Theoretical Study on Plasma Wind and Convection in Jovian Magnetodisc

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The macroscopic MHD equation for the dynamic balance of the flowing plasma in the Jovian magnetodisc region has been solved for the aligned rotator model where the magnetic field rotates with the symmetrical configuration with respect to the magnetic equator that coincides with the perpendicular direction to the rotation axis. The equatorial region of the magnetodisc studied here is restricted in a limited azimuthal extent called "flow region". The results indicate that the plasma is flowing out due to the centrifugal force forming the disc wind that blows outward with super magnetosonic velocity when the plasma approaches a critical region. The expansion of the disc current is also made as a result of the outflow of the plasma with slight differences of electron and proton velocity. In the region of the disc plasma, the magnetic field is frozen in the flowing plasma; there is a transient region, therefore, called here the internal magnetopause that separates the magnetic lobe of the Jovian magnetosphere from the flowing disc plasma. The plasma flow is interrupted at the inner boundary of the magnetopause forming the balance of the dynamic pressure between the solar wind and the disc wind. This inner boundary is sandwiching the intrinsic Jovian magnetic field with the outer boundary which is formed due to the solar wind interaction with the Jovian magnetic field.

When the solar wind pressure increases, the position of the magnetopause is compressed with heating up effects on the disc plasma. The plasma flow cannot exceed the magnetosonic velocity and no disc wind is formed. The balance feature of the disc wind and the solar wind, thus, controls the location of the Jovian magnetopause so called spongy nature after the observation by the field and particle instruments onboard Pioneer 10, 11, Voyager 1 and 2.

1. Introduction

After the arrival of two Pioneer spacecrafts, fascinating features of the Jovian magnetosphere have been revealed. The Jovian magnetosphere has wide variety of features which differ from the earth's magnetosphere; sometimes the magnetopause towards the sun is located at a position further than 100R_J from the center of Jupiter and makes a fast inward movement indicating the minimum distance of 40R_J (WOLFE et al., 1974). Even multi-crossing of the magnetopause
due to back and forth motions of the magnetospheric boundary has been discovered (Smith et al., 1974); that is called "spongy nature" of the Jovian magnetosphere. From Pioneer 10 and 11 results, it has also been indicated that the middle and outer regions of the Jovian magnetosphere have magnetodisc configuration. Due to current sheet in the magnetodisc the magnetic field is stretched outward in equator from the average dipole field configuration. The energetic particles are trapped in the magnetodisc region (Smith et al., 1974; Van Allen et al., 1974; etc.). Relating to this evidence, very thin magnetodisc structure was pointed out theoretically (Goertz, 1976). However, since the field in the magnetosphere is open in the previously proposed outflow model with super-Alfvénic velocity (Michel and Sturrock, 1974; Prakash and Brice, 1975; Kennel and Coroniti, 1975), it is difficult to give an explanation on the existence of energetic particles which are trapped in a region close to the equatorial plane. As has been detected by Pioneer 10 and 11 when the spacecrafts passed through the region of the equatorial current sheet, there was, however, significant intensity of the north-south magnetic field component. This evidence shows apparent discrepancy to the open field model. For the validity of the corotating plasma model with the closed magnetic field (Hill et al., 1974; Carberry et al., 1976; etc.), the number density \( N \) of the plasma is required to be a very low value not to exceed \( 10^{-4} / \text{cc} \), at about \( r = 100 R_J \), within the Alfvénic limit in the equatorial current sheet. For this estimated low density it is required to have an extraordinarily high temperature when we consider the pressure balance in the current sheet on the basis of the magnetic field configuration observed by Pioneer 10 and 11. Meanwhile, recent results of the Voyager observations (Scudder et al., 1981) have indicated that the disc plasma is much denser than the estimated value for the corotation model and much colder than the lobe plasma in the Jovian magnetosphere with a relation of anti-correlation between the density and temperature of the disc electrons. A simple model of the corotational disc plasma with closed magnetic field configuration, therefore, might also not be the real state.

In the present paper we have made a theoretical investigation on an outflowing plasma model in the closed magnetic field configuration. The theoretical development on the plasma behavior has been carried out for a simple case assuming an aligned rotator model where the axis of the magnetic dipole coincides with the rotation axis of Jupiter. By solving the basic equations, the existence of the super magnetosonic plasma flow that can be called the Jovian disc wind has been predicted. The present work is characterized by this prediction of the existing plasma flow with a super magnetosonic speed containing a perpendicular component of the magnetic field in the equatorial region. The situation treated in this paper is however only possible when the region of the blowing disc wind is restricted in an azimuthally localized region. In the equatorial magnetodisc, charge accumulation phenomena may be induced by the electric field due to outflowing plasma with velocity \( V \) containing the closed magnetic field \( B \) under the "frozen in" condition. The polarization field \( E_p \) makes balance with \( V \times B \).
field in the inertial system when no current is formed to cancel the charge accumulation \( q(r) \), that is distributed around the points given by \( r \). In this case we have the well known relationship \( \nabla E_p = q(r)/\varepsilon_0 \) for the dielectric constant \( \varepsilon_0 \) in vacuum.

The outward flow of the disc plasma is proposed already by the present authors (OYA and AOYAMA, 1982) first, and the possibility of the outward flow has been confirmed for the two dimensional model of the computer simulation by SATO and MURAKAMI (1983). The present paper has the integrated contents of the first paper (OYA and AOYAMA, 1982) considering the more realistic three dimensional feature including the concept of the stational flow.

2. Model and Basic Equations

2.1 Aligned rotator model

When the Jovian magnetic field is measured in a region of plasmasphere apart from the planetary surface, the magnetic field (SMITH et al., 1974; ACUNA and Ness, 1976) can be expressed using a tilted dipole. The dipole axis is inclined about 10° with respect to the rotation axis of Jupiter. This tilt has a significant effect on the spatial and temporal variation of the magnetic field in the Jovian plasmasphere and the magnetosphere. When we concentrate the discussion on the dynamic behavior, however, the basic nature of the rotation effect can be obtained using a more simplified model than the case of the tilted dipole. That is, to obtain the basic concept for the formation of the Jovian magnetospheric disc near the magnetic equator, we considered, in this paper, an aligned rotator where the axis of the magnetic dipole coincides with the rotation axis of the planet. A cylindrical coordinate system \( r, \theta \) and \( z \) (see Fig. 1) is used. There is no significant motion of the plasma in the \( z \)-direction that is set in parallel to the rotation axis.

