Pitch-Angle-Scattering Diffusion Coefficient for Particles with Pitch Angle near 90° in Simulated Magnetic Field Fluctuations

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Diffusion coefficient of pitch angle scattering near 90° pitch angle is extensively studied for particle rigidity of 1 GV $\sim$ 9 GV by the method of JONES *et al.* (1973). Our calculations are based on a slab model with transverse (x, y) field fluctuations varying only along the direction (z) of the average magnetic field $\langle B \rangle$. The root mean square of fluctuation field $\langle \Delta B^2 \rangle^{1/2}$ is fixed to 0.3 times the average field $\langle B \rangle$ through this calculation. The fluctuations have a power spectrum proportional to $k^{-\gamma}$ at large wave numbers, where k is a wave number and γ is a power spectral exponent varying from 1.5 to 2.3.

A big hump in the calculated pitch-angle-scattering diffusion coefficient near 90° pitch angle and its dependence on particle rigidity and γ are discussed in terms of test particle motion trapped in magnetic bottles formed in random magnetic field realizations. It is concluded that the hump does not mean a real enhanced pitch angle diffusion through 90° pitch angle.

1. Introduction

The main process of propagation of cosmic rays in the solar wind reference frame is a diffusion along the average magnetic field lines through scattering in pitch angle by the magnetic irregularities. The theory for scattering in pitch angle has been developed more than a decade ago (JOKIPII, 1966; HASSELMANN and WIBBERENTZ, 1968). However, the quasi-linear theory developed by these authors has a problem in predicting no scattering in pitch angle at $\mu = 0$, where μ is a pitch angle cosine. JOKIPII (1974) showed that the theory might be useful only for the magnetic field of small fluctuations, that is $\eta \ll 1$, where $\eta = \langle \Delta B^2 \rangle^{1/2} / \langle B \rangle$, $\langle \Delta B^2 \rangle^{1/2}$ and $\langle B \rangle$ are the perturbation field and the mean field respectively. The magnetic field fluctuations observed in interplanetary space, however, shows rather larger irregularities, that is $\eta = 0.3 \sim 0.5$ (MOUSSAS *et al.*, 1978).

As a next step, a different approach taking into account a nonlinear term of particle trajectories has been investigated (Jones *et al.*, 1973; VÖLK, 1973; OWENS,

1974; GOLDSTEIN, 1976, 1977). These efforts lead to a finite scattering through $\mu = 0$. However, their predictions do not agree quantitatively around $\mu = 0$, again.

Jones *et al.* (1973) developed a method calculating the scattering in pitch angle for charged particles in given magnetic field fluctuations by a computer to test the above theories. They calculated the pitch angle diffusion coefficient $D(\mu)$ for a fluctuation power spectrum with the spectral index of $\gamma = 2$, and indicated the incompleteness of quasi-linear theory by showing the large $D(\mu)$ around $\mu = 0$. By using a similar method of JONES *et al.* (1973), MOUSSAS *et al.* (1978) computed the scattering in pitch angle for both the magnetic field fluctuations observed in interplanetary space and the simulated field with $\gamma = 2$. OWENS *et al.* (1980) calculated $D(\mu)$ for larger fluctuation field of $\gamma = 1.5$. They showed stronger scattering takes place around $\mu = 0$ with decreasing rigidity of particles.

KATO *et al.* (1981), after calculating the scattering in pitch angle by using a similar method to that by JONES *et al.* (1973) and KAISER *et al.* (1978), pointed out that the large hump of $D(\mu)$ around $\mu = 0$ comes from the trapping of test particles in magnetic bottles and does not mean the real enhanced scattering in pitch angle.

In the present paper, to confirm the above argument we show the fact that some test particles are really trapped for a long time in some magnetic bottles constructed in the slab model of larger fluctuation fields with a larger value of γ (e.g. $\gamma \geq 2$). Further, it is pointed out that the Fokker-Planck formalism may have a problem in describing the scattering in pitch angle around $\mu = 0$ for specially such large field fluctuations (e.g. $\eta > 0.1$) with a large value of γ (e.g. $\gamma > 1.5$).

