A Comparison of the Proposed IGRF Models:
Internal and Relative Consistencies†

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To compare three sets of models submitted by IGS, USGS, and NASA as proposed revisions of the International Geomagnetic Reference Field, the root mean square amplitudes over the Earth’s surface were computed for the main field (MF) and secular variation (SV) models and for differences between them. The results obtained for the total field and for the nth-degree harmonics showed that the IGS 1975–80 SV model, the IGS 1980 MF model, and the NASA 1980–85 SV model differ significantly from the other corresponding models. It is proposed that IGRF main field models for epochs 1965, 1970, and 1975 be derived from means of all three candidate models; for 1980, the mean of the USGS and NASA models would be used. The secular variation model for 1980–85 would be taken as a mean of the submitted USGS and IGS SV models.

The main purpose of the present report is to analyse the internal as well as relative consistencies of three sets of proposed IGRF models: USGS set (Peddie and Fabiano, 1982), IGS set (Barracloough et al., 1982), and NASA set (Langel et al., 1982). The analysed quantity, amplitude \( A \), is defined as a root of the mean square value, over the Earth’s surface, of the magnitude of the magnetic induction vector \( B \)

\[
A^2(T) = \frac{1}{4 \pi} \int_0^{2 \pi} \int_0^\pi |B(a, \theta, \lambda, T)|^2 \sin \theta \, d\theta \, d\lambda,
\]

where \( a \) denotes Earth’s radius, \( \theta \) and \( \lambda \) are spherical polar coordinates of a point (colatitude and east longitude, respectively), and \( T \) denotes the epoch of the analysis.

According to Lowes (1966), the integral in Eq. (1) equals

\[
A^2(T) = \sum_{n=1}^{\infty} (n + 1) \sum_{m=0}^{n} [(g^m_n(T))^2 + (h^m_n(T))^2],
\]

where \( g^m_n(T) \) and \( h^m_n(T) \) are the coefficients of expansion of the potential \( V \) of the internal geomagnetic field which is given by

\[
V(r, \theta, \lambda, T) = a \sum_{n=1}^{\infty} \left( \frac{a}{r} \right)^{n+1} \sum_{m=0}^{n} [g^m_n(T) \cos m\lambda + h^m_n(T) \sin m\lambda] P^m_n(\cos \theta).
\]
Here \( r \) is the distance from the Earth's center and \( P_n^m(\cos \theta) \) are semi-normalized associated Legendre polynomials.

The comparison has been carried out not only for the total amplitude \( A(T) \) of the main field and secular variation (with \( g_n^m \) and \( h_n^m \) replaced by \( \hat{g}_n^m \) and \( \hat{h}_n^m \) in Eq. (2)), but also for the amplitudes \( A_n(T) \) of the \( n \)-th-degree harmonics, where

\[
(A_n(T))^2 = (n + 1) \sum_{m=0}^{n} [(g_n^m(T))^2 + (h_n^m(T))^2].
\] (4)

Moreover, the amplitudes \( A(T) \) and \( A_n(T) \) of the differences between models, representing the mean magnitude of the vector difference between two field models, have been computed. They are given by Eqs. (2) and (4) with \( g_n^m \) and \( h_n^m \) replaced by \( \Delta g_n^m \) and \( \Delta h_n^m \) for main field and by \( \Delta \hat{g}_n^m \) and \( \Delta \hat{h}_n^m \) for secular variation.

The results of computations, presented as plots of \( A \) and \( A_n \) versus time, are discussed briefly in the following. In the plots, \( U \) corresponds to the USGS set of models, \( I \) corresponds to the IGS set, and \( N \) to the NASA set.

In Fig. 1, the amplitude \( A \) of the total main field computed for the three proposed sets is shown. The values for 1985 are obtained by applying the 1980–85 SV terms to the 1980 MF models. The USGS and NASA models are very close to each other, especially at 1980, but with the difference increasing significantly at 1985. On the other hand, the IGS amplitude is about 15–30 nT higher for all but the last epoch. The greatest difference, 27 nT, occurs at 1980 indicating a large discrepancy between the IGS model and the other two models at this epoch.

![Main Field Models](image)

**Fig. 1.** Total amplitude \( A \) of the main field models. \( U \) denotes the USGS set of models, \( I \) denotes the IGS set, \( N \) denotes the NASA set.
Figure 2 shows the amplitudes $A_n$ of the $n$th-degree harmonics of the main field models. Generally, the lower the degree, the better the relative agreement. The IGS set shows the greatest variability, particularly for the higher degrees between 1975 and 1980.

The amplitudes of the differences between the main field models are shown in Fig. 3. The values obtained for the series truncated at $n = 8$ are also shown; the truncation was...
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Fig. 3. Total amplitude $A$ of the differences between the main field models. At 1965, $A$ for the series truncated at $n = 8$ is also shown for differences involving the IGS model.

necessary because the IGS 1965 model is of degree and order eight. Generally, the amplitudes of differences are much higher than the differences between amplitudes (see Fig. 1). Relatively lower differences between USGS and NASA models confirm their similarity, especially at the epoch 1980. Both differences involving the IGS set are much higher and very similar, being determined mainly by the behaviour of this set of models.

