text{rot}}$ on $P$ can be experimentally and theoretically expressed (e.g. Nagata, 1969b) as

$$J_{\text{rot}}(P) = \frac{J_{\text{rot}}^0}{1 + \beta^* P},$$  \hspace{1cm} (8)$$

where $\beta^*$ is a material constant and takes a numerical value of $1 \times 10^{-4}$ cm$^2$/kg in the order of magnitude for most igneous rocks and assemblages of ferromagnetic minerals. The observed general trend of a decrease of $J$ with an increase of $P$ is attributable to the change of $J_{\text{rot}}$ with $P$ expressed by (8). Since $J_{\text{rot}}(P)$ is reversible with respect to $P$, the irreversible changes of $J$ with $P$ shown in Figs. 1 and 2 must be attributed to the irreversible changes of $J_{90}$ with respect to $P$.

The Nagata-Carleton theory of piezomagnetization on the basis of irreversible movements of 90° domain-walls can reasonably well stand for the observed irreversibility between $J(H,P\uparrow)$ and $J(H,P,P\downarrow)$. In their theory, a pair of 90° domain-walls are always taken into account; a (plus) domain-wall for which the applied magnetic field ($H$) and uniaxial stress ($P$) cause wall movements in the same direction and a (minus) domain-wall for which the effect of $P$ on wall movement is opposite to that of $H$. The conditions are schematically illustrated in Fig. 3. The ($-J$) domain bounded by the (plus) 90° domain-wall should be reduced by applying $H$ and $P$ if the magnetostriction coefficient is positive, thus increasing the magnetization along the direction of $H$ for both $H$ and $P$, whereas the ($+J$) domain bounded by the (minus) 90° domain-wall should be increased by applying $H$ and $P$.
but reduced by applying $P$.

It is assumed in the original Nagata-Carleton theory that the magnitudes of $H$ and $P$ are sufficiently small so that the above-mentioned processes of the (plus) and (minus) 90° domain-walls can take place for any value of $H$ and $P$.

On the basis of the Rayleigh's laws of magnetic hysteresis phenomenon expressed by

$$J_{90}(H+) = XH + bH^2,$$  \hfill (9)

$J_{90}(H + P^\uparrow)$ and $J_{90}(H + P^\downarrow)$ in the ideal case illustrated in Fig. 3 are expressed by the Nagata-Carleton theory as

$$J_{90}(H + P^\uparrow) = XH + b\left(H^2 + HH_p + \frac{1}{4}H_p^2\right) \quad \text{for} \quad 0 \leq H_p \leq 2H,$$

and

$$J_{90}(H + P^\downarrow) = XH + b\left(H^2 + HH_{p+} + \frac{1}{4}H_{p+}^2\right) \quad \text{for} \quad 0 \leq H_{p+} \leq 2H,$$

where $J_s$ and $\lambda_s$ denote respectively the spontaneous magnetization and the isotropic magnetostriction coefficient.

These theoretical results show that $J_{90}(H + P^\uparrow)$ increases with an increase of $P$, but $J_{90}(H + P^\downarrow)$ is kept constant in a range of $0 \leq P \leq P_+$. If $0 \leq P \leq P_+$, the theory indicates that

$$J_{90}(H + P^\uparrow + P^\downarrow) = J_{90}(H + P^\downarrow + P^\uparrow) = J_{90}(H + P^\downarrow + P^\downarrow)$$

which is a function of only $H$ and $P_+$ and is independent of $P$.

The theoretical expressions of $J_{90}(H^\uparrow)$, $J_{90}(H^\downarrow)$, $J_{90}(H^\uparrow P^\downarrow)$, etc., given by (10), (11), and (13) respectively, are concerned with the ideal case where the 90° domain-wall planes make an angle of $\pi/4$ with the direction of $H$. In the experimentally examined samples, a large number of the domain-wall planes are at random angles to the $H$-direction. The piezomagnetization characteristics in such a case have been theoretically examined by Nagata and Carleton (1969b), the result showing that no qualitative difference is introduced by averaging the random orientations of wall planes, but the additional effect of $P$ or $P_+$ is quantitatively reduced to about 1/3. In the present theoretical discussions of the hysteresis phenomenon of piezomagnetization, therefore, the simple ideal model will be considered for the sake of mathematical simplicity.

