Periodicities in Sunspot Numbers

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The annual mean sunspot numbers during 1749–1971, are subjected to power spectrum analysis by the recentmost Least-Squares linear prediction method evolved by Barrodale and Erickson. For the two consecutive intervals 1749–1844 and 1845–1971, only one major peak at $T=9.7$ yr is common and many other peaks are different, indicating either unreliability of the earlier data, or a possibility that the two samples do not belong to the same population. In general, peaks near $T=5$ yr have only border-line significance, peaks near $T=11$ yr are highly significant and multiple and probably change with time. In the large periodicity region peaks definitely exist for $T=60$ – 100 yr range, but probably change with time. Attempts at prediction of future activity yielded a number less than 100 for cycle 21, much lesser than the observed value of 155 in 1979–1980.

1. Introduction

The annual mean values of the Zurich relative sunspot numbers show large fluctuations. Although systematic observations started only in 1894, data for earlier period are available (WALDMEIER, 1961). Figure 1 shows a plot of the Wolf numbers for the period 1749–1971. There is obviously a very prominent periodicity of about 11-yr. The conventional numbering of these 11-yr cycles is indicated at the top, with cycle 1 having a maximum in 1761. At present, we are in cycle 21 with a maximum in 1979–80. The years of maxima and minima of the various cycles are indicated. Also, the spacings between the successive maxima and successive minima are indicated. As can be seen, these are generally near 11 years, but can be as low as 7 or as high as 17. The amplitudes of the successive cycles also show large fluctuations.

Several attempts have been made in the past to study the periodicities in these annual Wolf numbers. After the earlier attempts of KIMURA (1913) and TURNER (1913) to use harmonic and periodogram methods, RAMANUJA RAO (1973) used the method of power spectrum analysis of BLACKMAN and TUKEY (1959) to study periods shorter than 11-yr and concluded that periodicities of 5.5 and 3.5 yr were indicated but were significant at slightly less than 95% confidence level. COLE (1973) used the same method and concluded that there was a basic period of 10.45 yr associated with the excitation of new solar cycles, and another period of 11.8 yr associated with the free-running length of a solar cycle. These

Fig. 1. The mean annual Wolf relative sunspot numbers for 1749-1971 (full lines) and for 1972-1980 (dots). The years of maxima and minima are indicated on the curve. At the top, the successive cycle numbers (1-21) and the spacings between successive maxima and successive minima (in years) are indicated.

two combined to give the mean cycle length of 11.06 yr and a phase variation of 190 yr. Similarly, amplitude variations of periods 88 and 59 yr were noticed by him, and these were attributed to an amplitude modulation of the solar cycle by a period of 11.9 ± 0.3 yr. In the Blackman and Tukey method, higher periodicities are resolved only with larger lags, and the peak resolution is then somewhat unsatisfactory. Burg introduced the MESA (Maximum Entropy Spectral Analysis) method for which ULRYCH and BISHOP (1975) have given a critical review. This method gives a much sharper resolution for higher periodicities, even those comparable to the data length. CURRIE (1973), as well as COHEN and LINTZ (1974), used MESA for studying the periodicities in sunspot numbers. RADOSKI *et al.* (1975) and KANE (1977) used the same method for geophysical and solar spectral studies. CURRIE (1973) reported major peaks at $T=9.9$, 11.1, 14.7 yr and sixteen minor peaks, while Cohen and Lintz (1974) reported a small peak at $T=8.0$ and larger peaks at $T=9.7$, 11.0 and ~ 100 yr. RADOSKI *et al.* (1975) and KANE (1977) located major peaks at about $T=9.6$, 10.7, 12.7 yr. In addition, KANE (1977) reported major peaks at about $T=17$, 20, 33, 51, 79 yr and eighteen minor peaks.

The exact values of the various periodicities can have profound theoretical implications. Modern work focuses attention on magnetohydrodynamical processes in the interior of the Sun. At present, there is no general agreement about the excitation mechanism for any one period or a large number of distinct periods. However, WOLFF (1976) proposed a regular system of beats between rigidly rotating, inertially oscillating modes in the Sun. Using only two adjustable frequencies in his model, he explained 20 of the 25 periodicities reported by CURRIE (1973) and COLE (1973). Planetary influences on variations of sunspot numbers have also been examined WOOD, 1972; SMYTHE and EDDY, 1977). Thus, the need of a precise determination of the frequencies in the sunspot spectrum can hardly be overemphasized.

