Two-Dimensional Magnetotelluric Modelling by the Boundary Element Method

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2-D magnetotelluric modelling is usually done by the finite element method and the finite difference method. When the anomalous body is of a complicated shape, division of the region into cells and initial data preparation are rather tedious. In this paper, the boundary element method is applied to 2-D magnetotelluric models. First, we convert the boundary value problem of electromagnetic field into the boundary integral equation. Then, we solve the integral equation with the boundary element method. The boundaries of the domain considered and the anomalous body are divided into elements. The integral along all boundaries is divided into a sum of integrals along each element. Linear interpolation and Gaussian quadrature formula are used to calculate these integrals. After these steps, the integral equation is converted into a set of linear equations. Solving the equation system with the Gaussian elimination method, the electromagnetic field can be obtained and the apparent resistivity can be derived. The boundary element method is suitable for a model with one anomalous body. When there are several anomalous bodies, formula derivation and programming are complicated. This method is applied to two examples. The results agree well with those obtained by the finite element method.

1. Introduction

In the past three decades, two-dimensional magnetotelluric modelling has received high attention. In order to get solutions, various methods have been used. BRYNELLI *et al.* (1963, 1969) and DOSSO (1966) used an electrolytic tank. TAHERI (1968) used RC-network. D'ERCEVILLE and KUNETS (1962) and RANKIN (1962) found analytical solutions for some specific models. COGGON (1971), REDDY and RANKIN (1973), ZHAO and XU (1983) and XU and ZHAO (1985) used the finite element method. DMITRIYEV and ZAKHAROV (1970) presented a method for reducing the boundary value problem of electrodynamics to integral equations. PARRY and WARD (1971) used the integral equation method to solve a 2-D electromagnetic scattering problem. ZAKHAROV (1975) applied the integral equation method to 2-D magnetic and magnetotelluric modelling.

In this paper, the boundary element method is used to solve the integral equations for H (or E) and its normal derivative. When the model consists of one anomalous body within a homogeneous space, this method is especially preferable.

Compared with the finite element method, the boundary element method is simpler in element division and initial data preparation and requires less memory space. Two examples illustrate that the results of the boundary element method agree well with those of the finite element method. Although Green's theorem is used, there is something different from the method used by PARRY and WARD (1971) and ZAKHAROV (1975), such as the integral of a Delta function on a boundary point.

2. Boundary Value Problem in Magnetotellurics

Let the x -axis be aligned in the direction of strike, the y -axis orthogonal to the x -axis and the z -axis vertically downward. Assuming that time factor is $e^{-i\omega t}$, the magnetotelluric apparent resistivity is expressed (CAGNIARD, 1953) for the H_x polarization as

$$\rho_{yx} = \frac{1}{\omega\mu} \left| \frac{E_y}{H_x} \right|^2, \quad \text{where} \quad E_y = \frac{1}{\sigma} \frac{\partial H_x}{\partial z}, \quad (1.1)$$

and for the E_x polarization as

$$\rho_{xy} = \frac{1}{\omega\mu} \left| \frac{E_x}{H_y} \right|^2, \quad \text{where} \quad H_y = \frac{1}{i\omega\mu} \frac{\partial E_x}{\partial z}. \quad (1.2)$$

μ is the magnetic permeability and σ the conductivity. In order to derive the apparent resistivities ρ_{xy} and ρ_{yx} , we must calculate H_x , E_x and their normal derivatives $\partial H_x/\partial z$ and $\partial E_x/\partial z$.

The governing equations for H_x and E_x are
 H_x polarization:

$$\left. \begin{aligned} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} &= i\omega\mu H_x, \\ \frac{\partial H_x}{\partial z} &= (\sigma - i\omega\epsilon)E_y, \\ -\frac{\partial H_x}{\partial y} &= (\sigma - i\omega\epsilon)E_z, \end{aligned} \right\} \quad (1.3)$$

E_x polarization:

$$\left. \begin{aligned} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} &= (\sigma - i\omega\epsilon)E_x, \\ \frac{\partial E_x}{\partial z} &= i\omega\mu H_y, \\ -\frac{\partial E_x}{\partial y} &= i\omega\mu H_z, \end{aligned} \right\} \quad (1.4)$$

where ε is the dielectric constant.

