Global Models of the Magnetic Field in Historical Times: 
Augmenting Declination Observations with Archeo- and Paleo-
Magnetic Data

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The earliest measurements of the Earth’s magnetic field were of declination. By 1600 these measurements were widespread, but widespread measurements of dip are available only after 1700, and of intensity only after 1832. Global models of the Earth’s magnetic field for these very early times must therefore be augmented by indirect measurements of dip and intensity using paleomagnetic methods.

Suppose that the declination is known everywhere and a potential field $B' = \nabla \psi'$ has been found that matches it. Then the ratio of the horizontal intensity of this field to the true value is found to be constant along the lines of constant $\psi'$. Thus one measurement of horizontal intensity on each line is sufficient to determine the horizontal field, and hence the complete field, uniquely.

Contours of $\psi'$ encircle the dip-poles, and so, for example, it would be sufficient to measure horizontal intensity along a single line joining the dip-poles to provide good global coverage.

The quantity of paleomagnetic information needed for the horizontal intensity is quite small. The Earth’s field at present has only two dip-poles, and measurements at some 10 sites spaced approximately evenly in latitude, and approximately 50y apart in time, should be adequate.

1. Introduction

Historical measurements of the Earth’s magnetic field, dating back to 1695AD, have been analysed to provide models of the global magnetic field (BLOXHAM and GUBBINS, 1985). These field models are surprisingly good; they give far greater detail than earlier models and can be used to examine the field at the core mantle boundary. Their quality reflects the good global distribution of measurements made at sea during certain periods when magnetic mapping was particularly intensive—at the beginning of the 18th. century, during Cook’s voyages later in the century, during the survey of the southern oceans in the 19th century, and by the research ship Carnegie early in the 20th. century. Other periods suffer from a shortage of inclination and intensity measurements.

With the exception of very early Chinese records, magnetic observations are documented from the sixteenth century. The very earliest records are restricted to
Europe and are unreliably reported. For example the records at London have been thoroughly researched by Malin and Bullard (1981) who were unable to find original references to some of the observations, and the title of their paper reflects the fact that reliable reports began only in 1570.

By the end of the 16th. century declination measurements had been made throughout the known world. Robert Norman invented the dip circle in London in 1586 and used it to measure inclination, but worldwide measurements did not become available until the 18th. century. There is therefore an interval of some 130 years for which we have good declination measurements but virtually no inclinations.

The first absolute intensity measurements were made by Gauss in 1832; relative intensities were made somewhat earlier, notably by Humboldt on his travels in America from 1798–1803.

The analysis of historical data therefore poses three different problems. After 1832 there is three component data; for certain intervals between about 1700 and 1832 there is good directional data but no intensity data; and for the earliest period from about 1570 to 1700, and certain later intervals, there is only declination data.

Conventional methods of analysis are designed to deal with three component data, and there are no special difficulties with the post-1832 observations. The second problem, of analysing good directional data without intensities, poses difficult questions of uniqueness that will be discussed in a separate paper (Proctor et al., 1985). The present paper is concerned with the third problem, that of declination measurements alone. The method of global field modelling is described first.

2. Global Field Modelling

Global models of the magnetic field are usually expressed as the gradient of a magnetic potential which satisfies Laplace’s equation; the potential itself is usually expressed in terms of the coefficients of a spherical harmonic series:

\[ \Phi = a \sum_{l=1}^{\infty} \sum_{m=0}^{l} \left( \frac{a}{r} \right)^{l+1} P_l^m(\cos \theta)(g_l^m \cos m\phi + h_l^m \sin m\phi), \] (1)

where \((r, \theta, \phi)\) are spherical polar coordinates, \(a\) is the Earth’s radius, and the \(\{P_l^m\}\) are Schmidt normalized spherical harmonics. The \(\{g_l^m\}\) and \(\{h_l^m\}\) are called the geomagnetic or Gauss coefficients. They have units of magnetic field. This representation contains two physical assumptions: that the Earth’s mantle and atmosphere are electrical insulators, and that monopole terms are absent.

The use of spherical harmonics is mathematically convenient but not a fundamental requirement of the physics. The potential nature of the field is embodied in the equations

\[ \mathbf{B} = \nabla \Phi, \] (2)

\[ \nabla^2 \Phi = 0, \] (3)
or equivalently

$$\nabla \cdot \mathbf{B} = \nabla \times \mathbf{B} = 0. \quad (4)$$

In order to determine \(\mathbf{B}\) everywhere in the potential region we must be able to find a unique solution of (3) for the potential \(\Phi\). A unique solution exists when one of the following quantities is known everywhere on a closed surface, in this case the Earth's surface:

1. \(\Phi\) (the Dirichlet problem),
2. \(\nabla \Phi \cdot \hat{r}\) (radial component of \(\mathbf{B}\), the Neumann problem),
3. a linear combination of (1) and (2),
4. the horizontal component of \(\mathbf{B}\).

(1) and (3) are not relevant to the present problem because magnetic potential is not measured directly; (2) and (4) show there is a redundancy in three component data because either the vertical or horizontal component would suffice.

These results require perfect knowledge of the relevant quantity everywhere, which is never achievable in practice. Nevertheless, such a uniqueness theorem is an essential prerequisite to analysing discrete data. No theorem in the standard literature on potential theory applies to directions. It is obvious that any result based on directions alone can be multiplied by an arbitrary constant scalar and is therefore non-unique. When only declination is known there is a further degree of non-uniqueness which can be illustrated by considering any axisymmetric field: the declination for all such fields is everywhere zero. Note that these are not the only manifestations of non-uniqueness; there are in fact more subtle problems.

