Some New Methods in Geomagnetic Field Modeling Applied to the 1960–1980 Epoch

R. A. LANGE^{*}, R. H. ESTES^{**} and G. D. MEAD^{*}

^{*}*NASA/Goddard Space Flight Center, Greenbelt, Maryland U.S.A.*

^{**}*Business and Technological Systems, Seabrook, Maryland, U.S.A.*

(Received February 17, 1982)

Models of the main (core) magnetic field are limited in accuracy by the nature of the data. Anomaly fields from the crust are a noise source in these calculations which restrict the accuracy to which the main field coefficients can be determined. For meaningful coefficients the degree and order of the model is limited to about eight for observatory and other surface data and to about 13 for satellite data. Utilizing satellite and surface data together has permitted the incorporation of a solution for the anomaly field at each observatory. These fields vary from a few tens of nT (nanotesla) to several thousand nT. The residuals of the observatory measurements to such models is no longer dominated by the anomaly fields and so is commensurate with the actual measurement accuracy (about 5 to 20 nT, depending on the observatory). Incorporation of the anomaly estimation has made possible the inclusion of stable time derivatives of the spherical harmonic coefficients up to the third derivative. Using these new techniques a spherical harmonic model, designated GSFC (9/80), was derived with degree and order 13 in its constant and first time derivative terms, six in its second time derivative terms, and four in its third derivative terms. The data used in deriving the model included (1) 15,206 Magsat observations, (2) 71,000 POGO satellite observations, (3) measurements from 148 magnetic observatories, (4) 300 measurements from filtered marine magnetic data, and (5) approximately 600 measurements from 150 selected repeat stations. The time span of the data was 1960 through 1980. RMS residuals of the model (including anomaly values) to the data are: Magsat scalar data, 10 nT; Magsat vector component data, about 7 nT; POGO data, 7 nT; observatory *X* component, 38 nT; observatory *Y* component, 19 nT; and observatory *Z* component, 17 nT.

1. Introduction

As shown by Gauss in 1839 the potential of the geomagnetic field can be represented by a spherical harmonic series of the form:

$$\begin{aligned} V = a \sum_{n=1}^{NMAX1} \sum_{m=0}^n \left(\frac{a}{r}\right)^{n+1} [g_n^m \cos m\phi + h_n^m \sin m\phi] P_n^m(\cos \theta) \\ + a \sum_{n=1}^{NMAX2} \sum_{m=0}^n \left(\frac{r}{a}\right)^n [q_n^m \cos m\phi + s_n^m \sin m\phi] P_n^m(\cos \theta) \end{aligned} \quad (1)$$

where: a is the mean radius of the earth; r, θ, ϕ are the standard spherical coordinates; and P_n^m (in “modern” methodology) are the Schmidt quasi-normalized form of associated Legendre functions. The magnetic field is then given by:

$$\bar{B} = -\nabla V = (B_r, B_\theta, B_\phi). \quad (2)$$

Theoretically, Eq. (1) applies exactly, at a given time, only when $NMAX1$ and $NMAX2$ go to infinity, under the assumption that the region under consideration, $a < r < b$, say, is source-free. The source-free assumption holds nearly exactly between the earth's surface and the ionosphere, but near-earth spacecraft pass through a region of "field aligned" currents in the auroral belt. The geometry of the field aligned currents is such that the field magnitude and vertical component are relatively unaffected (LANGE, 1974) but the horizontal components may have several hundred nT (nanotesla) contribution from these currents. This must be accounted for in deriving Eq. (1). In practice, the values of $NMAX1$ and $NMAX2$ are limited by the data accuracy, by finite computer capabilities and, for the external field, the nature of its temporal variability. The data accuracy aspect will be discussed in a later section. In Eq. (1) the terms in $(a/r)^{n+1}$ describe sources within $r < a$, or "internal" sources, and the terms in $(r/a)^n$ describe sources outside $r \geq b$, or "external" sources. Field measurements are used to derive the coefficients g_n^m , h_n^m , q_n^m , and s_n^m , usually by some form of least squares procedure.

Both the internal (g_n^m and h_n^m) and external (q_n^m and s_n^m) coefficients are known to vary with time. To date, temporal variations in the external terms have not been included in models. Variations in the internal field have been modeled by expanding the coefficients in Taylor series in time, e.g.:

$$g_n^m(t) = g_n^m(t_0) + \dot{g}_n^m(t_0)(t - t_0) + \ddot{g}_n^m(t_0) \frac{(t - t_0)^2}{2!} + \dots \quad (3)$$

Most models include only the constant and first derivative (secular variation) terms, although some more recent models have incorporated the second derivative (secular acceleration) also (e.g., CAIN *et al.*, 1967; BARRACLOUGH and MALIN, 1979). It should be noted that in some contexts the derivatives in Eq. (3) are combined with the factorials to produce the total coefficients which multiply the powers of time in the power series.

The principal sources of data for main field modeling have been (1) permanent magnetic observatories, (2) repeat measurements at selected sites with intervals between measurement of one to six years, (3) surveys from aircraft and ship, and (4) satellite measurements. Only the satellite surveys are truly global. Relevant surveys from which data are generally available were conducted by the Cosmos 49 spacecraft in October and November of 1964, by the OGO-2, -4, and -6 (POGO) spacecraft from October 1965 through July 1971, and most recently, by the Magsat spacecraft from November 1979 through June 1980. The Cosmos and POGO satellites measured only the field magnitude, which introduces an ambiguity in resulting spherical harmonic analyses (BACKUS, 1970; HURWITZ and KNAPP, 1974; STERN and BREDEKAMP, 1975; STERN *et al.*, 1980). The permanent magnetic observatories must still be regarded as the primary source of information regarding the temporal changes. Unfortunately, these data, and all data except the satellite data, are highly "contaminated" by fields originating in the crust of the earth. These anomaly fields can be tens to thousands of nT in any of the components and represent a large noise source when attempting to model the bulk of the geomagnetic field which originates in the earth's core.

This paper describes an attempt to utilize the observatory data in a more optimal way

by incorporating an estimation of individual observatory anomaly fields into the solution and, in so doing, to allow the derivation of a more accurate model of temporal variation. The results were presented as a possible contribution to the 1980 version of the IGRF (International Geomagnetic Reference Field) and the definitive Geomagnetic Reference Fields for 1965, 1970, and 1975 at the 1981 assembly of the International Association of Geomagnetism and Aeronomy (IAGA).

2. Model Degree and Order

In Eq. (1) n is the degree and m the order of any given term. $NMAX1$ is the maximum degree and order for internal terms and $NMAX2$ for external terms. Gauss' original model did not include external terms and used an $NMAX1$ of four because he concluded that the available data did not warrant the inclusion of further terms. In the years since Gauss the available data base has improved considerably and, accordingly, the degree/order of published models has increased. MALIN and POCOCK (1969) analyzed the question of the appropriate degree/order for models based on magnetic observatory data. Using data from 180 observatories they computed models from degree/order two to ten and compared the rms residuals. Their results are shown in Table 1. They note a rapidly decreasing rms from second to sixth order after which the decrease becomes very slow; this leveling off is attributed to the "crustal noise" in the data. They conclude that "at least six orders should be evaluated if the core field is to be fitted within 0.5%." Subsequent models not using satellite data are in reasonable agreement with their results and most of these models were of degree/order eight, nine or ten.

The situation changed drastically with the advent of satellite data, particularly the surveys by the POGO satellites from 1965 to 1971. These data are far enough above the earth that the crustal anomaly fields are less than 20 nT maximum, with rms below 5 nT. Further, the coverage is now truly global with no large gaps. The latest published models which include these data are mainly of degree/order 12 (BARRACLOUGH *et al.*, 1975; PEDDIE and FABIANO, 1976; BARKER *et al.*, 1981). Those of us who have been attempting to study crustal magnetic anomalies have been using a model of degree/order 13 (LANGEL *et al.*, 1980a). This choice is confirmed by LANGEL and ESTES (1982) who derived the spatial power spectrum (LOWES, 1966, 1974),

Table 1. RMS residuals for spherical harmonic analysis of observatory data.

Degree/Order	rms residual (nT)
2	3495.7
3	2218.2
4	889.0
5	538.7
6	388.5
7	364.5
8	359.5
9	354.3
10	351.9

$$R_n = (n + 1) \sum_{m=0}^n [(g_n^m)^2 + (h_n^m)^2] \quad (4)$$

for $n = 1$ to 23 using Magsat data. The derived spectrum showed a clear break near $n = 14$ which was interpreted to mean that the core field dominates for $n < 13$ and the crustal field for $n > 15$.

The situation for the temporal derivative terms is much more complicated. In the model to be presented, we have included first derivatives to degree/order 13, second derivatives to degree/order six, and third derivatives to degree/order four. For each coefficient we have calculated the ratio of coefficient magnitude to the standard error of the coefficient. As a rule of thumb, we assume that if this ratio is ≥ 2 , the coefficient has statistical significance above the 95% confidence level (see, e.g. BARRACLOUGH and MALIN, 1979).