Substituting E_y , E_z and H_y , H_z from the second and third equations of (1.3) and (1.4) into the first equations of (1.3) and (1.4), we obtain the equations governing H_x and E_x in a homogeneous medium as

$$\frac{\partial^2 H_x}{\partial y^2} + \frac{\partial^2 H_x}{\partial z^2} + (\omega^2 \mu \varepsilon + i\omega \mu \sigma) H_x = 0, \quad (1.5)$$

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} + (\omega^2 \mu \varepsilon + i\omega \mu \sigma) E_x = 0. \quad (1.6)$$

Rewriting (1.5) and (1.6) in a single form,

$$\nabla^2 U + k^2 U = 0, \quad (1.7)$$

where U represents H_x or E_x , ∇^2 is the 2-dimensional Laplace operator and $k^2 = \omega^2 \mu \varepsilon + i\omega \mu \sigma$. In free space, $\sigma = 0$, $k^2 = \omega^2 \mu \varepsilon$. In conductive rock at frequencies used in magnetotellurics, we have $k^2 \approx i\omega \mu \sigma$, since $\sigma / \omega \varepsilon \gg 1$.

Because the boundary conditions for E_x and H_x modes are different, we consider them separately.

2.1 Boundary condition for H_x polarization

2.1.1. Interface boundary condition

Let $H_x^{(1)}$, $H_x^{(2)}$ represent the magnetic field H_x on the two sides of an interface Γ_1 . From the continuity of tangential components, we have

$$H_x^{(1)}|_{\Gamma_1} = H_x^{(2)}|_{\Gamma_1}. \quad (1.8)$$

From Fig. 1, the tangential component of the electric field on Γ_1 is

$$\begin{aligned} E_t &= E_y \sin \alpha - E_z \cos \alpha \\ &= \frac{1}{\sigma} \left(\frac{\partial H_x}{\partial z} \sin \alpha + \frac{\partial H_x}{\partial y} \cos \alpha \right) \\ &= \frac{1}{\sigma} \frac{\partial H_x}{\partial n}, \end{aligned}$$

where α is the angle between the normal vector $\mathbf{n}$ and the y -axis. From the continuity of the tangential component of the electric field we have

$$\frac{1}{\sigma_1} \frac{\partial H_x^{(1)}}{\partial n} \Big|_{\Gamma_1} = \frac{1}{\sigma_2} \frac{\partial H_x^{(2)}}{\partial n} \Big|_{\Gamma_1}. \quad (1.9)$$

Fig. 1. The region and the boundary.

2.1.2. Outside boundary condition

Let σ_1 represent the conductivity of the medium around the anomalous body and σ_2 represent the conductivity of the anomalous body. We draw a semicircle Γ_∞ with a very large radius under the ground surface Γ_s (Fig. 1). Γ_∞ and Γ_s consist of a closed outside boundary surface Γ ($\Gamma = \Gamma_s + \Gamma_\infty$). From (1.3), $\partial H_x / \partial y = \sigma E_y$ and $\partial H_x / \partial y = -\sigma E_z$ in conductive rock. In free space, $\partial H_x / \partial z = -i\omega\epsilon E_y$ and $\partial H_x / \partial y = i\omega\epsilon E_z$. Because $\sigma / \omega\epsilon \gg 1$, we may approximate $\partial H_x / \partial z = \partial H_x / \partial y = 0$ at the free space side. This means that H_x is constant regardless of the distribution of the subsurface conductivity. We set $H_x = 1$ on Γ_s . Because Γ_∞ is far from the anomalous body, the electromagnetic field is the same as that for a homogeneous half space:

$$H_x|_{\Gamma_\infty} = e^{i\sqrt{i\omega\mu\sigma_1}z}. \quad (1.10)$$

2.2 The boundary condition for E_x polarization

2.2.1. Inside boundary condition

Let $E_x^{(1)}$, $E_x^{(2)}$ represent the electric fields on the two sides of a boundary. From the continuity of the tangential components of electric fields, we have

$$H_x^{(1)}|_{\Gamma_1} = E_x^{(2)}|_{\Gamma_1}. \quad (1.11)$$

From Fig. 1, the tangential component of H is

$$\begin{aligned} H_t &= H_y \sin \alpha - H_z \cos \alpha \\ &= \frac{1}{i\omega\mu} \left(\frac{\partial E_x}{\partial z} \sin \alpha + \frac{\partial E_x}{\partial y} \cos \alpha \right) \\ &= \frac{1}{i\omega\mu} \frac{\partial E_x}{\partial n}. \end{aligned}$$