3. Analysis of Declination Measurements

Consider the simple idealized case of perfect declination information. Declination is the angle between the horizontal magnetic field vector and true north, measured positive east (clockwise from north). We separate the magnetic field vector into its radial and horizontal components

$$\mathbf{B} = B_r \hat{r} + B_h. \quad (5)$$

Declination is the direction of the vector field \(\mathbf{B}_h\) on the spherical surface \(r=a\). The radial component of the second of Eq. (4) can be written as

$$\nabla_h \times \mathbf{B}_h = 0 \quad (6)$$

where \(\nabla_h\) is the horizontal gradient operator, given in spherical coordinates by

$$a \nabla_h = \left( 0; \frac{\partial}{\partial \theta'} \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \right).$$
(6) is a necessary condition on the horizontal field, and therefore the declination measurements, for consistency with a potential field vector $\mathbf{B}$. It follows from (6) that the horizontal field can be written as the horizontal gradient of a scalar potential $\psi(\theta, \phi)$. Comparing this expression with (1) shows that

$$\psi(\theta, \phi) = \Phi(a, \theta, \phi).$$  \hspace{1cm} (7)

If we can determine $\psi$ everywhere on the spherical surface $r=a$, then we can find $\mathbf{B}_h$ there, and by the uniqueness theorem (4) above we may then find $\mathbf{B}$ everywhere in the potential region. Unfortunately declination measurements alone serve only to determine the direction of $\mathbf{B}_h$ and not its magnitude; thus if $\mathbf{B}_h'$ is a known vector field that fits the declinations, they any vector field of the form $a\mathbf{B}_h'$, where the horizontal intensity function $a$ depends upon position, will fit equally well. The condition (6) places a restriction on $\mathbf{B}_h$ and the form of $a$.

Suppose we have found a vector field $\mathbf{B}_h'$ that satisfies (6) and the declination. (This is not a problem provided the declinations are consistent with a potential field. Experience with analysing real data suggests there are no practical difficulties. For references to methods and previous analyses of declination measurements see BARRACLOUGH, 1978). Then writing

$$\mathbf{B}_h' = \nabla_h \psi',$$ \hspace{1cm} (8)

and substituting into (6) and using the vector identity

$$\nabla \times (fa) = f \nabla \times a + \nabla f \times a,$$

where $a\cdot\dot{\mathbf{r}}=0$, gives

$$\nabla \times a \times \nabla_h \psi' = 0.$$ \hspace{1cm} (9)

Equation (9) shows that $\nabla \times a$ is parallel to the known vector $\nabla_h \psi'$. Level lines of $a$, therefore, coincide with the level lines of $\psi'$. This is the central result of the paper. Since $a$ is constant along the known contours of $\psi'$, it can be fixed by a measurement of horizontal intensity at just one point on each contour. The horizontal field vector is fixed once $a$ is known, and the complete field can then be found uniquely.

4. Conclusions

We now consider the practical implications of (9), and determine how to augment the declination measurements with paleomagnetic data in the best possible way.

The magnetic potential, $\psi'$, can be any function that provides a good fit to the declination data. The model at epoch 1715 of BLOXHAM and GUBBINS (1985) is
representative of the potential field in early times and is shown in Fig. 1. The units are arbitrary. The contours reflect the near-dipole nature of the magnetic field at the Earth's surface, but they also show clear departures from axial symmetry because they do not coincide with lines of latitude.

These contours show where the horizontal intensity function $\alpha$ must be determined to provide good global coverage. An example will make things clearer. The contour of $\psi'$ in Fig. 1 that passes through Sicily sweeps west to pass through northern Mexico, and east across central Asia to northern Japan. A determination of horizontal intensity from the lavas of Mt. Etna in Sicily would be equivalent to one from lavas erupted in Japan or one from early artefacts in Mexico. Duplicate measurements would provide a valuable check on the analysis, but the primary consideration must be to obtain a good global distribution of data.

How many measurements are required to provide good global coverage? In practice the declination data are far from perfect. In early times there will be many gaps in the data distribution map: the whole of the African and Antarctic continents will be blank for example. There is little point in aiming at a better distribution of horizontal intensity than that in declination. BLOXHAM and GUBBINS (1985) were able to produce acceptable historical models with gaps in the data distribution as large as 20°. This suggests that archeo- or paleo-magnetic measurements of inclination and intensity at some 10 sites separated by about 20° in latitude would be adequate.

Refer to Fig. 1 and consider how best to cover the globe with horizontal intensity measurements. There are 23 contour lines in all so that sites on alternate contours are required. The sites must have good historical records for the period 1500–1800 and

Fig. 1. Contours of magnetic potential at the Earth's surface at epoch 1715, from the model of BLOXHAM and GUBBINS (1985). Declination measurements from earlier times must be augmented with paleo- or archeo-magnetic determinations of horizontal intensity for a global model to be found. Only one horizontal intensity need be found on each contour line. Some possible sites are marked on the map.
have suitable material such as hearths or lavas; the samples must be oriented because both inclination and intensity are required.

In northern latitudes Iceland is an obvious candidate. Working south, results from France, Italy, Japan and the Southern U.S. might cover mid-latitudes; central America, Japan and China might provide some data. India and Peru cover equatorial latitudes. The southern hemisphere is always a problem; possible sites are in South America and Java, New Zealand and Australia. The contour through New Zealand passes into Antarctica, and is therefore very valuable. This leaves a gap of one contour in Antarctica which is unlikely to be filled unless use can be made of the magnetization of the ice.

Finally consider the required spread in time. The accuracy with which inclination can be determined is about 2°; this corresponds to 50y of typical secular change, and, therefore, sampling at 50y intervals would be adequate. The corresponding numbers for the more difficult intensity measurement is 3000 nT, again suggesting a 50y sample time.

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REFERENCES