2. Method

A slab model with the transverse field fluctuations ($\Delta B_x, \Delta B_y$) varying only the average field direction (z) is used to simulate the interplanetary magnetic field fluctuations. The average features of fluctuations are expressed approximately by a spatial power spectrum of a form

$$P(k) = \frac{A}{\{1 + (2\pi k L_c)^2\}^{\gamma/2}}, \quad (1)$$

where k is a wave number, γ an arbitrary constant (typically $1.5 \leq \gamma \leq 2.5$) and A a constant that should satisfy the relation

$$\eta^2 = 2 \int_0^\infty P(k) dk. \quad (2)$$

The quantity L_c is a correlation length of the magnetic field in the z -direction which is 6×10^{11} cm in our model after HEDGECOCK (1975), and η is $\langle \Delta B^2 \rangle^{1/2} / \langle B \rangle$, where $\Delta B^2 = \Delta B_x^2 + \Delta B_y^2$, where $\langle B \rangle$ ($= 5$ gamma) denotes the average field. A series of field fluctuations ("realizations") along the z -axis which satisfy Eqs. (1) and (2) is generated on grids with a spacing $d = L_c/1000$ by an auto regressive method

(ULRYCH *et al.*, 1975; KATO *et al.*, 1983) for given values of η and γ . In this paper, η is fixed to 0.3; a typical value in the interplanetary space for quiet times (MOUSSAS *et al.*, 1978). The realizations produced from the same Gaussian distributed numbers with different values of γ are shown in Fig. 1. It is clear that smaller scale irregularities become less frequent with the increasing value of γ . A realization without relatively smaller scale fluctuations may concern intimately with test particles trapped in a magnetic bottle for a long time and will be important to interpret the larger peak of $D(\mu)$ around $\mu = 0$.

The calculation of particle's trajectory, and the deriving pitch angle scattering diffusion coefficient $D(\mu)$ are similar to KAISER *et al.* (1978) and OWENS *et al.* (1980). Eight test particles are gyrotropically injected into a realization with a fixed pitch angle cosine μ_s at starting time $t = 0$. Here pitch angle cosine μ is defined as $\mathbf{B} \cdot \mathbf{V}/|\mathbf{B}||\mathbf{V}|$, where $\mathbf{V}$ is a velocity of particle, and $\mathbf{B}$ is the magnetic field at z . Trajectories of these particles are computed every $\Delta t = 0.02$ sec (physical time) by using the coordinate transformation in which during a step, the direction and the magnitude of magnetic field are supposed to be constant and particles gyrate about this field. These particles are traced until they will arrive at the boundaries μ_L or μ_R , where μ_L and μ_R denote the left side and the right side boundary from the μ_s . The pitch angle cosine μ of particles is binned in a histogram with $\Delta\mu = 0.02$ at the fixed time interval $T \approx 1$ sec (physical time) and the total accumulated histogram $H(\mu)$ is obtained for 1600 test particles which means 200 realizations. The distribution $f(\mu) = H(\mu)/\Delta\mu$ is regarded as a steady state distribution in μ -space by the steady injection of particles at $\mu = \mu_s$. In the steady state, the equation

Fig. 1. The dependence of characteristic scale of irregularities upon the values of γ are shown. Here γ is the value of spectral index. The magnetic fields are generated by auto regressive method every 0.001 step which is the thickness of the slab in unit of correlation length $L_c \sim 6 \times 10^{11}$ cm.

$$\begin{aligned}
 D(\mu) \frac{df(\mu)}{d\mu} &= -J_L, \mu_L \leq \mu < \mu_s \\
 &= -J_R, \mu_s < \mu \leq \mu_R
 \end{aligned} \quad (3)$$

should be satisfied. Here $D(\mu)$ is a pitch angle scattering diffusion coefficient, and J_L and J_R are the fluxes given by N_L/T and N_R/T respectively, where N_L and N_R are the number of particles that escaped at $\mu = \mu_L$ and $\mu = \mu_R$, respectively. $H(\mu)$, N_L , and N_R are directly and separately measured so that $D(\mu)$ can be determined readily from Eq. (3). The slope of $f(\mu)$; $df(\mu)/d\mu$, at a position μ is obtained from a linear least squares fit to the $f(\mu)$ distribution symmetrically bracketing μ . Typically, 5 points are used here.