We assume $\mu = \mu_0$, where μ_0 is the permeability of free space. From the continuity of tangential magnetic field components, we have

$$\left. \frac{\partial E_x^{(1)}}{\partial n} \right|_{\Gamma_1} = \left. \frac{\partial E_x^{(2)}}{\partial n} \right|_{\Gamma_1}. \quad (1.12)$$

2.2.2. Outside boundary condition

Since Γ_∞ is far from the anomalous body, the electric field on Γ_∞ is the same as that in a homogeneous earth:

$$H_x|_{\Gamma_\infty} = e^{i\sqrt{i\omega\mu\sigma_1}z}. \quad (1.13)$$

We draw a semi-circle Γ'_∞ with a very large radius (Fig. 1). Γ'_∞ and Γ_∞ constitute a closed outside surface. At Γ'_∞ , the anomalous field due to the anomalous body is zero. E_x consists of an incident field and a reflected field:

$$E_x|_{\Gamma'_\infty} = E_i e^{i\sqrt{\omega^2\mu\epsilon}z} + E_r e^{-i\sqrt{\omega^2\mu\epsilon}z}. \quad (1.14)$$

The first term is the incident wave and the second the reflected wave. E_i and E_r are coefficients to be determined. According to the boundary conditions at $z=0$ and $y=\pm\infty$, we find

$$\begin{aligned} E_i &= \frac{1}{2} \left[1 + (1+i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \right], \\ E_r &= \frac{1}{2} \left[1 - (1+i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \right]. \end{aligned}$$

Substituting E_r and E_i into (1.14), we derive

$$\begin{aligned} E_x|_{\Gamma'_\infty} &= \frac{1}{2} \left[1 + (1+i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \right] e^{i\sqrt{\omega^2\mu\epsilon}z} + \frac{1}{2} \left[1 - (1+i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \right] e^{-i\sqrt{\omega^2\mu\epsilon}z} \\ &= \cos(\omega\sqrt{\mu\epsilon}z) - (1-i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \sin(\omega\sqrt{\mu\epsilon}z). \end{aligned} \quad (1.15)$$

Now we summarize the boundary value problem as follows.
 H_x polarization:

$$\left. \begin{array}{l} \text{Basic equation:} \\ \text{Inside boundary condition:} \\ \text{Outside boundary condition:} \end{array} \right\} \begin{array}{l} \nabla^2 U + k^2 U = 0, \\ U^{(1)} = U^{(2)}, \\ \frac{1}{\sigma_1} \frac{\partial U^{(1)}}{\partial n} = \frac{1}{\sigma_2} \frac{\partial U^{(2)}}{\partial n}, \\ U|_{\Gamma_s} = 1, \\ U|_{\Gamma_\infty} = e^{i\sqrt{i\omega\mu\sigma_2}z}. \end{array} \quad (1.16)$$

E_x polarization:

$$\left. \begin{array}{l} \text{Basic equation:} \\ \text{Inside boundary condition:} \\ \text{Outside boundary condition:} \end{array} \right\} \begin{array}{l} \nabla^2 U + k^2 U = 0, \\ U^{(1)} = U^{(2)}, \\ \frac{\partial U^{(1)}}{\partial n} = \frac{\partial U^{(2)}}{\partial n}, \\ U|_{\Gamma_\infty} = e^{i\sqrt{i\omega\mu\sigma_2}z}, \\ U|_{\Gamma_\infty} = \cos(\omega\sqrt{\mu\epsilon}z), \\ - (1-i) \sqrt{\frac{\sigma_1}{2\omega\epsilon}} \sin(\omega\sqrt{\mu\epsilon}z). \end{array} \quad (1.17)$$

Now our task is to find $U(H_x$ or $E_x)$ and its normal derivative $\partial U/\partial n$ on the ground surface.