Table 2 shows the ratio distribution for these terms. On this basis we are justified in including some terms from each of the 13 degrees of the first derivative and, perhaps, should have extended the second and third derivative terms to still higher degree/order. Investigation of this question is beyond the scope of the present paper.

Table 2. Summary of distribution of ratio of coefficient magnitude to coefficient standard error.

Degree	Number of terms	Ratios			
		>100	10-100	2-10	<2
First derivatives					
1	3	1	2	0	0
2	5	3	0	2	0
3	7	0	6	1	0
4	9	0	6	3	0
5	11	0	8	3	0
6	13	0	10	3	0
7	15	1	9	4	1
8	17	0	12	4	1
9	19	0	12	6	1
10	21	0	8	8	5
11	23	0	6	15	2
12	25	0	2	17	6
13	27	0	4	11	12
Second derivatives					
1	3	0	3	0	0
2	5	0	1	4	0
3	7	0	6	1	0
4	9	0	5	3	1
5	11	0	7	3	1
6	13	0	12	0	1
Third derivatives					
1	3	0	3	0	0
2	5	1	3	1	0
3	7	0	7	0	0
4	9	0	5	3	1

Aside from the question of the accuracy to which individual coefficients are determined, one has to ask what the descriptive and predictive properties are for models with higher derivatives. We will address this question in a subsequent section.

There remains the question of external field (*NMAX2*). As MALIN and POCKOCK (1969 and references therein) point out, various attempts have been made to separate the internal and external parts of the field. However, the results have been widely different; in some cases unrealistically large external fields have been found, but have not been statistically significant. Again, a change has come with the global vector survey by Magsat. LANGELE *et al.* (1980b) used data from November 5–6, 1979 to derive the MGST(6/80) model. This model included statistically significant external terms of degree/order equal to one. Furthermore, plots of the Magsat data clearly demonstrated the need for such terms to adequately represent the data. However, it is known that the sources of the external fields, which are the magnetospheric ring current, magnetopause current, and magnetotail current, vary widely with time in a fashion not yet amenable to this type of model and, moreover, vary more strongly as functions of local time than of longitude as set forth in Eq. (1). While external terms may be valuable as an indication of the average quiet level of external fields, we have no compelling reason to include them in models describing extended periods of time.

3. Method of Analysis

The method of determining the parameters of Eq. (1) is essentially the same as that described by CAIN *et al.* (1967), with revisions to include higher temporal derivatives and to incorporate magnetic observatory data in a new way. Because of the data types involved, the problem is non-linear and must be solved iteratively. The Bayesian least squares estimation equations are as follows:

$$\delta\hat{\rho}_{n+1} = (A^T W A + \Omega_0^{-1})^{-1} [A^T W \delta y_n + \Omega_0^{-1}(\hat{\rho}_0 - \hat{\rho}_n)] \quad (5)$$

where

- A is the partial derivative matrix of the measurements with respect to the parameters,
 - $\hat{\rho}$ is the vector of adjusted parameters,
 - δy is the vector of residuals, i.e. measured data minus predicted value from the previous iteration,
 - W is the weight matrix for the measurements,
 - Ω_0 is the *a priori* parameter covariance matrix,
 - $\hat{\rho}_0$ is the *a priori* estimate of the parameters,
- and the estimate at the $(n + 1)$ st iteration is

$$\hat{\rho}_{n+1} = \hat{\rho}_n + \delta\hat{\rho}_{n+1}. \quad (6)$$

In the notation of CAIN *et al.* (1967), each measured quantity, say C , is a function of the coefficients g_n^m, h_n^m, q_n^m , and s_n^m from Eq. (1), denoted collectively by “ p ,” and of the standard r, θ, ϕ, t space-time coordinates:

$$C = C(p; r, \theta, \phi, t). \quad (7)$$

The partial derivatives in A are then the set $\partial C/\partial p$. The weight matrix W is assumed to be diagonal (uncorrelated data) and formed from the standard deviations of the various measurements, so that for the i th measurement with sigma σ_i

$$W_{ii} = \frac{1}{\sigma_i^2}. \quad (8)$$

As already noted, the data from magnetic observatories represent the most useful data set for determining the temporal variation of the internal field. The incorporation of such data directly into a main internal core field model, however, suffers from the fact that the magnetic field measured at the observatory may have a significant contribution due to local crustal fields. The field at an observatory is represented as the vector sum

$$\bar{B} = \bar{B}_i + \bar{B}_m \quad (9)$$

where $\bar{B}_i$ is the internal core field contribution from the scalar potential of Eq. (1) and $\bar{B}_m$ is the local anomaly field, which may change appreciably over a distance of a few kilometers. The time derivative of $\bar{B}_m$ is assumed to be negligible so that

$$\dot{\bar{B}} = \dot{\bar{B}}_i. \quad (10)$$

A global satellite data set, on the other hand, is comparatively free from the effects of crustal anomalies and is certainly free from the effects of the higher-amplitude, more localized anomalies. Assuming that for a main field model the crustal influence on satellite data may be treated as random noise, these data may then be used in conjunction with observatory data to isolate the non-core fields at the observatory. In terms of the algorithm of Eqs. (5), (7), and (8) this is accomplished by writing, for each measured component at each observatory

$$C = C(p, C_a; r, \theta, \phi, t), \quad (12)$$

where C_a is the anomaly in that component at that observatory. Terms such as $\partial C/\partial C_a$ are then added to the matrix, A , of partial derivatives. The vector, ρ , of adjusted parameters from Eq. (5) then includes p and all anomaly components. This procedure allows the data to properly distribute their influence among the temporal and constant parameters in a least square sense. Such a solution is well determined only when the satellite data, not strongly sensitive to the crustal fields, are included in the solution. A forward elimination technique is used for the anomaly components in accumulating the normal matrix $A^T W A$, so the cost of obtaining the solution for the model coefficients is greatly reduced. The values of the anomaly components, if desired, are then obtained by back substitution.

4. Data Set

The data utilized in the present analysis include data from (1) magnetic observatories, (2) repeat stations, (3) marine surveys, (4) the POGO satellite surveys, and (5) the Magsat satellite survey. The POGO data consisted of the 47,000 scalar observations used

for the POGO(8/71) model (LANGEL, 1974) augmented with 24,000 quiet OGO-6 observations from mid-1969 through early 1971. The Magsat data set, consisting of scalar and vector measurements from November 5 and 6, 1979, is identical to that used for the MGST (6/80) model (LANGEL *et al.*, 1980b). The measurement standard deviation used to weight the POGO data was 7 nT and for Magsat data was 10 nT, based on fits to these data alone.

Annual means data were taken from 148 observatories, selected on the basis of geographical distribution, longevity of measurement availability, and data quality. This resulted in our not using some high quality observatories in regions of higher observatory density. Moreover, only those annual means with three vector components were accepted. The observatories utilized are listed in Table 3 together with the anomaly vector bias from the solution, the time of data availability and the σ_i used in Eq. (8). The spellings used are those of the NOAA World-Wide Magnetic Survey of the National Geophysical and Solar-Terrestrial Data Center. In cases where a significant change at the observatory (e.g., shift of location, change of instrument) was known to have occurred, the data from that observatory were broken into subsets which were treated independently with respect to the anomaly bias vectors. In all cases the measured elements were converted to X , Y , and Z components where X is north, Y is east, and Z is down in a geodetic coordinate system assuming an equatorial radius of 6378.165 km and a reciprocal flattening of 298.25. To determine the σ_i in Table 3 a least squares quadratic curve was fit to each component at each observatory and the standard deviation of the data to the solution adopted as σ_i .

To fill in surface areas void of data, selected marine survey and repeat station data were utilized. In order to accommodate the non-observatory surface data in the solution in a consistent manner, techniques were used to remove the major parts of the crustal contributions. From the available scalar marine data over the years 1970–1974, 39 long, straight tracks of length greater than 1,200 km were selected. A low pass filter was applied to each track, removing anomaly wavelengths shorter than 500 km. Approximately 300 measurements were then taken along the filtered tracks. A measurement standard deviation of 10 nT was used in weighting the data in the solution, although analysis of crossing points for the 39 tracks indicated differences on the order of 50 nT when corrected for secular variation. The higher weighting reflects our regard for the relative importance of these data in an area devoid of other surface observations. Approximately 600 measurements from 150 repeat stations were utilized to fill sparse data regions in Central and South America, Africa, and Australia. Only stations with three or more occupations and good data quality were accepted. As there were generally insufficient data available for the repeat stations to solve for independent anomaly bias vectors, quadratic polynomial fits to these data were time differentiated to remove the assumed constant crustal influence. Measurement standard deviations for the “differentiated” repeat data were arrived at by utilizing the “differentiated” value, together with observatory and marine data, to derive a degree/order eight model. The standard deviations to that model were 0.2 deg/year for $\dot{D}$ and $\dot{I}$ and 5 nT/year for $\dot{H}$, $\dot{Z}$, and $\dot{B}$, taken collectively. These were adopted as measurement standard deviations for the repeat station data in the present analysis.

Table 3. Observatories used in solution.