As represented by (13), there is no hysteresis between $J_{90}(H^\uparrow P^\downarrow)$ and $J_{90}(H^\uparrow P^\downarrow P^\downarrow)$ or between $J_{90}(H^\downarrow P_0^\downarrow P^\downarrow)$ and $J_{90}(H^\downarrow P_0^\downarrow P_0^\downarrow P^\downarrow)$, so far as all (plus) and (minus) 90° domain-walls effectively behave for any large value of $H$ and $P$ in the
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However, individual volumes of the \((-J)\) and \((+J)\) domains in an actual sample should be finite. It can be considered, therefore, that when \(H + H_p\) exceeds a certain critical value \((H^*)\) in a \((-J)\) domain, its volume becomes zero, and further increase of \(H_p\) no longer affects its magnetization. The same situation should happen in a corresponding \((+J)\) domain when the magnetic field is reversed, because the presence of a pair of \((+J)\) domain and \((-J)\) domain of initially the same size is always assumed in the present theory.

The differential density spectrum of a pair of domains of \(H^*\) in their critical value is noted by \(n(H^*)\). It seems very likely from the viewpoint of domain structure in rockforming ferromagnetic minerals that the spectral form of \(n(H^*)\) is characterized by \(n(H^*)=0\) for \(0 \leq H^* \leq H^*_L\), \(n(H^*)>0\) for \(H^*_L < H^* < H^*_H\) and \(n(H^*)=0\) for \(H^*_H \leq H^*\). In Fig. 3, the cumulative spectrum given by

\[
N(H^*) = \int_{H^*}^{\infty} n(H^*) \, dH^* \tag{14}
\]

is schematically illustrated, where \(N(H^*)\) values on the right side of \(H^*=0\) represent an assemblage of the \((-J)\) domains, while those on the left-side represent that of the corresponding \((+J)\) domains. The \(N(H^*)\) versus \(H^*\) curve must be symmetric with respect to \(H^*=0\), because \(J(-H) = -J(H)\). In the original Nagata-Carleton theory of piezomagnetization, it is assumed that \(n(H^*)=0\) for any value of \(H^*\), because their experimental data are concerned only with small values of \(H\) (several Oersteds) and small values of \(P\) (several hundred bars). Let us consider in Fig. 3 such an example where \(H_p\) of \(H + H_p > H^*\) and \(H_p - H < H^*\) is applied on a sample in addition to \(H\). Then those \((-J)\) domains for which \(H^* < H + H_p\) are diminished to zero, whereas the corresponding \((+J)\) domains for which the effective field is given by \(H - H_p\) still keep their finite volumes. The entire piezomagnetic characteristics caused by the above-mentioned process can be derived as a sum of those of the individual components having \(H^*\) as their characteristics critical value. Then, an individual component can be defined by

\[
N=1 \quad \text{for} \quad -H^* \leq H' \leq H^* ,
\]

\[
N=0 \quad \text{for} \quad H' \leq -H^* \quad \text{and} \quad H' \geq H^* ,
\]

where

\[
H' = H + H_p \quad \text{for the} \ (-J) \text{ domains} ,
\]

\[
H' = H - H_p \quad \text{for the} \ (+J) \text{ domains} .
\]