Fig. 1. Aligned rotator model of the Jovian disc region in meridional plane and a coordinate system. Jupiter is located at the origin of this coordinate system with the disc region expanded outside of \( 20R_J \) as given by shaded area. There is a transient region with dimension \( \delta \) between the corotating plasmasphere and the disc region.
2.2 Basic equations

The behavior of the plasma is described by macroscopic MHD equations of fluids being coupled with Maxwell’s equation; i.e.,

\[ N_j m_j \left( \frac{\partial V_j}{\partial t} + V_j \nabla V_j \right) = e_j e N_j (E + V_j \times B) - \nabla P_j - m_j N_j g, \]

\[ \text{rot } E = - \frac{\partial B}{\partial t} \]  \hspace{1cm} (1)

and

\[ \text{rot } H = \frac{\partial D}{\partial t} + \sum_j e_j e N_j V_j. \]

In the above Eq. (1), symbols are used to express the following quantities:

- \( N_j \), number density;
- \( m_j \), mass of the particles;
- \( V_j \), vector of mean velocity;
- \( e \), the electric charge unit;
- \( e_j \), the sign of the electric charge;
- \( E \), electric field vector;
- \( H \), magnetic field vector;
- \( P_j \), pressure;
- \( D \), vector of the electric field flux density defined as \( D = \varepsilon_0 E \) using the dielectric constant in vacuum in the MKS rationalized unit;
- \( B \), vector of the magnetic field flux density defined as \( B = \mu_0 H \) where \( \mu_0 \) is the magnetic permeability in vacuum given for the MKS rationalized unit.

All the quantities with the suffix \( j \) denote that corresponding values belong to \( j \)-th species of the particle in the plasma. The term, \( N_j m_j g \) in Eq. (1) indicates the gravity as

\[ N_j m_j g = \frac{G m_j N_j M_p}{r^3} \]  \hspace{1cm} (2)

for a position at \( r(r=|r|) \) with the planetary mass \( M_p \).

When the plasma motion is described in a frame called the inertial system, we can rewrite \( V_j \) using the planetary rotation vector \( \Omega_p \), as

\[ V_j = v_j + \Omega_p \times r \]  \hspace{1cm} (3)

where \( v_j \) is a mean differential velocity from the corotation speed of the planet for the particles with \( j \)-th species which is given by \( \Omega_p \times r \); \( r \) is, here, defined as a radial vector that is set in the equatorial plane starting from the center of the planet.

We treat here a quasi stationary state of the planetary plasma by neglecting
the time derivatives of the considering quantities. Considering two component plasma that consists of protons \((j=p)\) and electrons \((j=e)\), we can rewrite the dynamic equation given in Eq. (1); i.e.,

\[ Nm(\nabla \times \mathbf{V}) = \mathbf{I} - \nabla \mathbf{P} - Nmg \]  

where

\[ \mathbf{V} = \frac{V_p m_p + V_e m_e}{m_p + m_e} \]

\[ I = N e (V_p - V_e) \]

\[ N = N_p = N_e \]

\[ m = m_p + m_e \]

and

\[ P = P_p + P_e. \]

In addition to Eq. (4), the equations of the continuity and the state of the fluid are here considered; i.e.,

\[ \text{div}(Nm \mathbf{V}) = 0 \]  

and

\[ P = N k T \]  

where

\[ T = T_p + T_e. \]

The last equation of Eq. (1) is rewritten for this two component plasma, as

\[ \text{rot} \ \mathbf{B} = \mu_0 \mathbf{I}, \]

where \( \mu_0 \) is the magnetic permeability of vacuum in the rationalized unit.

3. Jovian Magnetospheric Disc

To describe the behavior of the plasma in the disc region, the cylindrical coordinate system \((r, \theta, z)\) mentioned in Section 2 is used (see Fig. 1). In this model, it is assumed that the all quantities are independent of angle \( \theta \) within
an allowed azimuthal range. It is also assumed that the current flows only in $r$ and $\theta$ directions; i.e., $I_z = 0$. It follows, then, from Eq. (8) that

$$B_\theta = \frac{C_1(z)}{r},$$  \hspace{1cm} (9)

where $C_1(z)$ is an arbitrary function of $z$. Being related to this $B_\theta$ value, the magnetic field $B_r$ can be decided so as to satisfy the observed configuration of the magnetic field line,

$$\frac{B_r}{B_\theta} = \frac{1}{r} \left( \frac{dr}{d\theta} \right).$$  \hspace{1cm} (10)

That is, from Eqs. (9) and (10), $B_r$ is given by

$$B_r = C_1(z) \frac{1}{r^2} \left( \frac{dr}{d\theta} \right).$$  \hspace{1cm} (11)

Here $dr/d\theta$ value is selected to fit the observed quantity.

The Jovian magnetic field near the magnetic equator i.e., in the disc region forms a spiral configuration whose angle $\varphi$ is linearly proportional to the distance $r$ (SMITH et al., 1974); i.e.,

$$\frac{dr}{d\theta} = K(z),$$  \hspace{1cm} (12)

where $K(z)$ is a constant, with respect to $\theta$, that has possibility to vary with respect to $z$. From Eqs. (11) and (12), $B_r$ is given by

$$B_r = C_2(z)/r^2,$$  \hspace{1cm} (13)

where $C_2(z)$ is a function of $z$ given as

$$C_2(z) = K(z)C_1(z).$$  \hspace{1cm} (14)

Based on these results of $B_r$ and $B_\theta$, possible solution of $B_z$ can be obtained from $\text{div} B = 0$ as

$$B_z = \frac{1}{r^3} \left( \int C_2(z) \, dz \right) + C_3(r).$$  \hspace{1cm} (15)

Where $C_3(r)$ is an arbitrary function of $r$, which can be determined by solving the equation of motion.