Station	Lat	Long	Anomaly bias			Time interval	Measurement sigmas		
			x	y	z		x	y	z
ADDIS ABABA	9.03	38.76	489.5	15.1	148.6	1960.5-70.5	2.3	3.6	1.0
AGINCOURT	43.78	-79.27	-45.7	175.7	-140.4	1960.5-69.1	8.2	3.9	4.5
ALERT	82.50	-62.50	5.7	34.6	-143.2	1961.9-74.5	12.7	15.9	24.0
ALIBAG	18.64	72.87	-80.2	463.7	669.0	1960.5-73.5	16.3	7.5	9.4
ALMA ATA	43.25	76.92	125.9	42.2	-152.1	1963.5-76.5	5.6	14.0	16.2
ALMERIA	36.85	-2.46	-6.0	8.6	22.2	1960.5-76.5	6.7	6.2	4.3
AMBERLEY	-43.15	172.72	-44.0	-16.8	70.1	1960.5-76.5	5.0	4.5	6.6
ANNAMALAINAGAR	11.37	79.68	219.5	-80.4	-174.6	1960.5-74.5	11.3	9.5	17.3
APIA	-13.81	-171.78	-86.9	208.1	-1017.2	1960.5-76.5	7.6	6.5	3.7
AQUILA	42.38	13.32	-8.4	42.2	9.5	1960.5-76.5	6.2	6.4	5.8
ARGENTINE ISLND	-65.24	-64.26	69.3	-83.5	512.8	1960.5-67.5	1.4	1.3	2.5
BAKER LAKE	64.33	-96.03	149.0	-41.9	-103.8	1960.5-68.5	2.0	6.5	16.2
	64.33	-96.03	219.4	-109.7	-114.3	1969.5-76.5	8.0	19.1	3.1
BANGUI	4.44	18.56	-85.6	42.0	95.0	1960.3-68.5	5.4	12.7	5.2
	4.44	18.56	-119.3	36.6	107.3	1969.5-74.5	2.2	22.9	1.5
BARROW	71.30	-156.75	23.9	-66.1	-32.0	1960.5-77.5	11.5	8.2	26.6
BELSK	51.84	20.79	96.5	153.5	314.0	1960.5-76.5	4.7	9.0	3.1
BEREZNAVYKI	49.82	73.08	-423.8	14.7	338.9	1965.5-76.5	6.9	7.6	13.9
BINZA	-4.27	15.37	-144.4	-118.0	-177.5	1960.5-66.5	6.3	4.2	4.4
BJORNOYA	74.50	19.20	-107.6	40.7	37.8	1960.5-64.5	0.7	1.6	5.0
	74.50	19.20	-119.9	54.6	2.5	1965.5-69.5	0.8	1.5	2.1
	74.50	19.20	-122.1	47.1	28.0	1970.5-72.5	0.7	1.6	5.0
BOULDER	40.14	-105.24	-52.5	53.5	-160.3	1964.5-77.5	7.8	3.1	9.0
BYRD	-80.02	-119.52	-24.2	37.5	-147.2	1962.5-68.3	4.0	2.0	7.6
CASTLE ROCK	37.24	-122.13	-128.6	-16.5	5.7	1970.5-74.5	0.9	1.0	3.8
CHA PA	22.35	103.83	-119.6	-90.5	-2.9	1960.5-75.5	21.6	7.4	25.3
CHAMRON FORET	48.02	2.26	-79.7	-24.5	119.0	1960.5-75.5	6.1	5.9	3.4
CHELYUSKIN	77.72	104.28	-49.7	-95.0	-80.1	1960.5-76.5	11.1	4.4	21.6
COLMERA	40.22	-8.42	-5.4	-22.8	48.0	1960.5-76.5	9.9	7.5	8.6
COLLEGE	64.86	-147.84	-17.2	-60.7	-107.3	1960.5-77.5	5.0	8.3	3.4
DALLAS	32.99	-96.75	-15.1	23.8	-73.5	1964.5-74.5	10.6	2.8	3.2
DIKSON	73.55	80.57	-114.5	-140.0	-278.3	1960.5-62.5	9.0	5.2	7.4
	73.55	80.57	-104.5	-147.6	-248.6	1963.5-76.5	9.0	5.2	7.4
DOMBAS	62.07	9.12	-93.7	-79.0	-234.8	1960.5-74.5	5.3	8.2	2.7
DOURBES	50.10	4.59	-7.7	-16.8	98.4	1960.5-76.5	4.9	8.3	2.4
DUMONT DURVILLE	-66.66	140.01	-172.8	-389.2	-2687.6	1960.5-72.5	14.6	15.7	36.3
	-66.66	140.01	-192.0	-419.3	-2852.9	1973.5-75.5	14.6	15.7	36.3
DUSHETI	-42.09	44.70	-238.4	4.8	-82.7	1960.5-76.5	7.5	3.6	12.9
DYMER	50.72	30.30	-26.6	90.1	117.9	1964.5-76.5	4.0	5.7	3.7
FORT CHURCHILL	58.77	-94.10	-140.2	46.8	-255.8	1965.5-76.5	5.4	8.6	9.0
FREDERICKSBURG	38.21	-77.37	28.1	-69.7	120.5	1960.5-77.5	16.9	3.4	19.9
FUQUENE	5.47	-73.74	14.4	-88.7	215.8	1960.5-74.5	2.8	4.0	23.9
FURSTENFELDBRUCK	48.17	11.28	-31.2	-5.9	14.9	1960.5-76.5	5.6	7.8	4.5
GNANGARA	-31.78	115.95	-51.0	-127.8	122.5	1960.5-76.5	2.7	5.4	12.7
GODHAVN	69.24	-53.52	207.6	-257.3	469.0	1960.5-73.5	5.2	3.8	9.5
GORKHOTAYEZHAY	43.68	132.17	-13.3	-23.5	-37.8	1960.5-76.5	14.2	7.2	18.2
GREAT WHALE R	55.27	-77.78	258.3	113.5	-58.9	1966.5-75.5	10.0	4.3	14.2
GROCKA	44.63	20.77	-40.0	-48.9	-51.1	1960.5-76.5	5.6	7.3	6.0
GUAM	13.58	144.87	174.9	104.3	-2.7	1960.5-77.5	9.1	4.4	5.2
HALLEY BAY	-75.52	-26.68	-26.3	400.2	-7.4	1960.5-66.5	8.3	11.3	35.1
HARTLAND	51.00	-4.48	-54.6	7.0	64.2	1960.5-76.5	5.2	8.8	5.0
HEL	54.61	18.81	31.8	-157.2	-73.6	1960.5-68.5	1.3	2.1	5.2
HERMANUS	-34.42	19.22	-17.6	24.0	-5.7	1960.5-75.5	8.8	2.9	3.1
HONOLULU	21.32	-158.00	-174.4	112.9	-304.6	1961.5-77.5	6.2	2.2	4.1
HUANCAYO	-12.04	-75.34	90.3	48.5	246.8	1960.5-76.5	11.0	3.1	16.0
HYDERABAD	17.41	78.55	352.2	109.6	457.0	1965.5-76.5	6.1	22.3	15.8
ISTANBL KNDILLI	41.06	29.06	171.5	121.7	1.9	1960.5-75.5	5.9	7.8	7.7
KAKIOKA	36.23	140.19	-1.5	0.5	-89.8	1960.5-64.5	2.1	0.3	0.9
	36.23	140.19	-6.8	3.1	-64.3	1965.5-76.5	7.6	3.5	3.4
KELES	41.42	69.20	-215.5	-43.4	-12.4	1960.5-63.5	0.8	0.3	0.7
KERGUELEN	-49.35	70.20	177.2	206.1	633.0	1960.5-75.5	5.4	11.8	25.7

(continued)

Table 3. (continued)

