Analysis of the Effect Produced by Lateral Inhomogeneities in the Upper Mantle in the Peruvian Region

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An expression for the field induced by the ionospheric current system at equatorial latitudes, due to the presence of a mantle with a non-uniform boundary in the north-south direction was obtained using conformal mapping. The induced field was then calculated for the particular case of a semi-elliptical rise of the mantle, and the result applied to the equatorial zone of Peru, where a latitudinal gradient in the depth of the non-conducting layer had been found in previous work. Finally, the results were correlated with the tectonic features of the studied zone.

1. Introduction

In a previous work (DUHAU and OSELLA, 1982) the geomagnetic daily variations (g.d.v.) at the equatorial zone of Peru measured by FORBUSH and CASAVARDE (1961), were separated into their parts of external and internal origin and from them the ionospheric current system was inferred. Since no previous hypotheses about the induced field were made to separate the total field, the results are valid for an analysis of the internal structure of the earth.

Using a simple model for the distribution of the earth conductivity (a non-conducting layer of thickness p and a perfectly conducting halfspace below it), the field induced by the current system previously found was calculated (OSELLA and DUHAU, 1983) and a difference between the values of p which best fit the data to the north and to the south of the dip equator was found.

So, to get a good description of all the measured profile, it is necessary to propose a model for the distribution of earth conductivity which takes into account the presence of a latitudinal gradient in p .

SCHMUCKER *et al.* (1966), when analyzing magnetic bays in Peru, found an anomalous behaviour of the magnetic field which they supposed to be due to a localized rise of highly conductive material under the Andes, and named the "An-

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dean Anomaly”.

Therefore, taking into account the previous results, in the present paper the expression for the field induced due to the presence of a mantle with a non-uniform boundary in the north-south direction will be obtained and the results will be applied to the particular case of a semi-elliptical rise of the mantle. This boundary is chosen because it can reproduce not only a localized elevation but also an extended one, provided that the parameters of the model, the depth of the mantle and the width and height of the rise, are adequately varied.

Finally, the values of those parameters will be selected so that the resulting computed induced field reproduces adequately the internal part of the g.d.v. at the Peruvian zone.

2. Determination of the Induced Field

2.1 Theoretical model

To calculate the field induced by an external current system due to the presence of a mantle with a non-uniform boundary, a cartesian system with origin in the mantle is used, with the coordinates x, y, z in the south-ward, east-ward and outward directions, respectively. This system is chosen for convenience in order to perform the calculations; to present the results, the usual geomagnetic coordinates will be used.

As in the equatorial zone the current circulates in the east-ward direction, the following two-dimensional system is considered: a perfectly conducting half-space with a boundary given by a function $f(x, z)$, which represents the mantle, and a perfectly dielectric semi-space over it, all the properties and the fields being constant in the y -direction.

The vertical component of the magnetic field, Z , produced by a current density $J(x', z)$ can be obtained from

$$Z(x, z) = \int J(x', z') G(r, r') dx' dz' \quad (1)$$

where $G(r, r')$ is the Green's function which gives the vertical component of the field produced at $r(r=(x, z))$ by a line of current of intensity equal to unity located at $r(r'=(x', z'))$ in the given configuration. To represent the ionospheric current system, $J(x', z')$ is assumed to be of the form

$$J(x', z') = J(x') \delta(z' - (h+p)) \quad (2)$$

where p indicates the position, in the assumed coordinate system, of the earth's surface, h the height over it at which the current sheet is located and $J(x')$ the surface current density (see Fig. 1).

As the Green's function is generally difficult to find it is useful to transform the coordinates in such a way that the boundary of the conductor is transformed into a plane. To do so, since the present is a two-dimensional problem, a conformal

Fig. 1. Proposed model used to calculate the induced field.

mapping is used.

Let w , W be the complex variables defined as

$$w = x + iz, \quad (3a)$$

$$W = u + iv, \quad (3b)$$

and suppose there exists a conformal transformation. $W = g(w)$, which maps the boundary of the conductor (in the w -plane) into a line (the real axis in the W -plane), then the Green's function is given by (GREENHOUSE *et al.*, 1973)

$$G(w, w') = \text{Im} \left[-\frac{i\mu_0}{2\pi} \left(\frac{1}{W - W'} - \frac{1}{W - W'^*} \right) \left(\frac{\partial g(w)}{\partial w} \right)_w \right] \quad (4)$$

where μ_0 is the magnetic permeability and $W' = g(w) = g(x' + iz')$ and $W'^* = g^*(w')$.