Now, it is true that MESA shows a frequency resolution far better than that given by any of the earlier methods. However, MESA has some well-known defects. In analogy to the Blackman and Tukey method, MESA resolves higher periodicities only when larger Lengths of the Prediction Error Filter (LPEF) are used. But, at these high LPEF, the low periodicity peaks show splitting. Also, the large periodicity peaks may show errors (frequency shifts) as large as 20%, depending upon the initial phase of the wave (CHEN and STEGAN, 1974). The peak-splitting can probably be taken care of by the elaborate additional programming proposed by FOUGERE (1977). But the problem of frequency shifts is very serious. As such, all the high periodicities mentioned above become suspect.

The drawbacks of the Burg algorithm are caused mainly by its imposition of a Toeplitz structure on the matrix of the system of equations which yield the AR (autoregressive) parameters. Whereas this procedure is responsible for the computational efficiency of the Burg's algorithm, the resulting spectra so obtained are inferior (in accuracy of frequency determination) to those obtained by using Least-Square (LS) solutions to the AR model (ULRYCH and CLAYTON,

1976). The LS solutions needed a large computational effort and hence were unpopular so far. However, recently, BARRODALE and ERICKSON (1980 a and b) developed an algorithm for solving the Least-Squares Linear Prediction (LSLP) problem directly and claimed that their method (designated by them as FABNE) gives results superior to those of Burg.

In this communication, we propose to apply this new FABNE method to the series of sunspot numbers and compare the results with those of Burg's algorithm, for which we used the program given by ANDERSON (1974).

2. Artificial Samples

Before applying FABNE to the sunspot number series, we illustrate the superiority of FABNE over the Burg algorithm with the help of artificial samples.

Figure 2 shows the FABNE spectra (upper half) and Burg spectra (lower half) for a sample of 101 data points ($t=0$ to 100) of the sinusoid $\sin [2\pi(t/160) + \phi]$. Thus, only one peak at $T=160$ is expected; but this period is larger than the data length 101. With the earlier Blackman and Tukey method, resolution of such a peak was unimaginable. In Fig. 2, the FABNE spectra show a resolution even for LPEF as low as 6. And for LPEF=30 (about 30% of the data length 101), the period is accurately located as $T=160$. Note that the abscissa scale is not the usual, conventional, linear scale in frequency, but is logarithmic in period T , with successive steps $\Delta(\log T)=0.005$ so that, for all periodicities, the step accuracy is uniformly about 1.2% of T . The Burg spectra in Fig. 2 do show a peak for T in the high periodicity region; but the period indicated is inaccurate even for LPEF=40. For higher LPEF, the peak at $T=160$ is indicated; but false peaks appear at other locations for reasons not yet clear. It is clear, however, that the frequency accuracy is far better for the FABNE method. In the above sample, the phase ϕ was chosen as 60° . When we chose other values of ϕ viz. 0° and 120° , the FABNE spectra remained unaltered, but the Burg peaks showed inaccuracies of about 5%, in addition to false peaks.

COHEN and LINTZ (1974) have raised an interesting question by claiming that the long-term amplitude modulation of a period of about 180 yr seen in sunspot numbers (see Fig. 1) is not a genuine long-term periodicity, but a beating effect of two high frequencies (low periodicities). Thus, when two frequencies f_1 and f_2 are near each other,

$$\begin{aligned} f(t) &= \sin(2\pi f_1 + \phi_1) + \sin(2\pi f_2 + \phi_2) \\ &= 2 \sin\left(2\pi \cdot \frac{f_1 + f_2}{2} + \frac{\phi_1 + \phi_2}{2}\right) \cos\left(2\pi \cdot \frac{f_1 - f_2}{2} + \frac{\phi_1 - \phi_2}{2}\right) \end{aligned}$$

and the resulting combination would show an amplitude oscillating with a frequency of $(f_1 - f_2)/2$ or a period of $2/\Delta f$. In the case of sunspots, periods $T_1=9.7$ and $T_2=10.9$ would give $\Delta f=0.012 \text{ c yr}^{-1}$, resulting in a long term amplitude

Fig. 2. FABNE spectra for LPEF=6, 20, 30 (upper half, dashed lines) and BURG spectra for LPEF=20, 40, 60, 90 (lower half, full lines) for 101 data points of a single sinusoid of period $T=160$, for an initial phase $\phi=60^\circ$. Vertical line indicates the position where the peak is expected.

modulation periodicity of $T=\text{about } 170$.