To describe the $z$-dependence of the magnetic field $B$ in the disc, we select a simple model, that is given by a function
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where $D$ is a parameter which describes the thickness of the disc region, in other words, $C_1(z)$ indicates the feature of the concentration of the disc current density.

Further, we assume that the spiral angle $\varphi$ that can be estimated from Eqs. (10) and (12) to be $\varphi = \cot^{-1}(k(z)/r)$ is independent to $z$-value; i.e., it follows that

$$K(z) = K_0$$

(17)

for a constant value $K_0$.

The magnetic field $B_r$, $B_\theta$ and $B_z$ are finally given by

$$B_r = \left(\frac{r_0}{r}\right)^2 B_{r0} A(z),$$

(18)

and

$$B_\theta = \frac{r_0}{r} B_{\theta0} A(z),$$

$$B_z = \frac{r_0^2}{r^3} B_{\theta0} \int A(z) \, dz + C_3(r)$$

where

$$A(z) = \frac{2}{\pi} \arctan (Dz)$$

$$B_{r0} = B_r(r_0, \infty),$$

and

$$B_{\theta0} = B_\theta(r_0, \infty).$$

(19)

The magnetic field $B$ is obtained selfconsistently being coupled with the plasma motion which induces the disc current $I$. This process is made determining the unknown function $C_3(r)$ in Eq. (18) coupled with the equation of motion (see Section 5). We have also checked the validity of the relationship

$$\text{rot } B = \mu_0 I,$$

(20)

in the section of discussions.

The constant $r_0$ is a distance from the Jovian center to the inner edge of
the disc region where we assume as the starting point of the magnetospheric
disc. We estimate from Pioneer 10 outbound data of the magnetic field that
$r_0 = 20R_J$. From the data of the Pioneer 10 observation the boundary values of
the magnetic field at $r = r_0$ are estimated as $B_r(r_0, z) = 5.0 \times 10^{-8}$ Web/m$^2$ for
$z = \infty$ and $B_\theta(r_0, z) = -5.0 \times 10^{-9}$ Web/m$^2$ for $z = \infty$. Selecting $K_0$ value to fit
the observation result obtained on the outbound trajectory of Pioneer 10, the
spiral structure of the magnetic field line projected on the equatorial plane has
been calculated as is given in Fig. 2 (The spiral structure is illustrated for the
case observed from the north pole side of Jupiter). The same figure can be used
for all of the value of $D$ because we have assumed that the spiral angle does
not depend on the distance $z$ from the equator.

4. Region of Localized Plasma Flow

Outward flows of the plasma in the equatorial plane containing the vertical
component of the magnetic field $B_z$ generate the electric field $E_\phi$ in the azimuthal
direction. From the frozen in condition,

$$E + V \times B = 0,$$

it follows directly that

$$\frac{\partial B}{\partial t} = \text{rot} (V \times B).$$

![Fig. 2. Spiral structure of magnetic field lines in the Jovian disc region viewed from the north
pole side. A dotted region around the center is a corotation plasmasphere and solid lines indicate
magnetic field lines projected on the equatorial plane. The radial distance from Jupiter is in-
dicated by the unit of $R_J$ (Jovian radius).](image)
When we consider the quasi-stational model of the magnetosphere it results from Eqs. (21) and (22), that

$$\text{rot } (V \times B) = 0, \quad (23)$$

and

$$E = -\nabla \phi, \quad (24)$$

for a scalar function $\phi$. Existing electric and magnetic fields $E$ and $B$, respectively, in the inertial system can be observed as $E'$ and $B'$ in the frame fixed to the planet as

$$E' = E + (\Omega_p \times r) \times B, \quad (25)$$

and

$$B' = B.$$

When we consider the azimuthal component of the electric field $E_\theta$, it follows from Eq. (25) that

$$E_\theta = E_\theta. \quad (26)$$

Due to the existence of this azimuthal component of the electric field, we cannot take the whole azimuthal range of the disc as a homogeneously distributed region of the plasma flow; i.e., the electrical potential along a path encircling the planet reveals a discrepancy. The discrepancy is that there is a discontinuity at $\theta' = 2\pi$ as given by,

$$\phi' = \int_0^{2\pi} E_\theta r' d\theta' \neq 0. \quad (27)$$

Here the quantities with (') are indicating values in the frame fixed to the planet. We assume here that there is a space where we cannot apply the conditions of the homogeneous outward flow of the plasma. We define here the special area where the plasma is flowing outward and call it the localized flow region. This special area is given by an open region in Fig. 3. In boundary region between two localized flow regions (open and shaded regions in Fig. 3) the charge distribution $q(r')$ can exist so as to make balance with the fields generated in the outwards flow region. The charge accumulation $q(r')$ at the boundary that is fixed to the frame of the planet produces the field $E'$, so as to satisfy the equation,

$$\nabla' \cdot (\varepsilon_0 E') = q(r'). \quad (28)$$
Fig. 3. Schematic sketch of the localized flow region of the plasma flow in the equatorial magnetodisc. Regions shaded by oblique lines show the area where the charge separation is taking place. Curves with arrows express the stream lines of the disc wind; and current density vectors, at corresponding positions are indicated by arrows for the case of the disc wind condition corresponding to the result labeled by A in Fig. 6.

The field $E'$, due to accumulated charge is therefore observed as the field $E$ in the inertial system as given by Eq. (26).

In the localized flow region in Fig. 3, calculated stream lines and current vectors in the equatorial plane are also shown. The spatial extent of the flow region may be controlled by the various conditions of the Jovian magnetodisc plasma coupled with the condition of the solar wind plasma. In this localized flow region inside the boundary area, all physical quantities are assumed to have a symmetrical nature in the azimuthal direction.