3. Green's Theorem and Fundamental Solution

In the finite element method, the differential equations are transformed into a variation problem. Similarly, in the boundary element method, the differential equations are transformed into integral equations. This can be realized by using the Green's theorem:

$$\int_{\Omega} (U \nabla^2 \phi - \phi \nabla^2 U) d\Omega = \oint_{\Gamma} \left(U \frac{\partial \phi}{\partial n} - \phi \frac{\partial U}{\partial n} \right) d\Gamma, \quad (2.1)$$

where Γ is the boundary of domain Ω , $\mathbf{n}$ is the outside normal vector, U is a function to be solved and ϕ is the fundamental solution of the derivative equation:

$$\nabla^2 U + k^2 U = 0. \quad (2.2)$$

This means that ϕ is the Green's function of infinite space:

$$\nabla^2 \phi + k^2 \phi = -\delta(P), \quad (2.3)$$

where P is an arbitrary point in region Ω (Fig. 2) and $\delta(P)$ is the Dirac delta function centered at P . Substituting (2.2), (2.3) into the left side of (2.1), we obtain

$$\int_{\Omega} (U \nabla^2 \phi - \phi \nabla^2 U) d\Omega = - \int_{\Omega} U \delta(P) d\Omega. \quad (2.4)$$

When P is located on boundary Γ , from the property of the Dirac delta function (see Appendix),

$$\int_{\Omega} U \delta(P) d\Omega = \frac{\alpha_p}{2\pi} U(P),$$

where α_p is the angle subtended by Ω at P (Fig. 2). Therefore,

$$\frac{\alpha_p}{2\pi} U(P) = - \oint_{\Gamma} \left(U \frac{\partial \phi}{\partial n} - \phi \frac{\partial U}{\partial n} \right) d\Gamma. \quad (2.5)$$

This is the integral equation that U and its normal derivative $\partial U / \partial n$ at boundary points must satisfy. If U on the boundary is given, we can find $\partial U / \partial n$ with the boundary element method. The fundamental solution of (2.2) is

$$\phi = -\frac{1}{4} N_0(kr), \quad (2.6)$$

where N_0 is the zero order Bessel function of the second kind, r is the distance between point P and an arbitrary point in the region, $k = \sqrt{\omega^2 \mu \epsilon}$ in free space and $k = \sqrt{i\omega \mu \sigma}$ below the ground surface.

Fig. 2. Boundary point P and angle α_p subtended by Ω at P .

4. Solution for H_x Polarization

4.1 Integral equation

Let $U^{(1)}$ and $U^{(2)}$ represent H_x in Ω_1 and Ω_2 respectively (Fig. 3). For region Ω_1 , U at boundary point P is derived from (2.5) as

$$\begin{aligned} \frac{\alpha_p}{2\pi} U^{(1)}(P) &= -\oint_{\Gamma} \left(U^{(1)} \frac{\partial \phi}{\partial n} - \phi \frac{\partial U^{(1)}}{\partial n} \right) d\Gamma \\ &= - \left(\int_{\Gamma_s} + \int_{\Gamma_{\infty}} + \oint_{\Gamma_I} \right) \left(U^{(1)} \frac{\partial \phi}{\partial n} - \phi \frac{\partial U^{(1)}}{\partial n} \right) d\Gamma. \end{aligned} \quad (3.1)$$

The fundamental solution in Ω_1 is $\phi = -(1/4)N_0(k_1r)$, where $k_1 = \sqrt{i\omega\mu\sigma_1}$. Differentiating ϕ partially with respect to n , we get

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial r} \frac{\partial r}{\partial n} = \frac{1}{4} k_1 N_1(k_1r) \cos \beta,$$

Fig. 3. Division of elements.

where N_1 is the first order Bessel function of the second kind, β is the angle between r and $\mathbf{n}$ (Fig. 3).