Since the current flows at a height $(p + h)$ and the magnetic field is to be

calculated over the surface of the earth, then

$$W = g(x + ip), \quad (5a)$$

$$W' = g(x' + i(p + h)). \quad (5b)$$

So, the vertical component of the total magnetic field is given by

$$Z(x, p) = \int J(x') G(x, p; x', (p + h)) dx'. \quad (6)$$

If the external current system is already known the vertical component of the external field, Z_e , can be calculated and by subtracting it from Z , the induced field, Z_i , is obtained.

Finally, the horizontal component of the induced field, H_i , is calculated as the Hilbert transform of Z

$$H_i(x, p) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{Z_i(x', p)}{x - x'} dx'. \quad (7)$$

2.2 The field induced for a semi-elliptical boundary

The function

$$w = f(W) = \frac{aW + \sqrt{W^2 - b^2}}{a + 1} \quad (8)$$

maps the real axis in the W -plane into the real axis in the w -plane except for the semielliptical boundary around the origin (KOBBER, 1957), with a horizontal semi-axis, r_h , and a vertical one, r_v , given by

$$r_h = ab/(a + 1), \quad (9a)$$

$$r_v = b/(a + 1). \quad (9b)$$

The inverse function, i.e., $W = g(w)$, is given, for $a \neq 1$, by

$$W = g(w) = \frac{a(a + 1)w + \sqrt{b^2(1 - a^2) + w^2(a + 1)^2}}{a^2 - 1}. \quad (10)$$

If $a = 1$, the resulting profile is a semi-circle and the inverse function is

$$W_c = g_c(w) = w + \frac{R^2}{w} \quad (11)$$

with $R = b/2$.

Replacing $g(w)$ (Eq. 10) and its derivative in Eq. (4), the Green's function is obtained.

The current system in the equatorial zone has an extended part, J_p , and a

localized one, J_j , (the electrojet) that can be represented by (OSELLA and DUHAU, 1983)

$$J_p(x') = J_{op} \cos [k(x' - x_o - x_p)] + J_{oc} \quad (12)$$

$$J_j(x') = \begin{cases} J_{oj}(1 - (x' - x_o)^2/D^2) & -D + x_o \leq x' \leq x_o + D, \\ 0 & x' > x_o + D; x' < -D + x_o, \end{cases} \quad (13)$$

where J_{op} , J_{oc} and J_{oj} are the amplitudes of the surface current densities, k is the wave number, D is the width of the electrojet and x_o and x_p are the distance from the dip equator of the origin of the coordinate system and of the maximum of the horizontal component of the extended field, respectively.

The vertical components of the external field, produced by the extended system, Z_e^p , and by the electrojet, Z_e^j , are (OSELLA and DUHAU, 1983)

$$Z_e^p = \frac{\mu_0 J_{op}}{2} \sin [k(x - x_o - x_p)] \quad (14)$$

and

$$Z_e^j = \frac{\mu_0 J_{oj}}{2} \frac{1}{\pi} \left[\frac{1}{2} \left(1 - \frac{(x - x_o)^2}{D^2} + \frac{(p + h)^2}{D^2} \right) \ln \frac{(x - x_o + D)^2 + (p + h)^2}{(x - x_o - D)^2 + (p + h)^2} + \frac{2x}{D} - \frac{2d(x - x_o)}{D^2} \left(\arctan \frac{x - x_o + D}{p + h} - \arctan \frac{x - x_o - D}{p + h} \right) \right]. \quad (15)$$

Finally, the total induced field, $Z_i = Z_i^p + Z_i^j$ is calculated from

$$Z_i^p = Z^p - Z_e^p, \quad (16)$$

$$Z_i^j = Z^j - Z_e^j. \quad (17)$$

3. Application to the Peruvian Zone

At the Peruvian equatorial zone, the parameters of Eqs. (12) and (13) which give the external current systems were obtained from the external part of the g.d.v. and have the following values (OSELLA and DUHAU, 1983):

$$\begin{aligned} J_{op} &= 0.0352 \text{ Amp/m}, & J_{oc} &= 0.0144 \text{ Amp/m}, \\ K &= 7.6 \times 10^{-4} \text{ km}^{-1}, & J_{oj} &= 0.094 \text{ Amp/m}, \\ D &= 400 \text{ km}, & h &= 107 \text{ km}, & x_p &= 350 \text{ km}. \end{aligned}$$