We examine now an artificial sample composed of 201 data points ($t=0$ to 200) of

$$f(t) = \sin\left(2\pi \frac{t}{10}\right) + \sin\left(2\pi \frac{t}{12}\right).$$

Thus, genuine peaks are expected only at $T=10$ and $T=12$, and the question is: does a peak appear at $T=120$ also, as expected due to beating? Figure 3 shows (a) the FABNE spectra and (b) the Burg spectra. In FABNE, the peaks at $T=10$ and 12 are shown even for $LPEF=17$ (about 8% of the data length 201). For higher $LPEF$ upto 100, the results are similar. Surprisingly, even though the input signals are both of the same amplitude (unity), the peak heights for $T=12$ are consistently lesser than those for $T=10$, by several orders of 10 (note that the ordinate scale is logarithmic too). This is one aspect which has proved very baffling to us, viz. in the FABNE method, the heights of the peaks seem to bear no relation to the input powers. In the case of the Burg spectra, the peaks are shown with slight inaccuracies, viz. $T=10.1$ and 11.9 . However, in both the cases, viz. FABNE as well as Burg, there is no indication of a peak at $T=120$ (position marked by the vertical arrow). Thus, one may conclude that, if a large periodicity is due to a beating effect, it does not show as a separate peak, either in Burg spectra or in the FABNE spectra. Conversely, it may be reasonable to assume that, in case the Burg or FABNE spectra do show a large

Fig. 3. (a) FABNE spectra for $LPEF=17, 50, 100, 125$ and (b) BURG spectra for $LPEF=50, 125$, for 201 data points of an artificial sample containing two sinusoids of equal amplitudes and periods $T=10$ and $T=12$. The vertical arrow indicates the position where a beat periodicity $T=120$ was expected, but is not seen.

period, it should be considered as genuine, with better accuracy in the FABNE spectra. From Fig. 3, it also seems that it is not desirable to use large LPEF (exceeding about 50% of data length) for the FABNE spectra, as false peaks may appear. Also, the heights of the peaks do not seem to convey any useful information. However, this aspect needs further scrutiny. In case of Burg spectra too, the heights of the peaks at $T=10$ and $T=12$ are not equal. In earlier literature, LACOSS (1971) mentions that, in the Burg algorithm, it is not the height of the peaks, but the area under the peaks, which is expected to be proportional to the power of the signal. In our studies, we have not yet been able to confirm this finding of Lacoss. However, this needs further scrutiny.

3. Spectra of Sunspot Number Series

Since sunspot data are more reliable for 1849 onwards, and since COHEN and LINTZ (1974) have presented results for 1845–1971 separately, we also treat this interval separately.

Figure 4 shows (a) the FABNE spectra and (b) the BURG spectra for the sunspot series of 127 years (1845–1971). For FABNE spectra for LPEF up to

Fig. 4. (a) FABNE spectra for LPEF=3, 25, 35, 45, 55, 65, 75 and (b) BURG spectra for LPEF=75, 105, for 127 data points of annual sunspot numbers for the years 1845–1971.

35 (about 30% of the data length 127), only four peaks are indicated, a small one at $T=5.7$, a large one at $T=11.0$ and two small ones at $T=23.4$ and $T=100$. For higher LPEF, many more peaks appear. For LPEF=65 (roughly 50% of 127), a small peak at $T=5.7$, medium peaks at $T=7.9$, 12.2, 24.5 and 34.7 and major peaks at $T=9.8$, 10.8, 127.4 are indicated. For LPEF=75, an additional small peak appears at $T=16.8$. For higher LPEF (85 or more) the FABNE program fails. We feel that the peaks shown for LPEF=65, may be the most appropriate ones.