Station	Lat	Long	Anomaly bias			Time interval	Measurement sigmas		
			x	y	z		x	y	z
KIEV	50.72	30.30	-70.4	187.7	129.9	1960.5-63.5	0.1	1.4	1.5
KLYUCHI	55.03	82.90	159.1	-88.2	6.3	1967.5-76.5	4.3	1.4	4.7
KRASNAYA PAKHRA	55.48	37.31	108.3	-12.8	243.5	1960.5-76.5	4.6	9.4	7.4
KSARA	33.82	35.89	-67.1	67.4	-77.7	1960.5-70.5	8.8	2.9	36.2
LEIRVOGUR	64.18	-21.70	-299.9	594.9	-482.9	1960.5-75.5	6.4	10.8	11.9
LERWICK	60.13	-1.18	-137.3	169.3	37.7	1960.5-76.5	4.7	11.4	4.7
LOGRONO	42.46	-2.51	-2.3	0.6	56.4	1960.5-76.5	6.8	7.1	3.4
LOPARSKOYE	68.25	33.08	90.1	337.4	-481.6	1961.5-76.5	5.8	9.5	6.7
LOURENCO MARQUES	-25.92	32.58	328.5	34.2	-159.6	1960.5-71.5	8.9	3.8	7.7
LOVO	59.35	17.83	35.2	0.9	7.5	1960.5-76.5	5.1	9.6	3.7
LUANDA BELAS	-8.92	13.17	193.0	-6.4	50.0	1960.5-74.5	7.8	4.0	18.2
LUNPING	25.00	121.17	16.0	-34.6	79.9	1965.8-74.5	4.3	22.5	5.4
LYOV	49.90	23.75	128.9	125.3	153.7	1960.5-76.5	7.3	9.6	9.0
LWIRD	-2.25	28.80	246.5	130.7	19.6	1960.5-70.5	7.5	2.3	2.5
M BOUR	14.39	-16.96	143.0	-48.4	78.8	1960.5-71.5	7.5	3.6	4.3
MACQUARIE ISLAND	-54.50	158.95	236.3	-7.3	309.5	1960.5-76.5	4.9	5.6	6.0
MAGADAN	60.12	151.02	-1365.2	346.4	1273.7	1960.5-66.5	1.6	1.1	30.4
MAURITIUS	-20.09	57.55	475.5	-201.8	-441.5	1960.5-65.5	5.7	3.6	12.2
MAWSON	-67.60	62.88	16.7	25.0	197.7	1960.5-76.5	7.3	9.5	25.3
MEANOOK	54.62	-113.33	67.9	8.3	-160.3	1960.5-75.5	6.5	12.1	11.5
MEMAMBETSU	43.91	144.19	-239.3	133.1	77.2	1960.5-76.5	10.4	4.2	8.6
MIRNY	-66.55	93.02	-135.1	50.2	-428.5	1960.5-76.5	5.5	5.8	20.2
MISALLAT	29.52	30.89	-90.1	73.8	123.4	1960.6-73.5	7.9	4.5	4.8
MOCA	3.34	8.66	-102.6	1.4	151.6	1960.5-71.5	6.8	4.0	1.9
MOLODEZHNYAYA	-67.67	45.85	-40.6	-95.5	-207.4	1965.5-76.5	11.9	7.3	51.8
MOULD BAY	76.20	-119.40	-22.4	5.0	-40.7	1962.8-76.5	5.1	11.7	9.3
MUNTINLUPA	14.38	121.02	-64.2	-183.9	19.1	1960.5-75.5	12.7	6.2	9.6
NAIROBI	-1.33	36.81	47.4	62.0	-46.1	1964.5-76.5	7.4	3.7	10.9
NARSSARSSUAQ	61.10	-45.20	-373.2	273.1	575.5	1968.9-73.5	10.4	5.5	11.5
NEWPORT	48.26	-117.12	-71.3	110.2	-129.2	1966.6-77.5	3.6	4.1	3.6
NIEMEGR	52.07	12.67	-41.3	0.1	-68.0	1960.5-75.5	5.3	8.2	2.9
NOVOLAZAREVSKAY	-70.77	11.83	-310.9	72.4	194.6	1961.5-76.5	6.1	9.4	40.5
NURMIJARVI	60.51	24.65	271.1	-98.3	107.2	1960.5-75.5	5.8	9.6	4.3
OTTAWA	45.40	-75.55	104.7	-143.0	184.7	1968.7-74.5	2.8	2.7	3.0
PANAGYURISHTI	42.52	24.18	-197.1	-167.7	-182.1	1960.5-75.5	8.0	5.4	10.5
PARAMARIBU	5.81	-55.22	-25.1	-32.2	-1.5	1960.5-74.5	7.9	8.2	16.2
PARATUNKA	52.90	158.43	-356.5	225.1	271.6	1969.5-76.5	1.8	2.9	1.5
PATRONY	52.17	104.45	0.8	46.8	-79.2	1960.5-76.5	11.7	5.7	19.0
PILAR	-31.67	-63.83	77.2	-10.6	42.3	1960.8-74.5	11.7	12.0	5.4
PLAISANCE	-20.43	57.67	0.0	0.0	790.0	1966.5-76.5			6.9
PLESHEMITZI	54.50	27.88	264.9	166.4	-104.9	1962.5-67.5	2.0	1.3	3.8
	54.50	27.88	269.4	148.3	-119.6	1968.5-76.5	2.0	1.6	2.2
PODKAM TUNGUSKA	61.60	90.00	28.9	54.2	-293.3	1969.5-75.5	2.9	2.1	3.9
PORT MORESBY	-9.41	147.15	-40.2	59.8	163.4	1960.5-75.5	4.4	5.7	9.1
PRUMONICE	49.99	14.55	-51.4	25.4	-69.2	1960.5-71.5	4.6	2.0	6.2
QUETTA	30.19	66.95	-3.0	52.8	-36.4	1960.5-67.5	3.0	2.9	3.4
	30.19	66.95	-11.3	50.7	-54.6	1968.5-75.5	2.0	13.3	17.9
QUIACA	-22.10	-65.61	88.5	-54.4	160.9	1960.5-74.5	13.1	15.4	4.6
RESOLUTE BAY	74.70	-94.90	36.7	30.1	72.0	1960.5-75.5	10.4	7.2	8.2
ROBURENT	44.30	7.89	76.5	48.6	117.7	1964.8-65.5	5.2	3.3	0.9
	44.30	7.89	66.0	-56.3	143.9	1966.5-68.5	5.2	3.3	0.9
	44.30	7.89	81.3	59.1	89.1	1969.5-73.5	5.2	3.3	0.9
RUDE SKOV	55.84	12.46	19.1	-5.5	-33.4	1960.5-76.5	5.3	9.3	3.4
SABHAWALA	30.36	77.80	-15.2	-27.8	63.0	1964.5-73.5	4.8	7.1	13.8
SAN JUAN	18.38	-66.12	61.0	33.5	220.6	1960.5-64.5	1.2	0.9	2.7
	18.12	-66.15	-139.3	182.0	213.6	1965.5-77.5	6.6	2.6	15.0
SAN MIGUEL	37.77	-25.65	684.7	421.2	1711.4	1960.5-76.5	10.7	36.8	11.1
SANAE	-70.30	-2.37	-51.4	-19.1	28.6	1962.7-70.5	4.2	3.9	5.5
	-70.30	-2.37	-90.7	-72.7	-5.8	1971.7-75.5	0.4	1.2	6.7
SCOTT BASE	-77.85	166.78	-2278.1	-936.6	-3766.9	1960.8-76.5	7.9	7.9	11.2

(continued)

Table 3. (continued)