At Γ_∞ , the anomalous electromagnetic field is zero. From (1.16),

$$U^{(1)}|_{\Gamma_\infty} = e^{ik_1 z},$$

$$\left. \frac{\partial U^{(1)}}{\partial n} \right|_{\Gamma_\infty} = \left(\frac{\partial U^{(1)}}{\partial z} \frac{\partial z}{\partial n} \right) \Big|_{\Gamma_\infty} = ik_1 e^{ik_1 z} \cos(n, z),$$

where (n, z) is the angle between $\mathbf{n}$ and z . So the integral on Γ_∞ is

$$-\int_{\Gamma_\infty} \left(U^{(1)} \frac{\partial \phi}{\partial n} - \frac{\partial U^{(1)}}{\partial n} \phi \right) d\Gamma$$

$$= -\int_{\Gamma_\infty} \frac{k_1}{4} e^{ik_1 z} [N_1(k_1 r) \cos \beta + iN_0(k_1 r) \cos(n, z)] d\Gamma.$$

We divide Γ_∞ into elements, keeping the length of each element less than $\lambda_1/8$, where λ_1 is the wave length in Ω_1 , and $\lambda_1 = 2\sqrt{2\pi}/\sqrt{\omega\mu\sigma_1}$. Then we use Gaussian quadrature formula to integrate over each element (see (3.5)). We add together and denote as C_p , which is a function of P . Equation (3.1) can be rewritten as

$$\frac{\alpha_p}{2\pi} U^{(1)}(P) = -\int_{\Gamma_s} U^{(1)} \frac{\partial \phi}{\partial r} \cos \beta d\Gamma + \int_{\Gamma_s} \frac{\partial U^{(1)}}{\partial n} \phi d\Gamma$$

$$- \oint_{\Gamma_1} U^{(1)} \frac{\partial \phi}{\partial r} \cos \beta d\Gamma + \int_{\Gamma_1} \frac{\partial U^{(1)}}{\partial n} \phi d\Gamma + C_p. \quad (3.2)$$

For region Ω_2 , $U^{(2)}(P)$ at point P on Γ_1 is derived from (2.5) as

$$\left(1 - \frac{\alpha_p}{2\pi}\right) U^{(2)}(P) = \oint_{\Gamma_1} U^{(2)} \frac{\partial \phi}{\partial r} \cos \beta d\Gamma - \oint_{\Gamma_1} \frac{\partial U^{(2)}}{\partial n} \phi d\Gamma \quad (3.3)$$

where $(1 - \alpha_p/2\pi)$ is the angle subtended by Ω_2 at P . Because the normal vector of Γ_1 is defined from inward, the sign in front of the above integrals has been changed. The fundamental solution in region Ω_2 is $\phi = -(1/4)N_0(k_2 r)$, where $k_2 = \sqrt{i\omega\mu\sigma_2}$. Substituting the boundary condition (1.16) into (3.3), we obtain

$$\left(1 - \frac{\alpha_p}{2\pi}\right) U^{(1)}(P) = \oint_{\Gamma_1} U^{(1)} \frac{\partial \phi}{\partial r} \cos \beta d\Gamma - \oint_{\Gamma_1} \frac{\sigma_2}{\sigma_1} \frac{\partial U^{(1)}}{\partial n} \phi d\Gamma. \quad (3.4)$$

(3.2) and (3.4) are integral equations. Solving the equations with the boundary element method, we can find $\partial U/\partial n$ on ground surface Γ_s .

4.2 Boundary element method

We divide Γ_s and Γ_l into elements, approximate each element as a straight line segment (see Fig. 3) and make the length of element less than $\lambda_1/8$ and $\lambda_2/8$, where λ_1 and λ_2 are the wavelengths in Ω_1 and Ω_2 , respectively. The ends of the element are nodes. There are n nodes on Γ_s , whose ordinal numbers are from 1 to n . There are $m-n$ nodes on Γ_l , whose ordinal numbers are from $n+1$ to m . We divide the integral equation (3.2) into a sum of integrals on each element. For node i , equation (3.2) can be written as

$$\begin{aligned} \frac{\alpha_i}{2\pi} U_i^{(1)} = & - \sum_{\Gamma_s} \int_{\Gamma_E} U^{(1)} \frac{k_1}{4} N_1(k_1 r) \cos \beta \, d\Gamma - \sum_{\Gamma_s} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial n} \frac{N_0(k_1 r)}{4} \, d\Gamma \\ & - \sum_{\Gamma_l} \int_{\Gamma_E} U^{(1)} \frac{k_1}{4} N_1(k_1 r) \cos \beta \, d\Gamma - \sum_{\Gamma_l} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial n} \frac{N_0(k_1 r)}{4} \, d\Gamma + C_i. \end{aligned} \quad (3.5)$$