In Fig. 4, the magnetization curve of the \((-J)\) domain bounded by the (plus) walls \((J_+\) and that of the \((+J)\) domain bounded by the (minus) walls \((J_-\) are separately illustrated for a case where \(H + H_p > H^*\) but \(H - H_p > -H^*\). The \(J_+\) magnetization of \((-J)\) domain is saturated at \(H + H_p = H^*\) and no further change takes place with a further increase in \(H_p\). A Rayleigh hysteresis loop comprising \(J_+(H_p, P_+ P\) and \(J_-(H_p, P, P_0 P\) appears only for a range between \(H\) and \(H^*\). On the other hand, the \(J_-\) magnetization of \((+J)\) domain continuously changes up to \(H - H_p\), and a Rayleigh hysteresis loop comprising
Fig. 4. Magnetic hysteresis curves of the \( J_+ \) magnetization of \((-J)\) domain and the \( J_- \) magnetization of \((+J)\) domain. Case of \( 2H<H^*-H<H_p<H^*+H \).

\[ J_+(H+) \text{ and } J_-(H+P+P_0) \text{ extends for a range between } H-H_p \text{ and } H. \]

1) In Fig. 4, \( J(H+) \) is represented by point A, whereas \( J(H+P+P_0) \) is represented by the middle point between B and C. The illustrated difference between \( J(H+) \) and \( J(H+P+P_0) \) has been named the initial irreversibility of piezomagnetization (NAGATA, 1960).

2) Since the Rayleigh hysteresis loop of \( J_+ \) magnetization is smaller than that of \( J_- \) magnetization, \( \frac{J_+(H+P+P_0) + J_-(H+P+P_0)}{2} \) is always smaller than \( \frac{J_+(H+P+P_0P}) + J_-(H+P+P_0P)}{2} \), though these two values are identical to each other at \( H_p=0 \) and \( H_p=H_{p+} \). Thus, a hysteresis phenomenon of piezomagnetization appears with respect to a change of \( H_p \) between 0 and \( H_{p+} \). Since both Rayleigh hysteresis loops, \( J_+(H_+P+P_0P) \) plus \( J_+(H_+P+P_0P) \) and \( J_-(H_+P+P_0P) \) plus \( J_-(H_+P+P_0P) \), are reproducible in the further repeated processes of cyclic changes of \( H' \) between \( H \) and \( H+H_p \), the observed reproducibility of the hysteresis of piezomagnetization can be theoretically explained by the present model.

4. Example of Theoretical Simulation of Experimental Result

A theoretical model to interpret the hysteresis of piezomagnetization and its reproducibility is proposed in principle in Section 3. It seems that the theoretical model can qualitatively explain both the initial irreversibility of piezomagnetization and the hysteresis of piezomagnetization.

To examine the experimental data in a more realistic way, however, each process of magnetization changes caused by \( H \) and \( H_p \) must be more quantitatively expressed. Regarding even the elemental component having \( H^* \) of the critical field, mathematical expressions of magnetization in individual processes of increases and decreases of \( P \) in the
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presence of $H$ appear fairly complicated because the magnetization is fundamentally dependent on the mutual magnitudes of $H$, $H_p$, and $H^*$. The final mathematical expression of $J_{90}(H+P\downarrow)$, $J_{90}(H+P\uparrow+P\downarrow)$, and $J_{90}(H+P\uparrow+P\downarrow+P\downarrow)$ for various cases are summarized in the appendix, where $J_{90}(H+P\uparrow+P\downarrow)=J_{90}(H+P\downarrow)$ and $J_{90}(H+P\uparrow+P\downarrow+P\downarrow)=J_{90}(H+P\downarrow)$ throughout all cases. As expressed in the appendix, $J_{90}(H\uparrow)$, $J_{90}(H+P\uparrow)$, and $J_{90}(H+P\uparrow+P\downarrow)$ for given values of $H$ and $P$ (consequently $H$ and $H_p$) take considerably different numerical values for different magnitudes of parameter $H^*$.