For Burg spectra, LPEF=75 shows peaks roughly similar to LPEF=65 of FABNE. The high periodicity is however at $T=93.3$ in Burg, in contrast to $T=127.4$ in FABNE. For higher LPEF (=105) in Burg, lower periods show peak splitting. COHEN and LINTZ (1974) have not mentioned the LPEF they have used. Their answers, viz. a small peak at $T=8.0$, medium peaks at $T=9.7$ and $T=110.0$ and a big peak at $T=10.9$, seem to tally well with our LPEF=75 for Burg spectra, except that $T=12.2$ is not mentioned by them.

Figure 5 shows (a) FABNE and (b) BURG spectra for the earlier period (1749-1844) of 96 years. For FABNE, LPEF=45 (about 50% of 96) shows only one major peak at $T=9.7$. For higher LPEF=55, several other peaks occur.

Fig. 5. (a) FABNE spectra for LPEF=2, 25, 35, 45, 55 and (b) BURG spectra for LPEF=45, 55, 85 for 96 data points of annual sunspot numbers for the earlier period 1749-1844.

There is a peak at about $T=65-70$. The Burg spectra show similar characteristics, except that the large period peak is at $T=58$ yr and for very high $LPEF=85$, another peak appears at $T=151$ yr. If we consider only the FABNE spectra and compare $LPEF=65$ of Fig. 4 and $LPEF=55$ of Fig. 5, $T=9.7$ yr is the major peak in both; but other peaks are different. Therefore, unless one assumes that the earlier data are unreliable (McNISH and LINCOLN, 1949), the conclusion would be that the periodicities for the two successive intervals 1749–1844 and 1845–1971 are not completely similar, and hence the two are not samples of the same population. McNISH and LINCOLN (1949) arrived at a similar conclusion, while COHEN and LINTZ (1974) claimed the opposite. Unfortunately, the heights of the peaks are not reliable indices of their importances in both methods. Hence, homogeneity could not be ruled out by a consideration of the differences in heights in the two samples above. However, in the present case, even the frequencies are different. For example, the earlier interval (Fig. 5) shows $T=7.4$, 8.5 and 11.6 yr, while the latter interval (Fig. 4) shows $T=7.9$ and 11.0 yr. There are other differences, too.

If these differences are ignored and the two samples are considered homogeneous, spectra could be calculated for the whole interval 1749–1971. These are shown in Fig. 6, as (a) FABNE and (b) BURG. In FABNE, there is a single peak near $T=11$ yr for $LPEF$ up to 55. For $LPEF=65$ (about 30% of the data length 223), there are two small peaks at $T=4.8$ and 5.8 yr, three major peaks at $T=8.3$, 9.9 and 11.1 yr and an indication of a peak at $T=86$ yr. This pattern continues up to $LPEF=85$. For higher $LPEF$, several other peaks appear. For $LPEF=145$, the triplet 8.3, 9.9, 11.1 yr is replaced by other peaks, viz. 9.1, 10.8, 11.6 yr. It seems to us that results for $LPEF$ exceeding 115 (about 50% of the data length 223) are not reliable. In the Burg spectra, $LPEF$ up to 155 show results similar to $LPEF=115$ of FABNE. For higher $LPEF$, Burg spectra show an additional peak at $T=\text{about } 145$ yr. Such a high periodicity was seen only in the Burg spectra of the earlier period (Fig. 5). Its reliability is a moot question, in view of its appearance only in the Burg spectra.

4. Predictions

In spite of the better capacity of resolution in the high periodicity region in the FABNE method, the above results give some cause for apprehension, mainly for two reasons. Firstly, the height of the peaks is not necessarily indicative of their importance, and it is not possible to apply any statistical significance tests to these peaks in both FABNE and BURG spectra, as one could do in the Blackman and Tukey method. Secondly, the choice of the $LPEF$ is arbitrary. For determining the optimum $LPEF$ for Burg spectra, ULRICH and BISHOP (1975) suggested the use of AKAIKE's (1969) Final Prediction Error (FPE) criterion, which is well-known in the field of spectral analysis of stochastic processes. BARRODALE and ERICKSON (1980 a and b) have also recommended its use and have incorporated it in their computer program to give the optimum $LPEF$. We used the