Discussions about the total convection in the low latitude disc region relating to the charge accumulated regions are given in Section 6.

5. Jovian Wind in the Disc

The bulk motion of the plasma in the Jovian magnetospheric disc is, here, investigated using MHD equations. The governing MHD equations for protons and electrons are combined in the one-fluid expression as has been given in Eq. (4). In the disc region, we can neglect the effect of the gravity of Jupiter compared with the effect of the other forces. Then it follows that

$$Nm(V\nabla)V=I \times B - \nabla P$$  \hspace{1cm} (29)
where $V$ is the bulk velocity in the disc plasma. The continuity equation and the equation of state are given by Eqs. (6) and (7), assuming that the disc plasma is ideal gas. Here we assume that the temperature $T$ is constant without dependence on the distance $r$.

In this section, we consider the behavior of the disc plasma in the equatorial plane ($z = 0$) under the assumptions $V_z = 0$, $I_z = 0$ and $\partial/\partial \theta = 0$. Using these assumptions, Eq. (6) is rewritten as

$$m\left(\frac{\partial N}{\partial r} V_r + \frac{N}{r} \frac{\partial}{\partial r} (rV_r)\right) = 0$$

(30)

The relation of Eq. (30) gives the condition of the conservation of the flowing flux, as

$$NrV_r = \Phi(z)$$

(31)

with $\Phi(z) = r_0 N_0(z) V_{r0}(z)$, where the quantities with subscript 0, denote the values at $r = r_0$.

We assume here, $\text{rot}(V \times B) = 0$ from the condition of "frozen in", that is,

$$r V_r B_z = C$$

(32)

where $C$ is a constant that does not depend on the radial and vertical distances $r$ and $z$.

5.1 Critical line

The equation of motion given by Eq. (29) is divided into three components, as

$$Nm\left(V_r \cdot \frac{\partial V_r}{\partial r} - \frac{V_r^2}{r}\right) = I_\theta B_z - \frac{\partial P}{\partial r},$$

(33)

$$NmV_r\left(\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\right) = -I_z B_z,$$

(34)

and

$$0 = I_\theta B_z - I_\theta B_r - \frac{\partial P}{\partial z};$$

(35)

i.e., Equations (33), (34) and (35) are $r$, $\theta$ and $z$ components, respectively.

These equations of motion are finally rewritten as
being coupled with other basic equation, where

\[ C = rB_z(r, 0)V_r(r, 0), \]  

and

\[ \Phi(0) = rN(r, 0)V_r(r, 0). \]  

The z-component of the equation of motion obtained from Eq. (35) expresses the pressure balance between \( I \times B \) and \( \nabla P \) effects in z-direction and, therefore, is independent of the motions in r and \( \theta \) direction in equatorial disc region.

In the expressions of Eq. (36), we can find a singular point for the velocity

\[ V_r^2 = V_r^2 + V_A^2, \]  

where \( V_r = (K T/m)^{1/2} \) and \( V_A = (B_z^2/N\mu_0)^{1/2} \). At this point in the phase-space, \( dV_r/dr \) takes infinite value if the numerator of the right-hand side of Eq.(36) takes a finite value. When the numerator of the right-hand side of Eq.(36) tends to zero, there is, however, the possibility to take a finite value in \( dV_r/dr \) function.

To describe the functional behavior of \( dV_r/dr \) around the singularity, we rewrite Eqs.(36) and (37) as

\[
\frac{dV_r}{dr} = \frac{f_1(r, V_r, V_\theta)}{f_2(r, V_r)}, \tag{41}
\]

\[
\frac{dV_\theta}{dr} = g(r, V_r, V_\theta) = \frac{g_1(r, V_r, V_\theta)}{f_2(r, V_r)}, \tag{42}
\]

where

\[
f_1(r, V_r, V_\theta) = C r^2 V_r \frac{\partial B_r}{\partial z} + C^2 V_r + \Phi(0)\mu_0 r V_r^2(mV_\theta^2 + kT), \tag{43}
\]

\[
f_2(r, V_r) = \Phi(0)\mu_0 r^2 V_r^3 - \Phi(0)kT\mu_0 r^2 V_r - rC^2, \tag{44}
\]

\[
g_1(r, V_r, V_\theta) = \left( \frac{C}{\mu_0 mV_r \Phi(0)} \frac{\partial B_\theta}{\partial z} - \frac{V_\theta}{r} \right) \cdot f_2(r, V_r). \tag{45}
\]
The theoretical study on plasma wind and convection provides a curve in the three-dimensional phase space described by the coordinates \((r, V_r, V_\theta)\) called, here, "a critical line". We can obtain a solution indicating a possibility that the radial velocity \(V_r\) of the plasma motion exceeds a local speed of the magnetosonic waves and increases with distance after passing through this critical line towards the outside (in the direction where \(r\) makes an increase).

In order to find the behavior of the solution of Eqs. (41) and (42) near the critical line, it is convenient to expand the right hand side of Eq. (41) in a local domain surrounding the critical line. For this purpose, Eqs. (41) and (42) are rewritten for new variables \(S, U\) and \(V\) which are transformed from the variables \(r, V_r\) and \(V_\theta\), respectively as a convenient description for slight shift from the values on the critical line, as

\[
\frac{dU}{dS} = \frac{f_1(S, U, V)}{f_2(S, U)}, \quad (46)
\]

\[
\frac{dV}{dS} = \frac{g_1(S, U, V)}{f_2(S, U)}. \quad (47)
\]