The node numbers of the two ends of an element are taken as j, k with coordinates $(x_j, y_j), (x_k, y_k)$. We assume that $U^{(1)}$ and $\partial U^{(1)}/\partial n$ are linear functions in each element. Using $U_j^{(1)}, U_k^{(1)}, (\partial U^{(1)}/\partial n)_j$ and $(\partial U^{(1)}/\partial n)_k$ to represent $U^{(1)}$ and $\partial U^{(1)}/\partial n$ at j and k , $U^{(1)}$ and $\partial U^{(1)}/\partial n$ can be written as

$$\begin{aligned} U^{(1)} &= \xi_j U_j^{(1)} + \xi_k U_k^{(1)} = [\xi_j \quad \xi_k] [U_j^{(1)} \quad U_k^{(1)}]^T, \\ \frac{\partial U^{(1)}}{\partial n} &= \xi_j \left(\frac{\partial U^{(1)}}{\partial n} \right)_j + \xi_k \left(\frac{\partial U^{(1)}}{\partial n} \right)_k = [\xi_j \quad \xi_k] \left[\left(\frac{\partial U^{(1)}}{\partial n} \right)_j \quad \left(\frac{\partial U^{(1)}}{\partial n} \right)_k \right]^T, \end{aligned}$$

where ξ_j, ξ_k are linear functions and defined as

$$x = \xi_j x_j + \xi_k x_k, \quad y = \xi_j y_j + \xi_k y_k.$$

It is obvious that at point j , $\xi_j=1, \xi_k=0$ and at point k , $\xi_j=0, \xi_k=1$.

The integral over an element can be derived as follows:

$$\begin{aligned} \int_{\Gamma_E} U^{(1)} \frac{k_1 N_1(k_1 r)}{4} \cos \beta \, d\Gamma &= \int_{\Gamma_E} [\xi_j \quad \xi_k] \frac{k_1 N_1(k_1 r)}{4} \cos \beta \, d\Gamma \cdot [U_j^{(1)} \quad U_k^{(1)}]^T \\ &= [f_{ij} \quad f_{ik}] [U_j^{(1)} \quad U_k^{(1)}]^T, \end{aligned} \quad (3.6)$$

where

$$\begin{aligned} f_{ij} &= \int_{\Gamma_E} \xi_j \frac{k_1 N_1(k_1 r)}{4} \cos \beta \, d\Gamma, \\ f_{ik} &= \int_{\Gamma_E} \xi_k \frac{k_1 N_1(k_1 r)}{4} \cos \beta \, d\Gamma. \end{aligned}$$

All these integrals are complex and are accomplished with the Gaussian quadrature formula (BREBBIA, 1978). The integral over an element is derived similarly as

$$\int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial n} \frac{-N_0(k_1 r)}{4} d\Gamma = \int_{\Gamma_E} [\xi_j \quad \xi_k] \frac{-N_0(k_1 r)}{4} d\Gamma \cdot \left[\left(\frac{\partial U^{(1)}}{\partial n} \right)_j \left(\frac{\partial U^{(1)}}{\partial n} \right)_k \right]^T$$

$$= [d_{ij} \quad d_{ik}] \left[\left(\frac{\partial U^{(1)}}{\partial n} \right)_j \left(\frac{\partial U^{(1)}}{\partial n} \right)_k \right]^T, \quad (3.7)$$