the top curves in Figs. 4, 5, 6. Thus, using LPEF on the FPE criterion, one would have to conclude that only a periodicity of T near 11 yr was meaningful and all other periodicities were fake. Whereas such a possibility cannot be ruled out, one feels somewhat uncomfortable in rejecting all the other peaks and also ignoring the fine structure of the $T=11$ yr line, revealed at higher LPEF. ULRICH and BISHOP (1975) have suggested 50% of data length as an upper bound for LPEF; but, as shown in KANE (1979), large periodicities cannot be resolved at LPEF of this order, and hence, he suggested that low periodicities may be identified from low LPEF and higher periodicities from higher and higher LPEF, keeping in mind that, at very high LPEF, the high periodicities indicated may have frequency shifts (uncertainties) as large as 20–30%. In the case of FABNE, this uncertainty is expected to be much lesser.

A possible alternative to access the relative importance of the various peaks could be by the method outlined by KANE (1977), viz. to pick up the possible periodicities $T_1, \dots, T_n$ from any particular spectra (low T from low LPEF and higher T from higher LPEF) and, expressing a function $f(t)$ as :

$$f(t) = \sum_{k=1}^n \left(a_k \sin 2\pi \cdot \frac{t}{T_k} + b_k \cos 2\pi \cdot \frac{t}{T_k} \right) \quad (1)$$

$$= \sum_{k=1}^n r_k \sin \left(2\pi \frac{t}{T_k} + \phi_k \right) \quad (2)$$

use the given series of observed values of $f(t)$ for a statistical analysis by standard least-square fit method to estimate the various components (a_k, b_k), or the amplitudes and phases (r_k, ϕ_k) with their standard errors. This is a sort of curve-fitting procedure, and it could be used for producing the expected values of $f(t)$, by plugging into Eq. (1) the statistically estimated values of a_k and b_k , and comparing the expected and observed values to judge the goodness of fit. More important, Eq. (1) could be used for predicting further values of t , beyond the length of observations. For example, the observations up to 1971, shown in Fig. 1 as full lines, could be fitted into Eq. (1) and $f(t)$ for 1972, 1973 etc. could be predicted. Since the observed values for these years are now available (big dots in Fig. 1), the worth of the prediction can be judged.

For prediction, one could use the whole series 1749–1971, or, in view of the possible unreliability of the earlier data, one could use only 1845–1971. We prepared five groups of periodicities T_k , three for 1749–1971, and two for 1845–1971. Table 1 gives the amplitudes r_k and their standard errors σ_{r_k} , in units of sunspot numbers. The groups are chosen, somewhat subjectively, from the spectra in Figs. 4 and 6, mostly from FABNE, but occasionally from Burg spectra too, and at LPEF near 50%. For every group, the standard error is the same for all amplitudes, mainly because there is an inherent assumption in this statistical procedure that the back-ground is a white noise. As can be seen,

Table 1. Amplitudes ($r_k \pm \sigma_{r_k}$) for the Waves of Various Periodicities T_k (Years) in Sunspot Numbers.

Data for 223 years (1749–1871)						Data for 127 years (1845–1971)			
Group 1		Group 2		Group 3		Group 1		Group 2	
T_k (yr)	$r_k \pm \sigma_{r_k}$	T_k (yr)	$r_k \pm \sigma_{r_k}$	T_k (yr)	$r_k \pm \sigma_{r_k}$	T_k (yr)	$r_k \pm \sigma_{r_k}$	T_k (yr)	$r_k \pm \sigma_{r_k}$
4.6	2 ± 2	4.6	3 ± 3						
5.5	5 ± 2	5.5	6 ± 3	5.5	5 ± 3	5.7	7 ± 3	5.7	6 ± 2
		6.7	4 ± 3						
8.3	6 ± 2	8.1	7 ± 3	8.1	7 ± 3	7.9	8 ± 3	7.9	6 ± 2
		9.1	5 ± 3	9.4	17 ± 3	9.7	19 ± 3	9.5	20 ± 2
9.9	23 ± 2	9.9	26 ± 3					10.0	20 ± 2
11.1	29 ± 2	10.8	23 ± 3	10.7	17 ± 3	11.0	35 ± 3	11.1	36 ± 2
11.9	17 ± 2	11.6	17 ± 3			12.4	16 ± 3	12.2	17 ± 2
		16.7	5 ± 3			16.8	5 ± 3	17.8	5 ± 2
20.9	6 ± 2	20.4	5 ± 3	21.1	7 ± 3	24.3	6 ± 3	22.1	6 ± 2
27.5	5 ± 2	27.2	5 ± 3	27.9	5 ± 3			27.9	5 ± 2
42.2	8 ± 2	40.3	6 ± 3	42.7	7 ± 3			40.3	6 ± 2
61.7	14 ± 2	58.9	14 ± 3	57.5	11 ± 3				
84.1	21 ± 2	83.2	19 ± 3	90.2	19 ± 3			108.4	20 ± 2
				142.9	5 ± 3	133.4	22 ± 3		