Expressing the magnetization at a certain stage of the piezomagnetization process of the $H^*$ component by $J(H^*)$, the total magnetization $J_{90}$ can be given by

$$J_{90} = \int_0^{-\infty} J(H^*) n(H^*) dH^* \int_0^{\infty} n(H^*) dH^*,$$  \hspace{1cm} (16)

or approximately by

$$J_{90} \approx \sum J(H^*) n(H^*) \sum n(H^*).$$  \hspace{1cm} (16')

An example of a theoretical simulation on the basis of the present model for an experimental result illustrated in Fig. 1 will be described in the following. As for the distributions of magnetization ($J$) to the three categories, $J_{\text{rot}}$, $J_{180}$, and $J_{90}$ in (7), we may be able to reasonably assume that $\alpha=1/2$. Regarding $\beta$ and $\gamma$, experimental data for assemblages of fine grains (about $50\mu m$ in mean diameter) of natural magnetite have indicated that $\gamma \geq 2\beta$ (NAGATA and CARLETON, 1969b). Since the grain size of natural magnetites is about $50\mu m$ in their mean diameter, we may assume that $\beta=1/6$ and $\gamma=1/3$.

As experimentally and theoretically demonstrated, the rotational magnetization component ($J_{\text{rot}}$) reversibly changes with $P$ in a way as represented by (8). The coefficient $\beta^*$ in (8) is theoretically expressed (NAGATA, 1970) by

$$\beta^* = \frac{3\lambda_s}{NJ_s^2 + \frac{4K}{3\pi}},$$  \hspace{1cm} (17)

where $\lambda_s$, $N$, $J_s$, and $K$ denote respectively the isotropic magnetostriction coefficient, the average demagnetization factor of ferromagnetic grains, the spontaneous magnetization and the crystalline anistropic energy. As the natural titanomagnetite sample examined in Fig. 1 is approximately represented by $5\%$ Fe$_2$TiO$_4$ - $95\%$ Fe$_3$O$_4$, we can put $J_s=450$, $K=1.9 \times 10^5$ and $\lambda_s=4.7 \times 10^{-5}$ in cgs emu (SYONO, 1965) and $n=3.4$ (NAGATA, 1943) in (17), whence we get $\beta^*=1.8 \times 10^{-10}$ (cgs) = $1.8 \times 10^{-4}$ cm$^2$/kg. The numerical value of $\beta^*$ thus theoretically estimated is in good agreement with experimental data (e.g. NAGATA, 1970).

On the other hand, the numerical values of coefficients of $X$ and $b$ for the examined sample have been determined from its magnetic hysteresis curves as $X=0.22$ emu/gm/Oe and $b=7.2 \times 10^{-5}$ emu/gm/Oe$^2$. Although the $X$ and $b$ values thus experimentally determined represent $X=\alpha X_{\text{rot}} + \beta X_{180} + \gamma X_{90}$ and $b=(\beta/\beta+\gamma)b_{180}+(\gamma/\beta+\gamma)b_{90}$ respectively, it could be simply assumed that $X_{90}=X$ and $b_{90}=b$ for reasonable approximations.

The relationship between $P$ and $H_p$ is theoretically expressed by (12). Then, (12) for
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Fig. 5. Example of theoretical curve of hysteresis of piezomagnetization of titanomagnetite:
Theoretical curves of \( J(H, P_\uparrow) \), \( J(H, P_\downarrow P_\uparrow) \), and \( J(H, P_\downarrow P_\downarrow P_\uparrow) \).

the present sample is numerically given by \( H_p = 2.2 \times 10^{-7} \) (cgs emu). Thus, the parameters necessary for a theoretical simulation of the hysteresis of piezomagnetization such as \( \beta^*, X_{90}, \), b, and \( H_p/P \) can be evaluated mostly based on experimentally observed data. However, no information is available from independent experiments for the spectral form of \( n(H^*) \) for \( N(H^*) \). Hence, several different models of \( N(H^*) \) are arbitrarily assumed in the trial-and-error approach to theoretically simulate the observed hysteresis of piezomagnetization. Figure 5 illustrates an example of the theoretical hysteresis of piezomagnetization, which is a reasonably good approximation of the observed hysteresis (Fig. 1). It is assumed in this model that \( n(H^*) = 0 \) for \( 0 \leq H^* \leq 110 \) Oe and \( H^* \geq 990 \) Oe,

\[
n(H^*) = \frac{\gamma}{880(\text{Oe})} \quad \text{for} \quad 110 \leq H^* \leq 990 \text{ Oe}.
\]