Station	Lat	Long	Anomaly bias			Time interval	Measurement sigmas		
			x	y	z		x	y	z
SIMOSATO	33.58	135.94	-44.6	29.4	20.9	1960.5-75.5	15.7	3.7	5.6
SITKA	57.06	135.33	-11.6	-14.5	-82.9	1960.5-77.5	5.6	8.9	4.0
SODANKYLA	67.37	26.63	-164.6	-104.3	-578.9	1960.5-75.5	4.7	10.5	3.4
SOUTH POLE	-89.99	-13.32	-1342.7	-3469.6	79.8	1960.5-71.5	6.7	7.2	22.5
ST JOHN S	47.59	-52.68	9.0	26.5	-2.1	1968.8-76.5	1.7	3.8	4.1
STEKOLINIY	60.12	151.02	-289.9	-742.5	52.5	1966.5-76.5	2.8	1.7	6.8
STEPANOVKA	46.78	30.88	-123.1	-698.5	68.1	1960.5-76.5	4.8	7.6	6.9
SYOMA BASE	-69.01	39.59	-24.3	-82.9	-14.7	1960.6-68.5	6.1	18.4	10.5
TAHITI	-17.57	-149.58	-633.1	-1078.3	297.7	1966.2-72.5	7.1	2.6	4.1
	-17.57	-149.58	-675.8	-691.5	-209.7	1973.5-73.5	7.1	2.6	4.1
TAMANRASSET	22.79	5.53	126.4	-208.8	-78.8	1960.5-70.5	2.9	1.3	1.1
TAMANARIVE	-18.92	47.50	371.9	0.2	-447.3	1960.5-74.5	8.4	6.3	12.7
TATUOCA	-1.20	-48.51	31.4	-177.5	89.7	1960.5-71.5	6.9	4.5	12.6
TEHRAN	35.74	51.38	-114.1	13.9	-192.2	1960.5-70.5	8.7	3.6	13.4
TENERIFE	28.48	-16.28	-470.4	87.5	-1051.2	1960.5-75.5	13.4	7.9	75.0
TEOLOYUCAN	19.75	-99.18	-124.0	3.0	-11.9	1960.5-70.5	8.0	16.9	14.2
	19.75	-99.18	-67.0	38.1	-98.7	1971.5-75.5	23.2	6.5	3.6
THULE II	77.48	-69.17	-55.9	104.9	13.3	1960.5-73.5	6.3	1.5	5.4
TIMANY	46.90	17.89	-16.6	-16.6	-27.7	1960.5-76.5	7.3	6.2	5.2
TIKSI	71.58	129.00	-112.2	-159.0	-134.0	1960.5-67.5	3.6	2.9	13.3
	71.58	129.00	-86.1	-164.1	-86.0	1970.5-76.5	4.3	6.9	8.7
TOMSK	56.47	84.93	-21.7	-64.5	-239.8	1960.5-69.5	5.9	2.1	4.0
TOOLANGI	-37.53	145.47	-48.9	-22.6	42.6	1960.5-76.5	4.6	6.3	4.2
TRELEM	-43.25	-65.32	170.9	5.0	25.8	1960.5-70.5	4.8	3.5	3.4
TRIVANDRUM	8.48	76.95	215.3	99.0	79.5	1960.5-64.5	5.1	0.7	3.1
	8.48	76.95	255.0	195.2	73.0	1965.5-74.5	4.3	4.3	6.8
TROMSO	69.66	18.95	55.5	-402.0	197.6	1960.5-71.5	7.6	2.5	7.4
TSUMEB	-19.22	17.70	-5.9	-60.0	63.0	1964.8-75.5	4.3	2.4	3.4
TUCSON	32.25	-110.83	-116.1	-67.1	167.1	1960.5-77.5	7.5	3.7	8.7
UELEN	66.16	-169.84	-104.6	25.1	-117.4	1960.5-76.5	8.1	12.2	12.2
ULAN BATOR	47.85	107.05	-69.2	-17.3	-95.2	1966.5-75.5	4.2	1.4	8.2
VALENTIA	51.93	-10.25	118.3	-56.1	25.4	1960.5-76.5	4.7	9.3	6.3
VANNOVSKAYA	37.95	58.11	171.5	93.4	58.8	1960.5-74.5	8.7	5.5	12.4
VASSOURAS	-22.40	-43.65	44.5	-126.5	-71.9	1960.5-73.5	8.3	3.6	3.3
VICTORIA	48.52	-123.42	-17.6	-6.6	-328.2	1960.5-76.5	5.3	6.3	7.1
VOSTOK	-78.45	106.87	-17.9	129.6	111.0	1960.5-76.5	20.4	14.2	18.6
VOYEKOV	59.95	30.70	64.8	23.2	-261.6	1960.5-76.5	5.9	10.2	8.2
VYSOKAY DUBKAYA	56.73	61.07	-291.6	-123.5	-521.4	1960.5-66.5	2.3	1.3	1.4
	56.73	61.07	-297.5	-112.0	-482.8	1967.5-76.5	3.1	1.7	4.2
WILKES	-66.25	110.58	619.2	-296.5	29.4	1960.5-66.5	7.7	3.3	5.3
WINGST	53.74	9.07	44.4	46.1	-54.7	1960.5-76.5	6.0	8.8	3.0
YAKUTSK	62.02	129.72	40.5	-1184.3	113.7	1960.5-75.5	4.7	8.2	13.2
YANGI-BAZAR	41.33	69.62	-301.6	61.0	-96.3	1964.5-76.5	6.1	4.7	13.4
YUZHNO SAKHALSK	46.95	142.72	36.0	-165.6	-151.3	1960.5-69.5	4.8	3.7	5.2
	46.95	142.72	-84.3	-48.7	116.8	1970.5-76.5	1.8	0.6	1.8
ZAYMISHCHE	55.83	48.85	-294.5	-315.7	-240.2	1960.0-77.0	3.5	2.3	2.7

5. Results

In order to test the usefulness of solving for anomalies at the observatories and of including second and third time derivatives, a model *not* incorporating Magsat data was developed and its prediction capability tested by comparing it to the MGST(6/80) model based only on Magsat data. The test model, denoted PMAG (7/80), as well as the final model, denoted GSFC(9/80), was of degree/order 13 in its constant and first derivative terms, of degree/order six in its second derivative terms and of degree/order four in its third

derivative terms. Table 4a gives the coefficient values and their first derivatives (secular variation), and Table 4b the second and third derivatives, for the GSFC(9/80) model. The standard error is given in parenthesis beside each coefficient. This is to be used as an error estimate only with caution because its accuracy depends upon (1) the accuracy of the σ_i used to weight the data (and the validity of the assumptions that the data are uncorrelated and have Gaussian error) and (2) the validity (accuracy and completeness) of our model. In practice the standard errors tend to underestimate the actual error but are nonetheless useful as an indication of the magnitude of error in each coefficient and of the relative accuracies between coefficients.

Table 5 summarizes a statistical evaluation of these models and two other recent (pre-Magsat) models. Model AWC75 (secular variation part) was derived by PEDDIE and FABIANO (1976) using data from 1967 through 1974 and model WC80 was derived by BARKER *et al.* (1981) using data from 1950 through 1980, but their secular variation model is thought to be applicable mainly from 1974–1977. AWC75 and WC80 include constant terms through degree/order 12 and first time derivatives through degree/order eight. In Table 5 MGST(6/80) is included as a standard of comparison for the 1980 epoch. Of the pre-Magsat models it is seen that PMAG(7/80) is the best predictive model. This indicates that, at least over a three-year interval, its temporal derivatives are not wildly varying beyond the data span. It should be noted that a similar model which did not include the observatory anomaly solution performed badly when used as a predictor, i.e. the presence of the anomaly solution affects the solution for the temporal terms in a beneficial way. The GSFC(9/80) model represents the 1980 field well because it incorporates Magsat data. It is, however, slightly deteriorated from MGST(6/80). This is not unexpected since it applies to a 20-year time period whereas MGST(6/80) is a direct fit to the two days of data involved.

Comparison with observatories shows typically high residuals for the models which do not estimate observatory anomalies (AWC75 and WC80). On the other hand, the observatory residuals for PMAG(7/80) and GSFC(9/80) are of the range one would expect from the published accuracy estimates of the observations, the internal consistency of the data from individual observatories, and the characteristics of unmodeled temporal variations. Note that no measurements with extreme residuals have been eliminated from this calculation, as is often done. This is because such extreme values are often due to the anomaly values solved for in PMAG(7/80) and GSFC(9/80) and so to compare the models all measurements should be utilized.

For further comparison, and to get a quantitative measure of the predictive capability of the models, statistics were computed of each model versus observatory data on a year-by-year basis. In doing so, we adopted the statistical measure used by MEAD (1979) in a similar analysis, namely, half the width of the median 68% of the residual values. This is designated $\bar{\sigma}$, and would be equivalent to one standard error, σ , if the distribution were normal. The reason for adopting this measure is to avoid the situation where a few very large residuals dominate the statistics. Note that this is a different statistic than used in Table 5, and so the numbers are not directly comparable. In the plots to follow, some of the year-to-year changes in $\bar{\sigma}$ are due to a changing distribution of magnetic observatories. Figures 1–3 show the variation of $\bar{\sigma}$ with time for the X , Y , and Z components of observatory data relative to five field models: WC80, IGS75, AWC75, PMAG(7/80), and GSFC(9/80). IGS75 is the designation for the model derived by BARRACLOUGH *et al.* (1975) from all

Table 4a. GSFC (9/80) magnetic field model (mean radius of the earth is 6371.2 km; mean epoch is 1980.0).