the highly significant peaks of large amplitudes are near $T=11$ yr and there are definitely more than one. The next large amplitude group is at high periodicities, but here also more than one are significant and large. These amplitudes are probably more meaningful than the heights of the peaks seen in Figs. 4 and 6. The periodicities near $T=5$ yr seem to be only of border-line importance, in agreement with the earlier finding of RAMANUJA RAO (1973).

Using these amplitudes, five sets of predictions can be obtained for the years 1972–80. These are shown in Fig. 7. Only data from 1959 onwards are plotted. The observed values are shown as dots, the bigger dots representing 1972 onwards up to 1980. The crosses represent expected/predicted values. The upper three curves are for Groups 1, 2, 3 of the predictions from data for 223 years (1749–1971). The lowest two curves are for Groups 1, 2, of the predictions from data for 127 years (1845–1971). The upper three curves do not show good coincidence between observed and predicted values. Also, a solar minimum in 1983 is indicated, which is very unlikely. The lowest two curves look more sensible, though the expected values (crosses) fall far short of the observed values (dots) in the last few years. Thus, prediction from recent data is more satisfactory than from the whole data of 223 years.

Predictions from spectral analysis are not new. COLE (1973) and COHEN and LINTZ (1974) predicted very small values of sunspots for cycle 21, which turned out to be incorrect as the observed value is about 155. In contrast, HILL (1977)

Fig. 7. Observed (full lines and dots) and expected (crosses and dashes) values of annual sunspot numbers for 1959–1983. The top three curves are for predictions from 223 years data, choosing three groups of periodicities and the lowest two curves are for predictions from 127 years data, choosing two groups of periodicities, as indicated in Table 1.

used 2754 monthly sunspot values for the years 1749–1975 and predicted sunspot maximum between 130–200 for cycle 21, which proved to be true. GLEISSBERG (1971) had predicted a number between 56 and 96, while SMITH (1976) predicted a number exceeding 150. None of the predictions in Figure 7 of ours exceeds 100. Recently, DE MEYER (1981) attempted to represent the sunspot record for 1749–1977 by a “harmonic” model comprising a relatively large number of lines as also by an “impulse” model. The predictions from these two models gave controversial results, and De Meyer concludes that sunspot activity is probably unpredictable.

5. Conclusions

The recent FABNE algorithm of spectral analysis introduced by BARRODALE and ERICKSON (1980 a and b) is known to be superior to all previous methods,

specially for detecting high periodicities. Annual mean (Wolf) sunspot numbers for 1749–1971 were subjected to FABNE spectral analysis, separately for 1749–1844 (96 years), 1845–1971 (127 years) and for the whole period 1749–1971 (223 years). When the spectra for the first 96 years and later 127 years were compared, a major peak at $T=9.7$ yr was common to both; but several other peaks had different periodicities. If the earlier data are not considered unreliable, the two successive samples may not belong to the same population.

In sunspot data, the period near $T=5$ yr is probably not significant. But the main peak near $T=11$ yr definitely has a fine structure which probably changes with time. Periods exceeding 100 yr are shown only by Burg spectra, but not by FABNE spectra, and hence may be spurious. But periodicities between 60 and 100 yr definitely exist, probably changing with time.

Predictions using five groups of these peaks indicated sunspot numbers for cycle 21 to be less than 100, which is not borne out by observations which indicate about 155. Whether any combination of periods, other than what we chose for our five groups can lead to better predictions, needs to be seen.