Namely,

\[
N(H^*) = \gamma \quad \text{for} \quad 0 \leq H^* \leq 1.10 \text{ Oe},
\]

\[
= \gamma \left( 1 - \frac{H^* - 110}{880} \right) \quad \text{for} \quad 110 \leq H^* \leq 9.90 \text{ Oe},
\]

\[
= 0 \quad \text{for} \quad 990 \text{ Oe} \leq H^*.
\]

The theoretical curves of \( J(H_+P_\uparrow), J(H_+P_\downarrow P_\uparrow), \) and \( J(H_+P_\downarrow P_\downarrow P_\uparrow) \) thus calculated and shown in Fig. 5 are at least in qualitative agreement with the corresponding experimental curves given in Fig. 1. As already mentioned, the hysteresis loop comprising \( (H_+P_\downarrow P_\uparrow) \) and \( J(H_+P_\downarrow P_\downarrow P_\uparrow) \) is repeatable by repeated cycles of an increase and a decrease of \( P \) between \( P = 0 \) and \( P = P_+ (= 2 \text{ kbars}) \). This theoretical conclusion also is in agreement with the experimental result.
5. Concluding Remarks

It seems that the hysteresis phenomenon of piezomagnetization, which appears in an observable magnitude when comparatively large magnetic field and uniaxial stress are applied, is theoretically interpreted on the basis of the 90° domain-wall movement model in the present study. A significant difference of the present theory from the original Nagata-Carleton one is simply an explicit consideration of the small finite volume of individual magnetic domains concerned. As experimentally demonstrated with the domain pattern configuration changes in magnetites caused by uniaxial compressions (e.g. Nagata, 1969c), this assumption of the finite volume of individual magnetic domains seems to be very feasible. In the experimental demonstration, magnetic domains of several tens of μm wide and about 100 μm long are diminished to zero by a uniaxial compression of 100 kg/cm², just as theoretically considered in the present study.

Then, it could be repeated to conclude that the effects of mechanical stresses on the magnetization processes of magnetic grains in natural rocks can be classified into two main categories, namely (a) the rotation of spontaneous magnetization of individual magnetic domains to minimize the sum of field, magnetocrystalline, and magnetostrictive energies, and (b) the movement of 90° domain-walls to minimize the magnetostrictive energy in individual domains.

The present study has simply demonstrated the principle of a theoretical model to interpret the observed hysteresis of piezomagnetization in an approximate mathematical expression. As mentioned in Section 3, the actual condition of random orientation of 90° domain-walls in natural rocks must be taken into account in more precisely simulating a theoretical model of the hysteresis phenomenon of piezomagnetization.

6. Appendix: Theoretical Expressions of \( J_{90}(H+P\uparrow) \), \( J_{90}(H+P\downarrow+P\downarrow) \), and \( J_{90}(H+P\downarrow+P_{0}P\downarrow) \)

In the simple case that the direction of \( H \) is paralleled to the axis of \( P \) and the 90° domain-wall planes make an angle of \( \pi/4 \) with the direction of \( H \), theoretical expressions of \( J_{90}(H+P\uparrow) \), \( J_{90}(H+P\downarrow+P\downarrow) \), and \( J_{90}(H+P\downarrow+P_{0}P\downarrow) \) in the Nagata-Carleton model take different forms depending on relative magnitudes of \( H, H_{p}, \) and \( H^{*} \).

In the present model, \( P \) works into the same direction as \( H \) on the \((-J)\) domains. For the \((J+)\) magnetization of a \((-J)\) domain, therefore, \( H+H_{p}=H^{*} \) is only a critical point where the volume of \((-J)\) domain becomes zero. On the contrary, the direction of \( P \)-effect is opposite to that of \( H \)-effect on the \((+J)\) domains. Hence, characteristic expressions of the Rayleigh’s hysteresis loop of the \((J)\) magnetization of a \((+J)\) domain change at \( H_{p}=2H \) and \( H_{p}-H=H^{*} \). Thus, the three critical points, \( H_{p}=H^{*}-H, H_{p}=2H \) and \( H_{p}=H^{*}+H \), must be taken into consideration in mathematical expressions of the present piezomagnetization model.