n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
1	0	-29987.9 (0.6)		20.51 (0.16)	
1	1	-1967.4 (0.9)	5606.7 (1.0)	9.08 (0.19)	-9.04 (0.21)
2	0	-1996.7 (0.5)		-19.53 (0.14)	
2	1	3027.7 (0.5)	-2128.8 (0.5)	4.78 (0.13)	-18.74 (0.15)
2	2	1662.9 (1.0)	-196.6 (1.1)	9.86 (0.16)	-26.62 (0.19)
3	0	1260.2 (0.4)		4.51 (0.12)	
3	1	-2190.6 (0.5)	-336.5 (0.6)	-3.56 (0.12)	-5.57 (0.13)
3	2	1251.3 (0.5)	270.9 (0.5)	-2.19 (0.12)	1.64 (0.13)
3	3	832.9 (1.0)	-251.5 (1.0)	3.87 (0.16)	-6.62 (0.16)
4	0	937.1 (0.3)		-2.20 (0.10)	
4	1	782.3 (0.4)	211.8 (0.4)	-2.24 (0.09)	3.66 (0.10)
4	2	397.2 (0.5)	-256.8 (0.5)	-10.09 (0.10)	1.06 (0.10)
4	3	-419.6 (0.4)	52.4 (0.4)	-3.82 (0.11)	6.77 (0.10)
4	4	198.3 (0.9)	-298.0 (1.0)	-5.54 (0.12)	-2.73 (0.13)
5	0	-217.1 (0.2)		-1.04 (0.04)	
5	1	357.0 (0.2)	45.2 (0.3)	-0.86 (0.04)	3.30 (0.04)
5	2	261.5 (0.3)	149.9 (0.3)	-0.73 (0.04)	0.50 (0.04)
5	3	-74.3 (0.4)	-150.7 (0.4)	-4.81 (0.05)	-0.82 (0.05)
5	4	-161.5 (0.3)	-77.7 (0.3)	0.11 (0.05)	1.09 (0.06)
5	5	-47.7 (0.7)	91.8 (0.8)	1.49 (0.07)	0.87 (0.07)
6	0	49.1 (0.2)		0.75 (0.03)	
6	1	65.0 (0.2)	-14.6 (0.2)	-0.04 (0.03)	0.03 (0.03)
6	2	42.0 (0.2)	93.4 (0.2)	3.49 (0.04)	-1.20 (0.03)
6	3	-191.4 (0.3)	70.9 (0.3)	2.20 (0.04)	-0.96 (0.04)
6	4	3.9 (0.3)	-42.9 (0.3)	0.44 (0.04)	0.11 (0.04)
6	5	14.1 (0.4)	-1.9 (0.3)	1.78 (0.04)	0.56 (0.04)
6	6	-107.1 (0.8)	17.5 (0.8)	1.34 (0.06)	2.72 (0.07)
7	0	71.0 (0.1)		0.08 (0.01)	
7	1	-58.1 (0.1)	-83.2 (0.1)	-0.36 (0.01)	-1.44 (0.01)
7	2	1.3 (0.1)	-27.1 (0.1)	0.05 (0.01)	-0.03 (0.01)
7	3	20.1 (0.2)	-5.5 (0.2)	0.45 (0.01)	0.15 (0.01)
7	4	-13.0 (0.2)	15.9 (0.2)	0.94 (0.01)	0.58 (0.01)
7	5	0.8 (0.3)	17.8 (0.3)	0.17 (0.02)	-0.37 (0.02)
7	6	10.8 (0.3)	-23.6 (0.3)	-0.20 (0.01)	0.03 (0.02)
7	7	-2.7 (0.8)	-9.9 (0.7)	0.09 (0.04)	0.54 (0.04)
8	0	18.9 (0.1)		0.60 (0.01)	
8	1	7.2 (0.1)	7.5 (0.1)	0.08 (0.01)	-0.08 (0.01)
8	2	0.9 (0.1)	-17.7 (0.1)	0.22 (0.01)	-0.30 (0.01)
8	3	-10.4 (0.1)	3.2 (0.1)	0.08 (0.01)	-0.09 (0.01)
8	4	-7.1 (0.2)	-22.4 (0.2)	-0.24 (0.01)	-0.34 (0.01)
8	5	4.0 (0.2)	9.4 (0.2)	-0.35 (0.01)	0.31 (0.01)
8	6	3.7 (0.3)	16.3 (0.3)	0.53 (0.02)	-0.54 (0.02)
8	7	7.1 (0.3)	-13.4 (0.2)	-0.48 (0.02)	-0.67 (0.01)
8	8	-1.3 (0.7)	-15.2 (0.7)	-0.26 (0.03)	0.05 (0.04)
9	0	5.2 (0.1)		-0.31 (0.01)	
9	1	10.7 (0.1)	-21.8 (0.1)	0.17 (0.01)	0.08 (0.01)
9	2	1.0 (0.1)	16.0 (0.1)	-0.04 (0.01)	0.09 (0.01)
9	3	-12.0 (0.1)	8.9 (0.1)	0.02 (0.01)	0.24 (0.01)
9	4	9.2 (0.1)	-4.9 (0.1)	-0.12 (0.01)	-0.19 (0.01)
9	5	-3.9 (0.1)	-7.5 (0.1)	-0.27 (0.01)	-0.27 (0.01)
9	6	-1.1 (0.2)	9.5 (0.2)	-0.04 (0.01)	-0.02 (0.01)
9	7	7.1 (0.2)	10.8 (0.2)	0.27 (0.01)	-0.05 (0.01)
9	8	1.5 (0.2)	-5.3 (0.3)	-0.00 (0.01)	-0.28 (0.01)
9	9	-5.0 (0.6)	2.1 (0.6)	-0.45 (0.03)	0.15 (0.03)
10	0	-3.3 (0.1)		-0.12 (0.01)	
10	1	-4.1 (0.1)	1.0 (0.1)	-0.09 (0.01)	-0.07 (0.01)
10	2	2.7 (0.1)	-0.1 (0.1)	0.04 (0.01)	-0.08 (0.01)
10	3	-5.5 (0.1)	2.6 (0.1)	-0.09 (0.01)	0.05 (0.01)
10	4	-1.6 (0.1)	5.5 (0.1)	0.01 (0.01)	-0.06 (0.01)
10	5	5.2 (0.1)	-4.3 (0.1)	0.01 (0.01)	-0.01 (0.01)
10	6	2.6 (0.1)	-1.3 (0.1)	-0.19 (0.01)	-0.14 (0.01)
10	7	1.3 (0.2)	-1.1 (0.2)	0.18 (0.01)	0.12 (0.01)
10	8	2.4 (0.2)	4.4 (0.2)	0.07 (0.01)	0.14 (0.01)
10	9	3.2 (0.2)	-0.6 (0.2)	0.09 (0.01)	-0.09 (0.01)
10	10	-0.3 (0.5)	-6.2 (0.5)	0.03 (0.02)	-0.05 (0.03)
11	0	2.3 (0.1)		-0.02 (0.01)	
11	1	-0.6 (0.1)	1.1 (0.1)	0.03 (0.01)	-0.03 (0.01)
11	2	-2.3 (0.1)	2.4 (0.1)	-0.03 (0.01)	-0.04 (0.01)
11	3	2.1 (0.1)	-2.1 (0.1)	-0.13 (0.01)	-0.02 (0.01)
11	4	0.3 (0.1)	-2.3 (0.1)	0.08 (0.01)	0.05 (0.01)
11	5	-0.4 (0.1)	0.4 (0.1)	-0.02 (0.01)	-0.02 (0.01)
11	6	-0.5 (0.1)	-0.1 (0.1)	-0.01 (0.01)	0.02 (0.01)
11	7	1.4 (0.1)	-3.5 (0.1)	-0.01 (0.01)	-0.19 (0.01)
11	8	1.5 (0.1)	-0.2 (0.2)	-0.11 (0.01)	0.05 (0.01)
11	9	-0.8 (0.2)	-0.7 (0.2)	0.09 (0.01)	0.15 (0.01)
11	10	1.9 (0.2)	-1.7 (0.2)	-0.10 (0.01)	-0.08 (0.01)
11	11	3.4 (0.4)	-0.1 (0.4)	0.08 (0.02)	-0.39 (0.02)
12	0	-2.0 (0.1)		-0.04 (0.01)	
12	1	-0.2 (0.1)	0.5 (0.1)	-0.03 (0.01)	0.00 (0.01)
12	2	0.5 (0.1)	0.5 (0.1)	0.11 (0.01)	-0.00 (0.01)

(continued)

Table 4a. (continued)

n	m	g_n^m	h_n^m	$\dot{g}_n^m$	$\dot{h}_n^m$
12	3	-0.3 (0.1)	2.9 (0.1)	-0.02 (0.01)	0.08 (0.01)
12	4	-0.2 (0.1)	-1.9 (0.1)	-0.06 (0.01)	-0.13 (0.01)
12	5	1.1 (0.1)	0.5 (0.1)	0.04 (0.01)	0.04 (0.01)
12	6	-0.3 (0.1)	0.2 (0.1)	0.03 (0.01)	-0.00 (0.01)
12	7	0.1 (0.1)	0.2 (0.1)	0.04 (0.01)	0.03 (0.01)
12	8	0.2 (0.1)	-0.3 (0.1)	0.01 (0.01)	-0.10 (0.01)
12	9	-0.5 (0.1)	-0.5 (0.1)	-0.06 (0.01)	-0.09 (0.01)
12	10	0.5 (0.2)	-0.6 (0.2)	0.01 (0.01)	0.09 (0.01)
12	11	0.8 (0.2)	0.6 (0.2)	0.09 (0.01)	-0.07 (0.01)
12	12	0.3 (0.3)	-0.1 (0.3)	-0.03 (0.02)	-0.01 (0.02)
13	0	0.5 (0.1)		0.02 (0.01)	
13	1	-0.2 (0.1)	0.3 (0.1)	0.03 (0.01)	0.07 (0.01)
13	2	0.2 (0.1)	0.2 (0.1)	-0.05 (0.01)	-0.01 (0.01)
13	3	-0.6 (0.1)	1.5 (0.1)	-0.00 (0.01)	0.03 (0.01)
13	4	0.2 (0.1)	0.8 (0.1)	-0.00 (0.01)	0.09 (0.01)
13	5	0.9 (0.1)	-1.0 (0.1)	0.01 (0.01)	-0.08 (0.01)
13	6	0.2 (0.1)	0.2 (0.1)	0.03 (0.01)	0.06 (0.01)
13	7	0.3 (0.1)	1.2 (0.1)	0.03 (0.01)	0.06 (0.01)
13	8	-0.5 (0.1)	-0.2 (0.1)	0.00 (0.01)	-0.01 (0.01)
13	9	0.2 (0.1)	0.7 (0.1)	-0.01 (0.01)	0.01 (0.01)
13	10	-0.4 (0.1)	-0.1 (0.1)	0.00 (0.01)	-0.02 (0.01)
13	11	0.2 (0.2)	-0.5 (0.2)	0.02 (0.01)	-0.04 (0.01)
13	12	-0.0 (0.2)	0.4 (0.2)	0.09 (0.01)	0.08 (0.01)
13	13	0.6 (0.3)	-0.4 (0.3)	0.07 (0.02)	-0.01 (0.02)