3. Results

To check our programming and computing method, we show $D(\mu)$ for $\varepsilon = 9$ (~ 1 GV), $\gamma = 2$ and $\eta = 0.3$ ($\langle \Delta B_x^2 \rangle^{1/2} = \langle \Delta B_y^2 \rangle^{1/2} \simeq 0.21 \langle B \rangle$), where ε is defined as $L_c/\text{gyroradius of particle in } \langle B \rangle$. As shown by the previous workers (KAISER *et al.*, 1978; OWENS *et al.*, 1980), for ~ 1 GV particle in realizations of $\eta = 0.3 \sim 0.5$ and $\gamma = 2 \sim 1.5$, the result of computer simulations coincides with that of quasi-linear theory except around $\mu = 0$. Figure 2 shows that our result and the curve predicted by quasi-linear theory (e.g. GOLDSTEIN, 1976) are consistent each other except around $\mu = 0$. So we feel that our total processes are justified.

Fig. 2. The pitch angle scattering coefficient $D(\mu)$ obtained from our simulations is shown together with the curve from the quasi-linear theory. Even for fluctuations of $\eta \approx 0.3$, two results have almost the same μ dependence except $|\mu| < 0.2$ as other workers showed.

The dependence of $H(\mu)$ histogram upon the spectral index γ are shown for the fixed parameters of particle rigidity (~ 1 GV), $\mu_s = 0.275$, $\mu_L = -0.4$, and $\mu_R = 0.9$ in Fig. 3. It is obvious from the figure that the flatness near $\mu = 0$ becomes pronounced with increasing the value of γ . As long as Eq. (3) is used to derive $D(\mu)$ from the $H(\mu)$ histogram, the $D(\mu)$ obtained thus has a peak around $\mu = 0$ for $\gamma = 1.7$ and 2. The $D(\mu \approx 0)$ for $\gamma = 2$ is larger than that for $\gamma = 1.7$ by a factor of 4. Further, for $\gamma = 2.3$, a hump on the histogram comes out clear in the negative region of μ . That makes the negative $D(\mu)$ between μ_p , the value of μ at the peak which is except $\mu = \mu_s$ and μ_M , that at the minimum of $H(\mu)$.

As already shown in Fig. 1, a realization with larger value of γ does not relatively have the irregularities of smaller scale. In such a realization showing more smoothed field, the scattering in pitch angle seems not to occur very often specially for $\mu \approx 0$. So $D(\mu \approx 0)$ for $\gamma = 2$ is expected smaller than that for $\gamma = 1.7$. But the above $D(\mu)$ obtained from the simulation shows the opposite sense. Moreover, the negative $D(\mu)$ obtained for $\gamma = 2.3$ is not explained in terms of diffusion.

The dependence of histogram $H(\mu)$ for $\gamma = 2.3$ upon particle rigidities are shown in Fig. 4. Other parameters are the same in Fig. 3. The slope of histogram around $\mu = 0$ is changing lower with decreasing the rigidity of particles. That means that the peak of $D(\mu)$ becomes bigger and bigger with decreasing rigidity of particles. Finally, a hump on the histogram appears for 1 GV particles. That will give a negative $D(\mu)$ again.

Generally, lower rigidity of particles (≤ 1 GV) may more likely follow a line of force than higher rigidity of particles in the magnetic field having power spectrum as Eq. (1). It means that the scattering in pitch angle will become less effective for lower

Fig. 3. The dependence of $H(\mu)$ histogram upon the value of spectral index γ are shown. The change of slope around $\mu = 0$ with increasing the value of γ is clear. μ_s , η and particle rigidity ($\epsilon = 9$; corresponding to ~ 1 GV) are fixed.

Fig. 4. The dependence of $H(\mu)$ histogram upon particle rigidity are shown. The change of slope around $\mu = 0$ is obvious with decreasing particle rigidity. μ_s , η and γ are fixed.

rigidity of particles. While, as already mentioned in the above, the $D(\mu \simeq 0)$ from the simulation looks opposite sense and the negative $D(\mu)$ for 1 GV particles is difficult to be interpreted in terms of simple diffusion.