It is further assumed in the present model that \( H^{*} \geq H \), because experimental data seem to approximately satisfy this condition. Even on such an assumption, two cases of \( H^{*} \leq 3H \) and \( H^{*} \geq 3H \) must be separately considered, because \( H^{*}-H < 2H \) for \( H^{*} < 3H \) and \( H^{*}-H > 2H \) for \( H^{*} > 3H \).
Case 1 \((H \leq H^* \leq 3H)\)

(I-1) \(0 \leq H_p \leq H^* - H\)

\[
J_{90}(H \to P \uparrow) = XH + b \left( H^2 + HH_p + \frac{1}{4}H_p^2 \right)
\]

\[
J_{90}(H \to P \to P \downarrow) = XH + b \left( H^2 + HH_p + \frac{1}{4}H_p^2 \right)
\]

\[
J_{90}(H \to P_0 \to P \uparrow) = XH + b \left( H^2 + HH_{p+} + \frac{1}{4}H_{p+}^2 \right) = J(H \to P_0 \to P \downarrow)
\]

(I-2) \(H^* - H \leq H_p \leq 2H\)

\[
J_{90}(H \to P \uparrow) = XH + b \left( H^2 + HH_p + \frac{1}{4}H_p^2 \right) \quad (0 \leq H_p \leq H^* - H)
\]

\[
= \frac{X}{2} (H^* + H - H_p) + \frac{b}{2} \left( H^* + H^2 - \frac{1}{2}H_p^2 \right) \quad (H^* - H \leq H_p \leq 2H)
\]

\[
J_{90}(H \to P \to P \downarrow) = \frac{X}{2} (H^* + H - H_p) + \frac{b}{2} \left( H^* + H^2 + \frac{1}{2}H_p^2 - H_{p+}H_p \right)
\]

\[
= XH + \frac{b}{4} \left( (H^* + H)^2 - 2H_p(H^* - H - H_{p+}) \right) \quad (0 \leq H_p \leq H^* - H)
\]

\[
J_{90}(H \to P_0 \to P \uparrow) = XH + \frac{b}{4} (H^* + H)^2 \quad (0 \leq H_p \leq H^* - H)
\]

\[
= \frac{X}{2} (H^* + H - H_p) + \frac{b}{2} \left( H^* + H^2 - \frac{1}{2}H_p^2 \right)
\]

\[
(2H \leq H_p \leq H^* + H)
\]

(I-3) \(2H \leq H_p \leq H^* + H\)

\[
J_{90}(H \to P \uparrow) = XH + b \left( H^2 + HH_p + \frac{1}{4}H_p^2 \right) \quad (0 \leq H_p \leq H^* - H)
\]

\[
= \frac{X}{2} (H^* + H - H_p) + \frac{b}{2} \left( H^* + H^2 - \frac{1}{2}H_p^2 \right) \quad (H^* - H \leq H_p \leq 2H)
\]

\[
= \frac{X}{2} (H^* + H - H_p) + \frac{b}{2} (H^* - H^2 - 2HH_p - H_p^2)
\]

\[
(2H \leq H_p \leq H^* + H)
\]

\[
J_{90}(H \to P \to P \downarrow) = \frac{X}{2} (H^* + H - H_p)
\]

\[
+ \frac{b}{2} \left( H^* + H^2 - \frac{1}{2}H_p^2 + 2HH_{p+} + \frac{1}{2}H_p^2 - H_{p+}H_p \right)
\]

\[
(2H \leq H_p \leq H^* + H)
\]
The Hysteresis of Piezomagnetization

\[ J_{90}(H + P \perp P \perp) = XH + b \left( H^2 + H H_p + \frac{1}{4} H^2_p \right) \]
\[ J_{90}(H + P \perp P \perp) = XH + b \left( H^2 + H H_p + \frac{1}{4} H^2_p \right) \]