Table 4b. GSFC(9/80) magnetic field model (mean radius of the earth is 6371.2 km; mean epoch is 1980.0).

n	m	$\ddot{g}_n^m$	$\ddot{h}_n^m$	$\ddot{g}_n^m$	$\ddot{h}_n^m$
1	0	-0.815 (0.032)		-0.0906 (0.0027)	
1	1	-0.449 (0.041)	1.158 (0.046)	-0.0488 (0.0041)	0.1914 (0.0046)
2	0	0.588 (0.026)		0.0384 (0.0023)	
2	1	0.487 (0.026)	-2.880 (0.030)	0.0264 (0.0023)	-0.2874 (0.0026)
2	2	0.694 (0.037)	-1.215 (0.041)	0.0198 (0.0039)	-0.0804 (0.0042)
3	0	1.304 (0.023)		0.1140 (0.0020)	
3	1	1.337 (0.024)	-2.218 (0.026)	0.1134 (0.0022)	-0.1836 (0.0024)
3	2	0.352 (0.024)	-0.156 (0.026)	0.0534 (0.0022)	-0.0168 (0.0023)
3	3	1.147 (0.034)	-0.489 (0.033)	0.0840 (0.0034)	-0.0690 (0.0033)
4	0	-0.248 (0.019)		-0.0240 (0.0017)	
4	1	-0.085 (0.018)	-0.240 (0.021)	-0.0012 (0.0016)	-0.0366 (0.0018)
4	2	-0.806 (0.020)	0.067 (0.021)	-0.0504 (0.0019)	0.0090 (0.0020)
4	3	-0.264 (0.021)	1.108 (0.021)	-0.0054 (0.0019)	0.1092 (0.0019)
4	4	-0.448 (0.026)	-0.192 (0.029)	-0.0354 (0.0028)	-0.0186 (0.0030)
5	0	-0.160 (0.003)			
5	1	-0.096 (0.003)	0.173 (0.003)		
5	2	-0.198 (0.003)	-0.135 (0.003)		
5	3	-0.272 (0.004)	0.107 (0.004)		
5	4	0.031 (0.004)	-0.035 (0.004)		
5	5	0.019 (0.006)	0.018 (0.006)		
6	0	0.077 (0.002)			
6	1	-0.063 (0.002)	0.048 (0.003)		
6	2	0.156 (0.003)	-0.074 (0.003)		
6	3	-0.025 (0.003)	-0.135 (0.003)		
6	4	0.036 (0.003)	0.096 (0.003)		
6	5	0.078 (0.003)	0.005 (0.003)		
6	6	0.156 (0.005)	0.081 (0.006)		

available data from 1955 through about 1974. $\bar{\sigma}$ is rounded to the nearest 5 nT. The observatories used in this evaluation are those listed in Table 3 (i.e. those used in deriving GSFC(9/80) and those listed in the Appendix. The statistic for GSFC(9/80) is computed *without* taking into account the local anomaly solution.

Examination of these plots shows that over its “lifetime” the GSFC(9/80) model describes the observatory data as well as or better than the other models. The deterioration of its representation prior to 1960 is also apparent. Of the models plotted, IGS75 has the longest useful “lifetime,” presumably because the span of the data used to derive the model extended to the 1950’s.

Table 5. Statistical comparison of five models.

Data type	Statistic type	Model				
		MGST(6/80)	AWC75	WC80	PMAG(7/80)	GSFC(9/80)
Magsat scalar	r.m.s.	8.2	138.9	118.7	82.2	10.1*
	mean	0.1	60.0	-20.6	-24.0	-1.6
	std. dev.	8.2	125.2	116.9	78.8	10.0
X-component***	r.m.s.	7.6	100.1	91.7	65.5	9.0
	mean	-0.2	24.7	-33.4	-27.9	-1.4
	std. dev.	7.6	97.0	85.5	59.3	8.9
Y-component***	r.m.s.	7.4	79.6	59.7	62.7	8.5
	mean	0.1	-1.3	-1.1	-1.6	-0.1
	std. dev.	7.4	79.6	59.7	62.7	8.5
Z-component***	r.m.s.	6.9	157.4	113.2	98.3	9.2
	mean	-2.5	24.9	20.9	8.4	-2.6
	std. dev.	6.5	155.4	111.3	97.9	8.8
POGO scalar	r.m.s.	—	69.8	121.6	6.7	6.9
	mean	—	43.8	21.3	0.0	-0.15
	std. dev.	—	54.4	119.8	6.7	6.9
Observatory X-component	r.m.s.	—	(1967-1974)	(1974-1977)	(1960-1977)	(1960-1977)
	mean	—	284.5	302.7	37.7**	39.9**
	std. dev.	—	-24.7	41.5	3.5	5.6
Y-component	r.m.s.	—	283.4	299.9	37.6	39.6
	mean	—	352.8	240.9	18.7	19.3
	std. dev.	—	-48.3	-15.8	-0.06	0.4
Z-component	r.m.s.	—	349.5	240.5	18.7	19.3
	mean	—	515.6	584.3	16.5	15.2
	std. dev.	—	-23.7	-71.0	-0.05	-0.11
		—	515.0	579.9	16.5	15.2

*Magsat residuals to GSFC(9/80) were taken including the MGST(6/80) external terms with GFSC(9/80) because the Magsat data used to derive GSFC(9/80) was corrected for that external field.

**PMAG(7/80) and GSFC(9/80) include estimates of observatory "anomalies" which are taken into account when computing residuals.

***Does not include data at latitudes greater than 50° or less than -50°.

Fig. 1. Variation of $\bar{\sigma}_X$ with time for observatory data relative to five field models, not including observatory anomalies.

Fig. 2. Variation of $\bar{\sigma}_Y$ with time for observatory data relative to five field models, not including observatory anomalies.

Fig. 3. Variation of $\bar{\sigma}_z$ with time for observatory data relative to five field models, not including observatory anomalies.

Fig. 4. Variation of $\bar{\sigma}$ with time for GSFC(9/80), including observatory anomalies.

It is very apparent that these models, considered collectively, suffer from a data limitation such that a spherical harmonic analysis of reasonable degree/order cannot represent the data with a $\bar{\sigma}$ of better than about 100 nT in X , 80 nT in Y , and 140 nT in Z . We attribute this to the presence of “crustal noise” in the data and believe that it is the fundamental limiting factor both on the accuracy of models based on such data and on our ability to evaluate model accuracy using such data.

A clearer picture of the model degradation is found in Fig. 4. Here the statistics for GSFC(9/80) versus three components of observatory data are plotted taking into account the local anomaly solutions. The $\bar{\sigma}$ for each component is now in the 5–20 nT range from 1962.5 on. These values are commensurate with the accuracy of measurement at observatories; i.e., we believe we have very nearly eliminated the effect of crustal anomalies and that the statistic is now dominated by the actual measurement noise. Deterioration of the model begins at about 1961.5, within the data interval used by the model. This is attributed to the lack of satellite data, with their global coverage, prior to 1965. The amount of deterioration then increases rapidly outside the data interval. It is roughly doubled at 1958.5. Comparison of Fig. 4 with Figs. 1–3 indicates that the model deterioration becomes comparable to the “crustal noise” at about 1956. For many purposes this might be taken to be the useful limit of extrapolation of the model.

Fig. 5. Comparison of Alibag annual means (X , Y , Z) to values computed from GSFC (9/80), including the observatory anomaly. Data from this observatory was used in the solution for GSFC (9/80).

Fig. 6. Comparison of Boulder annual means (X , Y , Z) to values computed from GSFC (9/80), including the observatory anomaly. Data from this observatory was used in the solution for GSFC (9/80).

Fig. 7. Comparison of Gornotayezhnaya annual means (X , Y , Z) to values computed from GSFC (9/80), including the observatory anomaly. Data from this observatory was used in the solution for GSFC (9/80).

Fig. 8. Comparison of Guam annual means (X , Y , Z) to values computed from GSFC (9/80), including the observatory anomaly. Data from this observatory was used in the solution for GSFC (9/80).

Figures 5–8 show the yearly averages at a series of observatories together with the field predicted by GSFC(9/80). Examination of these plots shows the need for the third time derivative, particularly for the X component at Alibag, Boulder, Gornotayezhnaya and Guam, and the Y and Z components at Gornotayezhnaya and Guam.

Figures 9–10 show data from two observatories *not* utilized in obtaining the solution. We have not, then, calculated the crustal component for these stations and none is included in the plots. This is most evident at Kodaikanal. Examination of Figs. 9–10 shows that although the magnitude of the model differs from the data, the temporal change of the data is well represented throughout the 1960–1980 time period. Furthermore, the third time derivative is important for the Y component at Hurbanovo.