JONES *et al.* (1973) assumed that $f(\mu)$ ($=H(\mu)/\Delta\mu$) was constructed by a steady flow from the μ_s : source point. If it is correct, the slope of the distribution $f(\mu)$ should be independent of the source point except near μ_s (e.g. $|\mu - \mu_s| > \Delta\mu$). In Fig. 5, $H(\mu)$ histograms are shown with the various values of μ_s (0.07, 0.12, 0.275) for particle rigidity of 1 GV in realizations of $\gamma = 2$. It is clear that the slope around $\mu = 0$ is dependent on μ_s . Again, the negative $D(\mu)$ is expected for $\mu_s = 0.07$. So we feel that the distribution $f(\mu)$ at least around $\mu = 0$ can not be determined by the steady flow like Eq. (3).

In the slab model adopted here, the strength of magnetic field is not constant along the z axis and changes like $\sqrt{\langle B \rangle^2 + \Delta B^2(z)}$ depending on the z axis. This means that the field can produce something like magnetic bottle (or magnetic mirrors). Considering test particles periodically moving back and forth in those magnetic bottles for a while, the above γ -, rigidity of particles-, and μ_s -dependences of $f(\mu)$ distribution seem to be easily resolved. Because the probability of trapping particles for a long time becomes higher for the realizations of larger value of γ , for smaller rigidity of particles, and also for smaller value of μ .

Next, we will show how test particle is trapped for a long time in a magnetic bottle. Figure 6 is the case without any trapping around $\mu = 0$ in which a particle reached the boundary (μ_L) within 1 minute corresponding to travelling almost 1 correlation length along the z axis. Most of test particles behave like this case. Figure 7 shows just the case with a long trapped particle around $\mu = 0$. Also, the same event

Fig. 5. The dependence of $H(\mu)$ histogram upon μ_s are shown. The slope around $\mu = 0$ are changing with the value of μ_s .

Fig. 6. The trajectory of a test particle without any trapping is shown in μ - t plane. Most of particles behave like this, that is, within 1 minute, they escape from either side of the boundary.

Fig. 7. A test particle supposed to be trapped in a magnetic bottle is shown. It moves periodically in μ - t plane more than 10 minutes.

in μ - z plane is shown in Fig. 8. The particle oscillates periodically in μ - t plane like sinusoidal wave and moves back and forth in μ - z plane. Referring the upper figure in Fig. 8, we can see that the particle is clearly trapped between the stronger fields forming magnetic bottle.

Here, we would like to note that a particle with pitch angle cosine of $\mu = 0$ in the slab model can change its orbit in a short time without staying in $\mu = 0$ for a long time as quasi-linear theory predicts. $\mu \simeq 0$ will occur in rather stronger fields as shown in Fig. 8. The direction of the stronger magnetic field is inclined largely from the z axis, since the direction of $\mathbf{B}$ is given by $\mathbf{B} = \langle B \rangle \mathbf{e}_z + \Delta B_x \mathbf{e}_x + \Delta B_y \mathbf{e}_y$, where $\mathbf{e}_x$, $\mathbf{e}_y$, and $\mathbf{e}_z$ denote the unit vector of the x , y , and z axis respectively. Hence, the orbital plane of particle with $\mu = 0$ tilts from the z axis. Consequently, as the particle rotates about $\mathbf{B}(z)$, the magnetic field seen by it changes successively both in the direction and in the strength because of the deviation of $\mathbf{B}(z)$ from the z axis. Thus, even after a short time compared with the period of gyration, the pitch angle cosine changes from $\mu = 0$ to another μ and the particle can not stay at $\mu = 0$ for a long time.

In a similar reason, a trapped orbit in a magnetic bottle will change gradually so that the particle can not be trapped forever in the bottle (quasi-trapped). If we see this quasi-trapped orbit in μ - t plane, it will show a periodical behavior like sinusoidal wave during the trapping. But, by the lapse of time, the sinusoidal form will be gradually deformed by the change of orbit as shown in Fig. 7 and finally, the sinusoidal pattern will fade out by the particle's leaving the trapping region.

Fig. 8. The same event is shown in μ - z plane together with the transverse magnetic field component (the upper figure). The particle seems to be trapped between the stronger fields producing magnetic bottle. The numbers attached to the solid squares mean the time spent by the particle after the injection in minute (physical time).