Using perturbations \(S', U'\) and \(V'\) from \(S_0, U_0\) and \(V_0\) on the critical lines, respectively, and neglecting terms higher than the second order of the magnitude of \(S', U'\) and \(V'\) values Eqs. (46) and (47) are expressed, as

\[
\frac{dU}{dS} = \frac{f_{10} + \frac{\partial f_1}{\partial S} S' + \frac{\partial f_1}{\partial U} U' + \frac{\partial f_1}{\partial V} V'}{f_{20} + \frac{\partial f_2}{\partial S} S' + \frac{\partial f_2}{\partial U} U'}, \quad (48)
\]

\[
\frac{dV}{dS} = \frac{g_{10} + \frac{\partial g_1}{\partial S} S' + \frac{\partial g_1}{\partial U} U' + \frac{\partial g_1}{\partial V} V'}{f_{20} + \frac{\partial f_2}{\partial S} S' + \frac{\partial f_2}{\partial U} U'}, \quad (49)
\]

where, \(f_{10}, f_{20}\) and \(g_{10}\) express the values of \(f_1, f_2\) and \(g_1\) on the critical line, respectively; i.e.,

\[
f_{10} = f_1(S_0, U_0, V_0) = 0,
\]

\[
f_{20} = f_2(S_0, U_0) = 0,
\]

\[
g_{10} = g_1(S_0, U_0, V_0) = g(S_0, U_0, V_0)f_2(S_0, U_0) = 0,
\]

and

\[
S = S_0 + S', \quad U = U_0 + U', \quad V = V_0 + V'.
\]
By introducing a parameter $t$ independent of $S$, $U$ and $V$, Equations (48) and (49) are rewritten as

$$\frac{d}{dt} \begin{pmatrix} S' \\ U' \\ V' \end{pmatrix} = \begin{pmatrix} \frac{\partial f_2}{\partial S} & \frac{\partial f_2}{\partial U} & \frac{\partial f_2}{\partial V} \\ \frac{\partial f_1}{\partial S} & \frac{\partial f_1}{\partial U} & \frac{\partial f_1}{\partial V} \\ \frac{\partial g_1}{\partial S} & \frac{\partial g_1}{\partial U} & \frac{\partial g_1}{\partial V} \end{pmatrix} \begin{pmatrix} S' \\ U' \\ V' \end{pmatrix}. \tag{50}$$

The geometrical feature of the solution described in the phase space near the critical line, can be effectively given using eigen values of the matrix in the right hand side of the Eq. (50); i.e.,

$$\begin{vmatrix} a_1 - \lambda & b_1 & 0 \\ a_2 & b_2 - \lambda & c_2 \\ a_3 & b_3 & c_3 - \lambda \end{vmatrix} = 0, \tag{51}$$

where

$$a_1 = \frac{\partial f_2}{\partial S}, \quad a_2 = \frac{\partial f_1}{\partial S}, \quad a_3 = \frac{\partial g_1}{\partial S}, \quad b_1 = \frac{\partial f_2}{\partial U}, \quad b_2 = \frac{\partial f_1}{\partial U}, \quad b_3 = \frac{\partial g_1}{\partial U}, \quad c_2 = \frac{\partial f_1}{\partial V}, \quad c_3 = \frac{\partial g_1}{\partial V}.$$

The eigen equation (Eq. (51)) is rewritten as

$$\lambda^3 - (a_1 + b_2 + c_3)\lambda^2 + (a_1c_3 + b_2c_3 + a_1b_2 - b_3c_2 - a_2b_1)\lambda - a_1b_2c_3 - a_3b_1c_2 + a_1b_3c_2 + a_2b_1c_3 = 0. \tag{52}$$

Considering that the Equation (50) expresses the local solutions which pass through the critical line (concurrency of the singular points), we can find that

$$c_3 = 0, \quad a_1 + b_2 + c_3 = 0, \quad -a_1b_2c_3 - a_3b_1c_2 + a_1b_3c_2 + a_2b_1c_3 = 0. \tag{53}$$

(see Appendix for detailed mathematical manipulations). The Equation (52), therefore, is rewritten as

$$\lambda(\lambda^2 + a_1b_2 - b_3c_2 - a_2b_1) = 0.$$
and the eigen values are obtained as

$$\lambda = 0, \pm \sqrt{-\beta},$$  \hspace{1cm} (54)

where

$$\beta = a_1b_2 - b_3c_2 - a_2b_1.$$  

Equation (37) which describes the $r$ dependence of $V_\theta$ has no singular point. This is the reason why one of the eigen values obtained in Eq. (54) equals to zero. The geometry of the solutions in the local phase space near the critical line are characterized by saddle points of the $X$-type configuration and the center of $O$-type configuration corresponding to the negative and positive value of $\beta$, respectively.

### 5.2 Critical solutions

The solutions for Eqs. (36) and (37) can be obtained by numerical methods. The calculated results are shown in Figs. 4 and 5 for the case of $X$-type critical solution. The solutions which pass through the critical line are here defined as critical solutions. The used parameters for the calculations are $C^* = 1$, $D = 1$ and $F = 0.4$, where parameter $C^*$ expresses the initial value of $V_rB_z$ electric field i.e., $C/r_0$ at $r_0 = 20\text{R}_J$, in the unit of $5 \times 10^{-5} \text{V/m}$, and parameter $F$ shows an initial flux density $N/V_r$ at $r = 20\text{R}_J$ in the unit of $2 \times 10^{11} \text{1/m}^2\text{S}$.

In Fig. 4 the three dimensional description of the solutions in $r$, $V_r$, and $V_\theta$ phase space is given. A dashed curve expresses the critical line discussed in Section 5.1. In a wide range of the phase space the numerically calculated solutions are connected with the corresponding local solutions near the critical line. On this critical line, a pair of solutions (curves labeled A and a, or curves labeled B and b for example) cross each other at given points on the critical line having finite $dV_r/dr$ value. In Fig. 5, the upper panel shows a two dimensional projection in $r$-$V_r$ phase space for the same solutions given in Fig. 4. A dashed curve in the center region of the figure corresponds to the two dimensional projection of the critical line. The geometrical feature around the critical lines changes from the $O$-type to the $X$-type in corresponding regions as has been indicated by $O$ and $X$ with the segments of bar at the top of the panel. The lower panel expresses the two dimensional projection of the critical solutions in $r$-$V_\theta$ phase space. The projection of the critical line on the $r$-$V_\theta$ plane is not indicated here to simplify the illustrations.