The parameters that determine the upper boundary of the mantle (p , r_v and r_h) and the position of the dip equator (x_o) are varied in order to get a good fit between the vertical components of the computed induced field, Z_i , (from Eqs. (16) and (17)) and the internal part of the geomagnetic daily variations found in a previous work (DUHAU and OSELLA, 1982). The resulting set of values are

$$\begin{aligned} p &= (450 \pm 50) \text{ km}, & r_v &= (350 \pm 30) \text{ km}, \\ r_h &= (5000 \pm 500) \text{ km}, & x_o &= (4200 \pm 400) \text{ km}, \end{aligned}$$

which give the horizontal and vertical components of the induced field shown in Fig. 2.

Notice that the measured profile which covers a latitudinal interval of approximately 2400 km around the dip equator (see Fig. 2), is a half of the horizontal semi-axis of the ellipse and the dip equator is located almost in the middle of the zone of sharp decrease (see Fig. 3). This fact confirms the previous result (OSELLA and DUHAU, 1983) since it means that the depth of the non-conducting layer has a latitudinal discontinuity around the equator and is almost constant at both sides, being approximately 450 km at the north and 200 km at the south, which coincide roughly with the previous values of 450 km and 150 km, respectively.

Notice the good agreement for the horizontal component except in the central interval of approximately (-400 km, 600 km) around the dip equator. For the vertical component the measured profile is well reproduced in the ends of the intervals, and the anomaly is also detected there, its effect being more extended than in the horizontal component, as expected.

Notice, also, that the anomaly for this component is almost symmetrical around the point where it has its maximum value and, consequently, the anomaly in the vertical one is antisymmetrical around the same point.

The difference observed in the central interval may be due, partly, to the fact that the analytical function we have assumed to represent the latitudinal variation of the depth of the non-conducting layer does not allow to reproduce a sharp discontinuity, but also it may be due to the presence of an anomaly which affects not only the vertical but also the horizontal component. This anomaly may be associated to the "Andean Anomaly", found by SCHMUCKER *et al.* (1966). Notice that the interval where the anomaly in the horizontal component of geomagnetic bays was found by these authors, coincides with the zone where the localized anomaly appears in the same component of the geomagnetic daily variation, as found here. Moreover, this localized anomaly cannot be explained by the presence of the ocean, since this effect contributes at most with 2 or 3 nT to the total field (DUHAU and OSELLA, 1984).

Summing up, it can be concluded that there exist, at least, two superposed effects; an extended one, due to a large latitudinal gradient in p , and a localized one that, according to the results of SCHMUCKER *et al.* (1966) may be due to the presence of a localized and, eventually, a more superficial zone of high conductivity. Further analysis will be needed to study the central interval.

Figure 4 shows a simple tectonic map of the South-American equatorial zone, with the position of the geomagnetic dip equator shown. Notice the existence of a volcanic zone located in an interval around the dip equator which may be associated with this localized anomaly.

4. Summary

An expression for the field induced by the ionospheric current system, at equatorial latitudes, due to the presence of a mantle with a non-uniform boundary

Fig. 2. Horizontal, H_i , (a) and vertical, Z_i , (b), components of the internal field. Solid line: determined from the geomagnetic daily variations (DUHAU and OSELLA, 1982). Dotted-dashed line: computed with the model described in the present work.

Fig. 3. Profile of the upper boundary of the highly conducting region of the mantle that gives the best fit between the computed and measured induced field.

Fig. 4. Tectonic map of the South-American equatorial zone (from WYLLIE, 1971).

in the north-south direction was obtained and the model was used for the particular case of a semi-elliptical rise of the mantle.

This model was then applied to the equatorial zone of Peru, where a latitudinal gradient in the depth of the non-conduction layer has been found in a previous work. The presence of this gradient in p has been confirmed here and, also, a localized anomaly around the dip equator has been found. This anomaly could coincide with the "Andean Anomaly" and could be associated with the tectonic features of the studied zone.

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