4. Discussions

Generally, the probability of trapping test particles in magnetic bottles in some realizations may be low as mentioned in Section 3, although it depends greatly upon value of γ and rigidity of particles. However, if test particles are once trapped, they will spend much longer time around $\mu = 0$ than no trapped particles by oscillating periodically in μ - t plane as shown in Fig. 7. So they can contribute very much to form $H(\mu)$ histogram around $\mu = 0$.

On reference to Fig. 7, probably, the contribution of periodically oscillating particles may approximately be expressed as a sinusoidal distribution $S_i(\mu)$ on μ plane which is defined as $S_i(\mu) = 1/\pi\sqrt{A_i^2 - \mu^2}$, where A_i is the amplitude of an oscillation in μ - t plane.

When value of γ is smaller or rigidity of particles is higher, the probability of trapping particles is generally considered low. Even if some particles are trapped, the duration of trapping time must be shorter and smaller amplitude A_i in μ - t plane may more frequently occur than larger one because of many smaller scales of irregularities as shown in Fig. 1. Then, $\sum_i S_i(\mu)$ will show a bell shaped distribution having a

maximum at $\mu = 0$. Here $\bar{S}_i(\mu)$ is defined as $\bar{S}_i(\mu) = \int_{\mu}^{\mu + \Delta\mu} S_i(\mu) d\mu$. This contribution to $H(\mu)$ histogram must actually be small because the probability of trapping is low, but not negligible. So if this $\sum_i \bar{S}_i$ is superimposed to the distribution constructed by the flow $J = -D(\mu)(df(\mu)/d\mu)$, then $H(\mu)$ histogram will show a gentle slope or a flat slope around $\mu = 0$ as shown in Figs. 3 or 4. That will produce a hump in $D(\mu)$ near $\mu = 0$.

On the other hand, when γ is larger or rigidity of particles is smaller, a trapped particle may have a longer durable oscillation with a typical larger amplitude A_i because the parameters γ and ε considered here will increase the probability trapping particle for a long time as expected from Fig. 1. Then, $\bar{S}_i(\mu)$ exhibits typically something like M shaped distribution having two peaks ($\mu \simeq -A_i$, $\mu \simeq A_i$) and a center at $\mu = 0$. If this distribution is added to the one obtained from the steady state flow $J = -D(\mu)(df(\mu)/d\mu)$, a peak around $\mu \simeq -A_i$ on $H(\mu)$ histogram will be pronounced as the upper two histograms in Figs. 3 or 4. On the other hand, since we inject test particles at $\mu_s > 0$, a peak around $\mu \simeq A_i$ is not so pronounced because of the large numbers of particles coming from μ_s . Thus, a negative $D(\mu)$ between μ_p and μ_M is expected, where μ_p and μ_M denote the value of μ at the peak which is except $H(\mu)$ at $\mu = \mu_s$, and that at the minimum of $H(\mu)$ respectively.

Hence, on the view point of particles trapped in magnetic bottles produced in some realizations, the change of histogram depending on the values of γ and the rigidity of particles as shown in Figs. 3 and 4 will be easily interpreted.

Such same effect due to particles trapped for a while in magnetic bottles as ours may exist in the results of other workers (OWENS *et al.*, 1980; KAISER *et al.*, 1978), although the negative $D(\mu)$ did not appear in their results because their parameters used there are the rigidity ≥ 2 GV for $\gamma = 2$ (KAISER *et al.*, 1978) and ≥ 0.4 GV for $\gamma = 1.5$ (OWENS *et al.*, 1980). Actually, larger peak of $D(\mu)$ around $\mu = 0$ are seen with decreasing rigidity of particles in their results and this dependence of $D(\mu \simeq 0)$ on rigidity of particles has not been reasonably explained. Therefore, we would say that their larger $D(\mu \simeq 0)$ also comes from particles trapped in magnetic bottles. If this argument will be allowed, we can say that the large $D(\mu = 0)$ obtained from the simulations so far does not mean the enhanced scattering around $\mu = 0$ but particles can not escape from some trapped regions for a while.

JOKIPII (1974) shows that Fokker-Planck formalism governing pitch angle scattering can be useful only for $\eta \ll 1$. So, for such small field of fluctuations, the method using the steady state flow $J = -D(\mu)(df(\mu)/d\mu)$ developed by JONES *et al.* (1973) to obtain $D(\mu)$ will also be appropriate.