Case II \((3H \leq H^*)\)

\((II-1)\) \(0 \leq H_p \leq 2H\)

\[ J_{90}(H + P \perp P \perp) = XH + b \left( H^2 + H H_p + \frac{1}{4} H^2_p \right) \]

\[ J_{90}(H + P \perp P \perp) = XH + b \left( H^2 + H H_p + \frac{1}{4} H^2_p \right) \]
\[ J_{90}(H+P+P_0P^\dagger) = XH + b \left( H^2 + HH_p + \frac{1}{4} H_p^2 \right) = J(H+P+P\downarrow) \]

\[(II-2) \quad 2H \leq H_p \leq H^* - H \]

\[ J_{90}(H+P^\dagger) = XH + b \left( H^2 + HH_p + \frac{1}{4} H_p^2 \right) \quad (0 \leq H_p \leq 2H) \]

\[ J_{90}(H+P+P\downarrow) = XH + 2bHH_p \quad (2H \leq H_p \leq H^* - H) \]

\[ J_{90}(H+P+P_0P^\dagger) = XH + 2bHH_p^+ \quad (0 \leq H_p \leq H_p^+) \]

\[ J_{90}(H+P+P_0P\downarrow) = XH + 2bHH_p^+ = J(H+P+P\downarrow) \quad (0 \leq H_p \leq H_p^+) \]

\[(II-3) \quad H^* - H \leq H_p \leq H^* + H \]

\[ J_{90}(H+P^\dagger) = XH + b \left( H^2 + HH_p + \frac{1}{4} H_p^2 \right) \quad (0 \leq H_p \leq 2H) \]

\[ = XH + 2bHH_p \quad (2H \leq H_p \leq H^* - H) \]

\[ = \frac{X}{2} (H^* + H - H_p) + b \left\{ \frac{H^*^2}{2} - (H_p - H)^2 \right\} \quad (H^* - H \leq H_p \leq H^* + H) \]

\[ J_{90}(H+P+P\downarrow) = \frac{X}{2} (H^* + H - H_p) \]

\[ + b \left( \frac{H^*^2 - H^2}{2} - \frac{1}{2} H_p^2 + 2HH_p^+ + \frac{1}{2} H_p^2 - H_p^+ H_p \right) \quad (H^* - H \leq H_p \leq H_p^+) \]

\[ = XH + \frac{b}{4} \left\{ H^*^2 - 3H^2 - H_p^2 + 4HH_p^+ \right\} \]

\[ + 2H^*H + 2H_p (H^* - H - H_p^+) \}

\[ (0 \leq H_p \leq H^* - H) \]

\[ J_{90}(H+P+P_0P^\dagger) = XH + \frac{b}{4} \left\{ H^*^2 - 3H^2 - H_p^2 + 4HH_p^+ + 2H^*H \right\} \]

\[ (0 \leq H_p \leq H^* - H) \]

\[ = \frac{X}{2} (H^* + H - H_p) \]

\[ + \frac{b}{2} \left( \frac{H^*^2 - H^2}{2} - \frac{1}{2} H_p^2 + 4HH_p^+ - \frac{1}{2} H_p^2 \right) \quad (H^* - H \leq H_p \leq H_p^+) \]

\[(II-4) \quad H^* + H \leq H_p \]

\[ J_{90}(H+P^\dagger) = XH + b \left( H^2 + HH_p + \frac{1}{4} H_p^2 \right) \quad (0 \leq H_p \leq 2H) \]
In the above expressions, \( J(H + P + P_{\perp}) \) and \( J(H + P + P_0P_{\perp}) \) in the case of (1-4) are exactly the same as those in the case of (II-4). The present theoretical idea on the basis of Rayleigh’s hysteresis characteristics can be applied in a similar way on the case of \( H \geq H^* \). In this case, however, the initial irreversibility of piezomagnetization does not take place, though the hysteresis of piezomagnetization represented by \( J(H + P + P_{\perp}) \) is associated.

REFERENCES