### 5.3 Further numerical results

The numerical solutions have been obtained for the case $z = 0$. The solution that can be applicable to the motion of the disc plasma should satisfy boundary conditions at the inner boundary at $r = 20\text{R}_J$ and the outer boundary located close to the magnetopause ($r > 40\text{R}_J$). We impose the relation
Fig. 4. Plots of critical solutions, in three dimensional phase space \( (r, V_r, V_\theta) \), that pass through the critical line described by a dashed curve. Pairs of the critical solutions labeled A-a, B-b, C-c, D-d, E-e and F-f make X-type (saddle point) configurations along the critical line. \( C = 1 \) is equivalent to \( C^* = 1 \) in text.

\[ V_r^2 + V_\theta^2 = V_c^2 \]  

(55)

at the reference level \( r = 20 \text{R}_J \) and \( z = 0 \) as the inner boundary condition, where \( V_c \) is a corotation speed \((\approx 250 \text{ km/s})\) at the reference level.

Solutions of Eqs. (36) and (37) which satisfy the inner boundary condition described in Eq. (55) are shown in Fig. 6. In the left side of the figure, the upper and the lower panels show the three dimensional display of the solutions (in the phase space) and the projection of the solutions into the \( r-V_r \) space, respectively. The right side panel shows the solutions in the \( r-V_\theta \) space. Solutions in the three panels with the same labels, A, B and C indicate corresponding projection of the same solutions in each phase space. The solution labeled A is a critical solution which passes across the critical line. The speed of the plasma flow becomes faster than the magnetosonic speed (super magnetosonic speed) in the region beyond \( r = 51 \text{R}_J \) in the case of the curve A solution. This solution corresponds, therefore to the Jovian disc wind with the super magnetosonic velocity. The curves labeled B and C, and curves without label below the curve C in the phase spaces express example cases of breeze solutions (slower than the magnetosonic speed).

There are another group of solutions that are given by curves above the curves of the critical solution in the phase space. These solutions are related to the initial condition of the velocity \( V_r \) at \( r = r_0 \); that is, \( V_c > V_r > V_{rc} \) where \( V_{rc} \) is the velocity, at the inner boundary, which gives the critical solution (curve A). The solutions above the critical line are not plotted for the case of non-
Fig. 5. Two dimensional projections into $r-V_r$ phase space (upper panel) and $r-V_e$ phase space (lower panel) of the critical solutions corresponding to the solutions given in Fig. 4. Used labels of the plots correspond to those used in Fig. 4. $C=1$ is equivalent to $C^* = 1$ in text.
critical solutions.

Among these possible solutions which satisfy the inner boundary condition, only one solution which satisfies the outer boundary condition can be selected as an applicable solution for the description of the behavior of the plasma in the Jovian magnetodisc. The outer boundary condition is raised as the pressure balance of the Jovian disc plasma with the solar wind plasma under the strong control of the intrinsic magnetic field of Jupiter. In the dayside region of the Jovian magnetodisc, the pressure of the disc plasma has to make balance with the solar wind pressure at the magnetopause.

The high speed Jovian disc wind corresponding to curve A in all the panels of Fig. 6 in the outer Jovian magnetodisc is possible only in the case of the

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**Fig. 6.** Selected solutions for the bulk motion of the plasma in the equatorial Jovian magnetodisc, which satisfy the inner boundary condition. Upper panel on the left side shows a three dimensional display of the solutions in phase space \( (r, V_r, V_	heta) \). Lower panel again on the left side indicates the projection of the three dimensional solutions into \( r-V_r \) phase space with a critical line described by a dashed curve. Right side panel shows a projection of the three dimensional solutions into \( r-V_	heta \) phase space. Solutions with same labels A, B and C in the three panels correspond to the plots of the identical solution of each other. \( C = 1 \) is equivalent to \( C^* = 1 \) in text.
lower plasma pressure than the case of the slower velocities as have been given by solutions corresponding to curves B and C in Fig. 6. These cases of B and C, for example, correspond to the breeze conditions where the flow velocity is in a range of sub-magnetosonic velocity. In the case of the compressed magnetosphere, there is the breeze condition of the disc plasma providing high pressure that makes balance with high solar wind pressure.

In Fig. 7, the stream lines of the disc plasma flow corresponding to the solutions given by curves A and C in Fig. 6 are shown. In the case of the disc wind A (Fig. 7(a)), the stream line is directed outward with increasing distance because of the increasing $V_r$ value with distance. On the other hand, the direction of the stream line corresponding to the breeze condition C (Fig. 7(b)) is dominated in azimuthal direction in the outer region of the magnetodisc due to decreasing radial flow velocity $V_r$.

In our model, however, no homogeneous flow condition with respect to the azimuthal direction is allowed as has been discussed in Section 4. We should, therefore, select only a part of the stream lines from the picture in Fig. 7 forming a limited region in the azimuthal angle.