However, in the case of slab model with larger γ and larger η , as already shown in this simulations, magnetic bottles will be occasionally formed in good shape having much stronger field at both ends. There, a test particle will be trapped for a long time and shows a periodical motion around $\mu = 0$ as in Fig. 7. This behavior seems to be rather deterministic change than diffusive one in pitch angle. Hence, Fokker-Planck formalism and also the method of JONES *et al.* (1973) seem not to be applicable in the vicinity of $\mu = 0$ for such fields with larger γ and η . For instance, for $\gamma = 2$ and η

$= 0.3$, the applicable range of this formulation in μ will be limited to $|\mu| \geq 0.2$, almost corresponding to a typical amplitude of oscillation in μ - t plane.

The real magnetic field fluctuations in interplanetary space shows rather $\eta \approx 0.3 \sim 0.5$ instead of $\eta \ll 1$ (MOUSSAS *et al.*, 1981), and also shows $\gamma \approx 2$ sometimes (SARI *et al.*, 1969). Hence, if we suppose the fluctuations change total magnetic field strength like the slab model, the trapped effect shown in our simulation may occur even in interplanetary space. Then, the scattering in pitch angle of cosmic rays in interplanetary space may not simply be described by Fokker-Planck equation in the range of $|\mu| \lesssim 0.2$. Further, if this is true in interplanetary space, the spatial diffusion coefficient $k_{\parallel}$ can not simply be obtained from $D(\mu)$ by the relation derived by JOKIPII (1966), or HASSELMANN and WIBBERENTZ (1968). Because, basically, the relation is derived by assuming that the change in pitch angle is statistical even near $\mu = 0$, and because the large $D(\mu)$ around $\mu = 0$ means that a particle can not escape from trapped regions.

5. Summary and Conclusions

Pitch angle scattering diffusion coefficients are calculated for particle rigidity of 1 GV $\sim$ 9 GV under the condition of magnetic field irregularities with $\eta = 0.3$ and $\gamma = 1.5 \sim 2.3$ by following the method given by JONES *et al.* (1973) and KAISER *et al.* (1978). The larger $D(\mu)$ around $\mu = 0$ are obtained with decreasing rigidity of particles and/or increasing value of γ . Also, the negative $D(\mu)$ near $\mu = 0$ is shown for the case of 1 GV particles in the magnetic field irregularities of $\gamma = 2.3$. Further, the $D(\mu)$ around $\mu = 0$ are found to depend upon values of μ_s ; the pitch angle cosine of injection point. These facts can be easily understood by considering that some test particles are trapped in magnetic bottles existing in some realizations and move almost periodically in μ - t plane.

On the basis of these thoughts, it is concluded that the enhanced $D(\mu)$ around $\mu = 0$ obtained thus does not mean actual pitch angle diffusion coefficient. Moreover, it is questionable that Fokker-Planck formalism can describe the scattering in pitch angle near $\mu = 0$ in the field fluctuation model which can change the total magnetic field strength, since the changes in pitch angle seems to include some deterministic process in the vicinity of $\mu = 0$ by trapping effects. Also, it is noted that spatial parallel diffusion coefficient $k_{\parallel}$ may not be simply calculated from $D(\mu)$ obtained here by using the ordinary relation (JOKIPII, 1966; HASSELMANN and WIBBERENTZ, 1968), if Fokker-Planck formalism is not appropriate around $\mu = 0$ as mentioned in Section 4.

If we suppose that the trapping effects exhibited in our simulations also occur even in interplanetary space for fields of large γ and $\eta = 0.3 \sim 0.5$, the theoretical formalism governing the scattering in pitch angle developed so far seems not applicable to behavior of cosmic rays in interplanetary space. Probably, a new formalism including trapping effect around $\mu = 0$ must be needed.

On this situation, we feel that observing time evolution of pitch angle distribution of solar cosmic rays and also magnetic field fluctuations simultaneously are important to understand cosmic rays in interplanetary space. Further, much

better understanding will be obtained by observing the above two phenomena on two spacecrafts distant ~ 0.1 AU each other on nearly the same magnetic line of force.

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