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REFERENCES

- AKAIKE, H., Fitting auto regressive models for prediction, *Ann. Inst. Stat. Math.*, **21**, 243–247, 1969.
- ANDERSON, N., On the calculation of filter coefficients for maximum entropy spectral analysis, *Geophysics*, **39**, 69–72, 1974.
- BARRODALE, I. and R. E. ERICKSON, Algorithms for least-squares linear prediction and maximum entropy spectral analysis—Part I: Theory, *Geophysics*, **45**, 420–432, 1980a.
- BARRODALE, I. and R. E. ERICKSON, Algorithms for least-squares linear prediction and maximum entropy spectral analysis—Part II: Fortran program, *Geophysics*, **45**, 433–446, 1980b.
- BLACKMAN, R. B. and J. W. TUKEY, *The Measurement of Power Spectra*, Dover, New York, 1959.
- CHEN, W. Y. and G. R. STEGEN, Experiments with maximum entropy power spectra of sinusoids, *J. Geophys. Res.*, **79**, 3019–3022, 1974.
- COHEN, T. J. and P. R. LINTZ, Long term periodicities in the sunspot cycle, *Nature*, **250**, 398–400, 1974.
- COLE, T. W., Periodicities in solar activity, *Solar Physics*, **30**, 103–110, 1973.
- CURRIE, R. G., Fine structure of the sunspot spectrum—2 to 70 years, *Astrophys. Space Sci.*, **20**, 509–518, 1973.
- DE MEYER, F., Mathematical modelling of the sunspot cycle, *Solar Physics*, **70**, 259–272, 1981.
- FOUGERE, P. F., A solution to the problem of spontaneous line—splitting in maximum entropy power spectrum analysis, *J. Geophys. Res.*, **82**, 1051–1054, 1977.
- GLEISSBERG, W., The probable behaviour of sunspot cycle 21, *Solar Physics*, **21**, 240–245, 1971.
- HILL, J. R., Long term solar activity forecasting using high—resolution time spectral analysis, *Nature*, **266**, 151–153, 1977.
- KANE, R. P., Power spectrum analysis of solar and Geophysical parameters, *J. Geomag. Geoelectr.*, **29**, 471–495, 1977.
- KANE, R. P., Maximum entropy spectral analysis of some artificial samples, *J. Geophys. Res.*, **84**, 965–966, 1979.

- KIMURA, H., On the harmonic analysis of sunspot relation numbers, *Monthly Notices R. Astron. Soc.*, **73**, 543-548, 1913.
- LACOSS, R. T., Data adaptive spectral analysis methods, *Geophysics*, **36**, 661-675, 1971.
- MCNISH, A. G. and J. V. LINCOLN, Predictions of sunspot numbers, *Trans. Am. Geophys. Union*, **30**, 673-685, 1949.
- RADOSKI, H. R., P. F. FOUGERE, and E. J. ZAWALICK, A comparison of power spectral estimates and applications of the maximum entropy method, *J. Geophys. Res.*, **80**, 619-625, 1975.
- RAMANUJA, RAO K., Short periodicities in solar activity, *Solar Physics*, **29**, 47-53, 1973.
- SMITH, F. M., Some new insights into the mechanism of the sunspot cycle, *Radio Commun (GB)*, **52**, 494-500, 1976.
- SMYTHE, C. M. and J. A. EDDY, Planetary tides during the Maunder Sunspot Minimum, *Nature*, **266**, 434-435, 1977.
- TURNER, H. H., On the expression of sunspot periodicity by a Fourier sequence and on the general uses of the Fourier sequence in similar problems, *Monthly Notices R. Astron. Soc.*, **73**, 714-732, 1913.
- ULRYCH, T. J. and T. N. BISHOP, Maximum entropy spectral analysis and autoregressive decomposition, *Rev. Geophys.*, **13**, 183-200, 1975.
- WALDMEIER, M., The Sunspot—Activity in the years 1610-1960, Schulthess & Co. AG, Zurich.
- WOLFF, C. L., Timing of solar cycles by rigid internal rotations, *Astropys. J.*, **205**, 612-621, 1976.
- WOOD, K. D., Sunspots and planets, *Nature*, **240**, 91-93, 1972.
- ULRYCH, T. J. and R. W. CLAYTON, Time series modelling and maximum entropy, *Phys. Earth Planet. Int.*, **12**, 188-200, 1976.