6. Discussion and Conclusions

Observatory and other surface data are inadequate in space and time for determining accurate field models. Hints of the limitations of these data for defining the main field have repeatedly surfaced, as for example in the study already cited by MALIN and POCKOCK (1969). Comparing Fig. 1–3 with Fig. 4 brings this limitation into clearer focus. The fact is that there is a “noise” with an “rms” ($\bar{\sigma}$) of about 100–150 nT in the observatory data set. We attribute this to the existence of crustal anomalies, although it is conceivable that error sources such as poor knowledge of data location, instrument inaccuracy or local magnetic contamination are contributing factors at some locations. Crustal anomalies are worldwide

Fig. 9. Comparison of Hurbanovo annual means (X , Y , Z) to values computed from GSFC (9/80). No observatory anomaly was computed because data from this observatory was not used in the solution for GSFC (9/80).

Fig. 10. Comparison of Kodaikanal annual means (X , Y , Z) to values computed from GSFC (9/80). No observatory anomaly was computed because data from this observatory was not used in the solution for GSFC(9/80).

in distribution, have amplitude as high as several thousands of nT, and have a broad spectrum of spatial wavelengths. Because of the poor spatial distribution of surface data, any attempt to even partially model the anomaly field with such data will be plagued with aliasing problems. We conclude that these data alone provide sufficient accuracy only for models up to a degree and order of about eight. Satellite data are not immune to these limitations, although they are not as severe. LANGE and ESTES (1982) showed that crustal anomaly fields dominate the Magsat data for wavelengths shorter than those corresponding to degree and order fifteen. This means that with present methods of modeling, we cannot determine the main field representation beyond degree and order 13 or 14, regardless of the quality of our data.

What must be appreciated is that the limitations of the data are limitations not only on modeling but also on the evaluation of models. For example, in considering Fig. 1, the differences between models A, I, P, and G between 1960 and 1975 are at the “noise” (anomaly) level of the observatory data and are not likely to be truly significant. To generalize this conclusion, a model derived from satellite data with accuracies of, say, 10–20 nT, cannot be accurately evaluated using observatory data unless the model is in error by several hundred nT.

The method of solving for observatory biases or anomalies presented here offers a partial solution to this data limitation. Further work is required to determine the dependence of the bias determination on other model parameters such as the time span involved, the degree and order of the constant and temporal terms, etc. One step in this direction has been taken in that we have determined the biases for several models of differing degree and order with only small changes in the values of the biases so determined.

Further such tests need to be performed. Moreover, it needs to be determined if the calculated biases are in reasonable accord with, say, aeromagnetic anomaly data. This is complicated by the fact that the untangling of the aeromagnetic anomalies from the main field model used to reduce the anomaly data is not straightforward.

Adequate representation and prediction of the temporal change of the earth’s main field has, in our minds, not been achieved. In this paper, we have taken two steps which we believe improve the situation. *First*, we have included a solution for the localized observatory anomaly fields and, *second*, we have utilized third time derivatives. The success of the second step depends upon inclusion of the first.

The use of second and third derivatives has certainly resulted in a model of good accuracy for a 20 year period. The advantages of this over the use of several individual models for a shorter period of time are continuity and that the highly accurate satellite data are allowed to provide some constraint at times remote from the data epoch and so, hopefully, increase the overall accuracy. Tests indicated that PMAG (7/80) predicted two to three years forward in time more accurately than other Pre-Magsat models. Comparison with observatory data (Figs. 1–3, 5–10) indicates that GSFC (9/80) “predicts” (backward in time from 1960) within the 100–200 nT level for about four years. However, for longer prediction, such models are clearly not suitable because the higher derivative terms begin to dominate and the resulting error increases at a more rapid rate than for models with, say, only first derivatives. A solution might be to taper the higher temporal derivatives to zero a few years beyond the existing data interval. This, however, is quite ad hoc and has little real justification. Another approach would be to continue to add temporal terms, including higher order derivatives, until, hopefully, some sort of convergence is achieved. From the

discussion of Table 2, it is likely that higher degree and order second and third derivatives are significant. It is not clear, however, that temporal convergence will ever be achieved within any reasonable computer limitations or even that it is possible. The problem, of course, is that model constraints based on the physics of the core dynamo are not built into these models. In our view, until this is done it will not be possible to derive truly adequate "forecast" models.

The authors wish to acknowledge the interesting discussions and contributions made by E. R. Lancaster of Goddard Space Flight Center in helping to develop some of the new modeling techniques presented in this paper.

Appendix

Observatories used for the statistics of Figs. 1–3 but not included in the model solution are: Abinger, Abisko, Acacias, Aso, Averroes, Baguio, Bouzareah, Budakeszi, Budkov, Cambridge Bay, Castellaccio, Centro Geofisico, Cheltenham, Davao, Dehra Dun, Druzhnaya, Ebro, Eights, El Abiod Sidi, Elisabethville, Eskdalemuir, Gibilmanna, Gonzalez Videla, Hallett Station, Heard Island, Heiss Island, Helwan, Hollandia, Hurbanovo, Ibadan, Isla da Pascua, Jassy, Julianehaab II, Kanoya, Kanozan, Karavia, Katuura, Kiruna, Kodaikanal, Kuyper, L. America III, L. America V, Lazarevo, Luanda Golf, Manhay, Maputo, Marion Island, Mizusawa, Monte Capellino, Murmansk, Nagycenk, Nantes, Nitzanim, Norway Station, Novo-Kazalinsk, Orcadas del Sur, Patrick, Pendeli, Port-Alfred, Regensberg, Roi Baudouin, San Fernando, Simferopol, Srednikan, Stonyhurst, Surlari, Swider, Taipei, Tangerang, Thule I, Tikhaya Bay, Toledo, Tulsa, Voroshilov, Vykhnodnoy, Watheroo, Wien Auhof, Wien Kobenzl, Witteveen, Yellowknife.

REFERENCES

- BACKUS, G. E., Non-uniqueness of the external geomagnetic field determined by surface intensity measurements, *J. Geophys. Res.*, **75**, 6337–6341, 1970.
- BARKER, F. S., D. R. BARRACLOUGH, and S. R. C. MALIN, World magnetic charts for 1980—Spherical harmonic models of the geomagnetic field and its secular variation, *Geophys. J. R. Astr. Soc.*, **65**, 525–533, 1981.
- BARRACLOUGH, D. R. and S. R. C. MALIN, Geomagnetic secular acceleration, *Geophys. J. R. Astr. Soc.*, **58**, 785–793, 1979.
- BARRACLOUGH, D. R., J. M. HARWOOD, B. R. LEATON, and S. R. C. MALIN, A model of the geomagnetic field at epoch 1975, *Geophys. J. R. Astr. Soc.*, **43**, 645–659, 1975.
- CAIN, J. C., S. J. HENDRICKS, R. A. LANGE, and W. V. HUDSON, A proposed model for the international geomagnetic reference field—1965, *J. Geomag. Geoelectr.*, Kyoto, **19**, 335–355, 1967.
- HURWITZ, L. and D. G. KNAPP, Inherent vector discrepancies in geomagnetic main field models based on scalar F, *J. Geophys. Res.*, **79**, 3009–3013, 1974.
- LANGE, R. A., Near-earth magnetic disturbance in total field at high latitudes 1. Summary of data from OGO 2, 4, and 6, *J. Geophys. Res.*, **79**, 2363–2371, 1974.
- LANGE, R. A. and R. H. ESTES, A Geomagnetic Field Spectrum, *Geophys. Res. Lett.*, **9**, 250–253, 1982.
- LANGE, R. A., R. L. COLES, and M. A. MAYHEW, Comparison of magnetic anomalies of lithospheric origin measured by satellite and airborne magnetometers over Western Canada, *Can. J. Earth Sci.*, **17**, 876–887, 1980a.
- LANGE, R. A., R. H. ESTES, G. D. MEAD, E. B. FABIANO, and E. R. LANCASTER, Initial geomagnetic field model from Magsat vector data, *Geophys. Res. Lett.*, **7**, 793–796, 1980b.
- LOWES, F. J., Mean square values on sphere of spherical harmonic vector fields, *J. Geophys. Res.*, **71**, 2179, 1966.

- LOWES, F. J., Spatial power spectrum of the main geomagnetic field, and extrapolation to the core, *Geophys. J. R. Astr. Soc.*, **36**, 717–730, 1974.
- MALIN, S. R. C. and S. B. POCKOCK, Geomagnetic spherical harmonic analysis, *Pure Appl. Geophys.*, **75**, 117–132, 1969.
- MEAD, G. D., An evaluation of recent internal field models, in *Quantitative Modeling of Magnetospheric Processes*, edited by W. P. Olson, Geophysical Monograph, **21**, pp. 110–117, American Geophysical Union, 1979.
- PEDDIE, N. W. and E. B. FABIANO, A model of the geomagnetic field for 1975, *J. Geophys. Res.*, **81**, 2539–2542, 1976.
- STERN, D. P. and J. H. BREDEKAMP, Error enhancement in geomagnetic models derived from scalar data, *J. Geophys. Res.*, **80**, 1776–1782, 1975.
- STERN, D. P., R. A. LANGE, and G. D. MEAD, Backus effect observed by Magsat, *Geophys. Res. Lett.*, **7**, 941–944, 1980.