The amplitudes $A_n$ of the differences between the main field models, not shown, confirm the general behaviour seen in the previous figure. For the epochs 1965, 1970, and (in few cases) 1975 all differences for a given degree are of similar magnitude. At the epoch 1980, however, differences USGS-NASA decrease to the level of about 10 nT, while differences involving the IGS model increase significantly; maximum (126 nT) occurs for the difference IGS-USGS at $n = 3$. The situation changes at 1985. Differences involving the IGS model grow at a much slower rate and some of them even decrease ($A_1$ for NASA-IGS
and IGS-USGS, $A_4$ for IGS-USGS and, especially, $A_3$ for NASA-IGS). The 1985 differences USGS-NASA, on the other hand, grow considerably, indicating disagreement in the predicted secular variation. The behaviour of $A_1$ is notable—during the whole period it does not exceed 45 nT for any difference.

In the next series of plots the results for secular variation models, defined as the rate of change between the main field models, are presented. In Fig. 4, the amplitude of the total field secular variation is shown for the three proposed sets of models, as well as for the IGS 1965–70 model truncated at $n = 8$. The most obvious feature is the very high amplitude of the IGS 1975–80 SV field, related to the difference between the IGS 1975 and IGS 1980 main field models. The second prominent feature is the increase of the NASA model amplitude for the interval 1980–85. Apart from these irregularities, the growing intensity of the secular variation during the period 1965–1980 is clearly seen.

The amplitudes $A_n$ for the secular variation models are shown in Fig. 5. $A_9$ and $A_{10}$ for all USGS SV models as well as for the IGS 1970–75 and 1980–85 SV models are not plotted since the corresponding MF coefficients are constant during these periods. The large values of $A_9$ and $A_{10}$ for the IGS 1965–70 SV model do not reflect an actual change of the field—they result from the change of the maximum degree of the MF model from eight to ten. The

Fig. 4. Total amplitude $A$ of the secular variation models. The open triangle represents the amplitude of the IGS 1965–70 SV model truncated at $n = 8$. 
Fig. 5. Amplitudes $A_n$ of the $n$th-degree
nth DEGREE HARMONICS OF SECULAR VARIATION MODELS (cont.)

harmonics of the secular variation models.
IGS set varies most, mainly because of the abrupt change between the 1970–75 and 1975–80 SV models seen for all but the third and fourth degrees. For 1980–85, the IGS and USGS SV models are close (except for $n = 6$), but the NASA model differs significantly from them, particularly for lower degrees (except for $n = 3$).

The amplitudes of the differences between secular variation models are shown in Fig. 6. The computation has been done both for $n \leq 8$ (not shown) and for $n \leq 10$. The truncation gives significant change of amplitude only for differences involving the IGS model for the interval 1965–70. After rejecting the spurious ninth- and tenth-degree IGS terms, resulting from the change of the maximum degree of the model, all three differences for the interval 1965–70 lie in the range of $12–13$ nT · yr$^{-1}$. Later, the differences involving IGS models increase rapidly until 1975–80 and then decrease significantly, especially in the case of 1980–85 value for IGS-USGS. The difference USGS-NASA attains minimum for 1970–75 and then grows steadily.

The behaviour of $A_n$ for the differences between secular variation models, not shown, confirms the general conclusions of the previous paragraph. All differences involving the IGS set of models have their maximum at 1975–80, except $A_1$ for the difference NASA-
IGS: its maximum is masked by the much larger discrepancy for 1980–85 (compare with Fig. 5). For the interval 1980–85, the differences IGS-USGS lie in the range 3.5–10.5 nT·yr⁻¹, which indicates good agreement between these SV models. The differences involving the NASA SV model are much greater for \( n \leq 4 \); they lie in the range 8.5–20.2 nT·yr⁻¹ with the maximum for \( n = 3 \).

From the above discussion, four general conclusions follow, concerning the internal and relative consistencies of the proposed sets of models.

(i) There is a significant discrepancy between the IGS main field model and the two other models at the epoch 1980. It probably stems from the fact that the IGS 1980 model, conceived primarily as a tool for preparing 1980 world declination charts (BARKER et al., 1981), was derived mainly from declination and inclination data. Therefore it represents closely the geometrical features of the field at the Earth’s surface, but is much poorer at describing its intensity. On the other hand, both the USGS and NASA models for 1980 are based mainly on the scalar and vector Magsat data.

(ii) The amplitude of the IGS 1975–80 secular variation model is exceptionally high. It is due to the transition between the ‘general’ IGS 1975 main field model and the ‘specialized’ IGS 1980 model.

(iii) The NASA 1980–85 secular variation model differs significantly from the two other models.

(iv) The change of the length of the spherical harmonic series representing the geomagnetic reference field is very inconvenient, although sometimes necessary. It disturbs slightly the total field amplitude and causes the spurious secular variation of the harmonics involved.

On the grounds of the foregoing conclusions, the following scheme regarding derivation of the new set of IGRF models can be proposed.

(i) Main field models for the epochs 1965, 1970 and 1975 would be computed from means of the USGS, IGS and NASA models for the appropriate epochs, with the ninth- and tenth-degree coefficients for 1965 taken as the mean of the USGS and NASA models. The main field model for 1980 would be computed as a mean of the USGS 1980 and NASA 1980 models.


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REFERENCES