6. Discussions

6.1 Internal magnetopause

When we consider the “frozen in” plasma flow in the disc region of the plasma we meet a bewildering situation; i.e., how the topological completeness of the magnetic field can be obtained in the Jovian magnetosphere with non-corotating $z$-component of the magnetic field. In Fig. 8, a concept of the internal magnetopause that separates the region of the disc flow from the region of the Jovian magnetosphere is given; the Jovian magnetosphere, here, is considered to be the region where the intrinsic Jovian magnetic field is dominating with low density plasma. In this boundary region, the physical state might be very similar to the earth’s magnetopause where the earth’s magnetic field merges with the moving interplanetary magnetic field. If the model invoked here expresses the actual state, there should exist the turbulent magnetic field over a transient range from the magnetosphere to the portion of the disc where $B_z$ component is moving with flowing plasma. The preliminary result of the Voyager observation of the Jovian magnetic field (NESS et al., 1979) indicates that there exists such a range of turbulence (see Figs. 2 and 4 associated with the disc where the value $\lambda$, in the paper by NESS et al. (1979), changes from 0° to 180° or vice versa.). Along the internal magnetopause large turbulence of the magnetic field may be produced. For an example, the velocity shear may produce the instability of the MHD wave as has been discussed in many works (CHAN-DRASEKHAR et al., 1961; SEN, 1963, 1964; SOUTHWOOD, 1968; etc.). Furthermore, these turbulent states of the magnetic field, coupled with the plasma flow, may induce strong plasma waves due to the existing microscopic instabilities. These strong plasma waves that may be associated with this kind of turbulence have
Fig. 7. Stream lines for the solution of the disc wind condition (a) and the breeze condition (b) corresponding to the cases labeled by A and C in Fig. 6, respectively. A dotted region located in the center portion in each case indicates the corotation region with radius $r = 20R_J$. 
already been reported by SCARF et al. (1979). The merging of the moving field to the intrinsic Jovian magnetic field can thus take place through the internal magnetopause. The frozen in flow in the disc region of the Jovian magnetosphere is possible under the condition of the constant supply of the magnetic field at the origin of the disc flow, i.e., \( r = 20R_J \) for this case. This supply can be achieved by the plasma that is originally trapped by the Jovian intrinsic magnetic field. The outward flow of the plasma from this semitrapped area thus starts to carry the magnetic field which is mainly induced by the disc current towards the outward region.

6.2 Convection pattern in the Jovian magnetodisc

In the plasma disc, the plasma can move freely containing the \( B_z \) component of the magnetic field being separated from the intrinsic Jovian magnetic field in the high latitude region apart from the disc having transient region at the internal magnetopause. We here consider the convection pattern of the plasma in the magnetodisc. As has already been discussed, we cannot admit the homogeneously distributed plasma flow in the whole azimuthal range but the flow region should be limited in a given range of the azimuthal angle. The extent of the allowed flow region may depend on the environment associated with various plasma parameters. Here, we have selected two example cases of the spreading convection region, i.e., the narrowly limited situation (Fig. 9(a)) and the widely open situation (Fig. 9(b)). Where, the boundary expressed by dashed curves indicates the outer boundary of the Jovian disc wind. At the edges of the plasma flow region there exist the region of the charge separation to cancel the generated electric field component, \(- V \times B_z\). The occurrence of the charge separation results in the generation of the electric field in the outside range of the limited flow region (see Fig. 9(b)).
Fig. 9. Patterns to indicate the convection of the plasma in the Jovian magnetodisc in the case of the narrow flow region (a) and widely extended flow region (b). Curves with arrows outside of the corotation region indicated by the dotted area in each case express the stream lines of the disc plasma. Two shaded regions in each diagram show charge up regions that give limitation to the region of the Jovian disc wind. In the case of (a), two diagrams are used to illustrate the process to obtain the resulted stream lines given in (2); i.e., in (1), local stream lines surrounding the charge up regions are indicated separately from the corotation components.
Surrounding the regions of the charge accumulation, there should be inward motions of the plasma to form the electric field oppositely directed to that in the outward flow region. This internal plasma flow may, however, be limited in a region close to the charge accumulation region. For the case of the narrow range flow, there is dominated corotation motion of the plasma in the region outside of the outward flow region. The total motion of plasma flow is, therefore, almost equal to this corotation motion, though there is a very slight inward twist of the flow direction near the charge accumulation region.

In the case of the wide range of the outward flow, the remaining region is restricted to a narrow range of area. In this case, we may see the inward flows of the plasma which are again making vortices surrounding the charge accumulation region. To depict the whole feature of the convection pattern in Figs. 9(a) and 9(b) we also applied the concept of the tailward flow of the plasma taking the hypothesis of the solar wind-magnetosphere interaction (VASYLIUNUS, 1983).

HILL et al. (1981) have proposed a convection model of the Jovian magnetospheric plasma called “corotating magnetospheric convection” on the base of the observation of anomaly of the intrinsic Jovian magnetic field (DESSLER and HILL, 1975, 1979; HILL and DESSLER, 1976; VASYLIUNAS and DESSLER, 1981). They considered that the longitudinal asymmetry of the Io plasma torus provides a driving mechanism of the corotating convection. Therefore, only in the relatively narrow flow region called “active sector” that spans about 100° in longitude centered around λHI = 225°, the plasma can flow outward. In the present model a more widely expanded flow region (see Fig. 9(b)) than the case of HILL et al.’s proposal discussed above, however, is possible because we consider that the starting point of the outflowing disc plasma is at about 20RJ where the corotation speed exceeds the local Alfvén speed not only in the “active sector” but in all longitudes. However, it is possible that the existence of the longitudinal asymmetry of the Io plasma torus controls the flow region of the disc plasma through the initial condition at R = 20RJ that decides the characteristics of the disc plasma motion.

6.3 Self consistency

When the magnetic field vector $B$ is decided as has been given in Eq. (18), the possible current can be obtained from Eq. (20); i.e.,

$$I = \frac{1}{\mu_0} \text{rot} B.$$

This expression can be rewritten by component equations as

$$I_r = -\frac{1}{\mu_0} \frac{r_0}{r} B_{\phi 0} \frac{\partial A(z)}{\partial z},$$

(56)
These results have already been used to obtain Eqs. (36) and (37). The results of \( V_r \) and \( V_\theta \) from Eqs. (36) and (37) show that the plasma flows in \( r-\theta \) plane indicating that the current \( I^* \) is also in \( r-\theta \) plane. Then, \( I^* \) is written by using \( V(V_r, V_\theta, 0) \) as

\[
I^* = Ne(V - V_e), \tag{57}
\]

assuming \( V = V_p \); where \( V_p \) and \( V_e \) are the average speeds of protons and electrons, respectively. From this current \( I^* \), we can decide the magnetic field intensity \( B^* \) using vector potential \( a \) which is related to the current as

\[
\text{rot} (\text{rot} a) = \mu_0 I^*, \tag{58}
\]

i.e., this can be rewritten as

\[
\nabla^2 a = -\mu_0 I^*,
\]

which has components as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial a_r}{\partial r} \right) + \frac{\partial^2 a_r}{\partial z^2} - \frac{a_r}{r^2} = -\mu_0 I^*_r, \tag{59}
\]

and

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial a_\theta}{\partial r} \right) + \frac{\partial^2 a_\theta}{\partial z^2} - \frac{a_\theta}{r^2} = -\mu I^*_\theta,
\]

and

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial a_z}{\partial r} \right) + \frac{\partial^2 a_z}{\partial z^2} = 0.
\]

As one of example cases of \( I^* \), we can select \( I^* = I \) in Eq.(56); then, it follows that

\[
\text{rot} B = \text{rot} (\text{rot} a) \tag{60}
\]

that is, \( B = \text{rot} a + \text{grad}\psi \) for an arbitrary scalar function \( \psi \).
Therefore, the magnetic field $\mathbf{B}^*$ can be expressed using this vector potential as

$$\mathbf{B}^* (\equiv \text{rot } \mathbf{a}) = \mathbf{B} - \nabla \psi.$$ 

Since $\psi$ can select arbitrarily, we can find a solution $\mathbf{B}$ given in Eq. (18) from Eq. (59). i.e., the magnetic field given in Eq. (18) can be one of the solutions which satisfy the relations given in Eq. (59); i.e., the processes used in this theory has a self-consistency.

7. Conclusion

The Jovian magnetospheric disc structure with the super magnetosonic outflow of the plasma has been studied. The magnetic field configuration used in this study is consistent with results of the in situ observations by using Pioneer and Voyager spacecrafts. In order to obtain the bulk motion of the disc plasma, we have solved MHD equation of the one fluid in the frame of the magnetic field where the generated current in the disc region controls the magnetic field configuration.

The result of the calculation of the bulk motion of the disc plasma indicates three kinds of solutions. In addition to the disc wind solution and the breeze solution, there is also a group of solutions that approach to $dV_r/dr = \pm \infty$ at a given point. We have to select the solutions, which satisfy the inner and outer boundary conditions, among these three kinds of solutions to fit the actual state. The flow speed of the plasma observed in the inertial frame at the inner boundary ($r=20R_J$) is selected to be almost equal to the corotation speed there. The balance between solar wind pressure and the disc plasma pressure at the Jovian magnetopause is used as the outer boundary condition. Considering the above boundary conditions, the solution for the dynamic and the continuity equations gives the super magnetosonic Jovian disc wind in the middle and outer magnetosphere of Jupiter except for the case of the largely compressed magnetosphere that gives the breeze solution.

Frozen in condition imposes the generation of the azimuthal electric field $E_\theta$ on the outflowing plasma due to the existing perpendicular component of the magnetic field $B_z$. To balance with the generated electric field, two charge accumulated regions are generated in the Jovian magnetodisc plasma. Under these conditions, outflow of the plasma cannot be permitted in the symmetric form in the whole range of the azimuth; that is, the plasma outflow takes place only in the localized flow region. Relating to this inhomogeneity of the plasma flow, we have proposed that the plasma in the Jovian magnetodisc has a large scale convection. For consideration of the convection pattern, there is the problem of how the topological completeness of the magnetic field configuration can be obtained in the Jovian magnetosphere. This problem is settled by introducing the new concept of the internal magnetopause. In the region of the internal magnetopause, the plasma instability due to the velocity shear may generate the
strong plasma waves which may destroy the frozen in condition. The existence
of this turbulent layer can give the possibility that the disc plasma flows outward
containing the perpendicular component of the disc magnetic field without giving
a large change to the magnetic field configuration in the outside region of the
plasma disc.

APPENDIX

As has already been discussed in the text, the quantities $S_0$, $U_0$ and $V_0$ must
satisfy the following conditions, i.e.,

$$f_1(S_0, U_0, V_0) = CS_0^2U_0^2 + \frac{r_0^2}{S_0^2} \frac{\partial A(z)}{\partial z} + C^2U_0 + \Phi(0)\mu_0 S_0 U_0^2 (mV_0^2 + kT) = 0,$$

(A-1)

and

$$f_2(S_0, U_0) = \Phi(0)\mu_0 S_0^2 U_0^3 - \Phi(0)kT\mu_0 S_0^2 U_0 - S_0 C^2 = 0.$$  (A-2)

Considering that all of the coefficients of the eigen equation (Eq.(52)) are defined
on the critical line, we can easily obtain, from Eq.(45) in the text, that

$$C_3 = f_2(S_0, U_0) \frac{\partial g}{\partial V}.$$  

This shows us from Eq. (A-2) that

$$C_3 = 0.$$  (A-3)

Also using the relationships described in Eqs.(A-2), (A-3) and (45) in the text,
the constant terms with respect to $\lambda$ in Eq.(52) can be written as

$$-a_1b_2c_3 - a_3b_1c_2 + a_1b_3c_2 + a_2b_1c_3 = f_2(S_0, U_0) \left( -\frac{\partial f_2}{\partial U} \frac{\partial g}{\partial S} + \frac{\partial f_2}{\partial S} \frac{\partial g}{\partial U} \right) \frac{\partial f_1}{\partial V}.$$  

This also results that

$$-a_1b_2c_3 - a_3b_1c_2 + a_1b_3c_2 + a_2b_1c_3 = 0.$$  (A-4)

The coefficient of the first order of eigen value $\lambda$ in Eq.(52) can be obtained
using the following relations,

$$a_1 = 2\Phi(0)\mu_0 S_0 U_0 (mU_0^2 - kT) - C^2,$$
and
\[ b_2 = 2C B \phi_0 r_0^2 \frac{\partial A(z)}{\partial z} U_0 + C^2 + 2\Phi(0)\mu_0 S_0 U_0 (mV_0^2 + kT). \]

Considering Eqs. (A-1), (A-2) and (A-3), it follows that
\[ a_1 + b_2 + c_3 = \frac{2}{S_0} f_2(S_0, U_0). \]

The result also shows that
\[ a_1 + b_2 + c_3 = 0. \]  \hspace{1cm} (A-5)

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