where

$$d_{ij} = \int_{\Gamma_E} \xi_j \frac{-N_0(k_1 r)}{4} d\Gamma,$$

$$d_{ik} = \int_{\Gamma_E} \xi_k \frac{-N_0(k_1 r)}{4} d\Gamma.$$

The elements containing node i are singular, since $r \rightarrow 0$, $N_0(k_1 r) \rightarrow \infty$, and $N_1(k_1 r) \rightarrow \infty$ in these elements. However, the integrals f_{ij} , f_{ik} , d_{ij} and d_{ik} on these elements are definite. For instance (see Fig. 4),

$$f_{ij} = \int_{\Gamma_E} -\xi_j \frac{k_1}{4} N_1(k_1 r) \cos \beta d\Gamma = 0, \quad (\because \cos \beta = 0)$$

$$d_{ij} = \frac{1}{4} \int_{\Gamma_E} -\xi_j N_0(k_1 r) d\Gamma$$

$$\approx \int_0^l \frac{l-r}{l} \frac{1}{2\pi} \ln \frac{k_1 r}{2} dr, \quad \left(\because \xi_j = \frac{l-r}{l}, \lim_{r \rightarrow 0} N_0(k_1 r) = -\frac{2}{\pi} \ln \frac{k_1 r}{2} \right)$$

$$= -\frac{l}{4\pi} \left(\frac{3}{2} - \ln \frac{k_1 r}{2} \right),$$

where l is the length of the element.

The sum of all integrals over each element on Γ_S is obtained as

$$\sum_{\Gamma_S} \int_{\Gamma_E} \frac{U^{(1)} - k_1 N_1(k_1 r)}{4} \cos \beta d\Gamma$$

$$= [F_{i1} \dots F_{ij} \dots F_{in}] [U_1^{(1)} \dots U_j^{(1)} \dots U_n^{(1)}]^T = \mathbf{F}_{is} \mathbf{U}_S^{(1)} \quad (3.8)$$

Fig. 4. Integral over the element containing node i .

$$\begin{aligned} & \sum_{\Gamma_s} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial \mathbf{n}} \frac{-N_0(k_1 r)}{4} d\Gamma \\ &= [D_{i1} \cdots D_{ij} \cdots D_{in}] \left[\left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_1 \cdots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_j \cdots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_n \right]^T = \mathbf{D}_{is} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s, \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} \mathbf{U}_s^{(1)} &= [U_1^{(1)} \cdots U_n^{(1)}]^T, \\ \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s &= \left[\left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_1 \cdots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_n \right]^T, \end{aligned}$$

are the column vectors formed by $U^{(1)}$ and $\partial U^{(1)} / \partial \mathbf{n}$ at each node on Γ_s , F_{ij} and D_{ij} are the sum of f_{ij} , d_{ij} at the elements around node j :

$$\mathbf{F}_{is} = [F_{i1} \cdots F_{ij} \cdots F_{in}],$$

$$\mathbf{D}_{is} = [D_{i1} \cdots D_{ij} \cdots D_{in}].$$

Only those elements on Γ_s have contribution to $\mathbf{F}_{is}$, $\mathbf{D}_{is}$.

Using the same method, we can obtain the sum of integrals over elements on Γ_I as

$$\sum_{\Gamma_I} \int_{\Gamma_E} U^{(1)} \frac{-k_1 N_1(k_1 r)}{4} \cos \beta d\Gamma = [F_{i(n+1)} \cdots F_{im}] [U_{n+1}^{(1)} \cdots U_m^{(1)}]^T = \mathbf{F}_{iI} \mathbf{U}_I^{(1)} \quad (3.10)$$

$$\begin{aligned} & \sum_{\Gamma_I} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial \mathbf{n}} \frac{-N_0(k_1 r)}{4} d\Gamma \\ &= [D_{i(n+1)} \cdots D_{im}] \left[\left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_{n+1} \cdots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_m \right]^T = \mathbf{D}_{iI} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_I, \end{aligned} \quad (3.11)$$

where

$$\begin{aligned} \mathbf{U}_I^{(1)} &= [U_{n+1}^{(1)} \cdots U_m^{(1)}]^T, \\ \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_I &= \left[\left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_{n+1} \cdots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_m \right]^T, \end{aligned}$$

are column vectors formed by $U^{(1)}$ and $\partial U^{(1)} / \partial \mathbf{n}$ of each node on Γ_I :

$$\mathbf{F}_{iI} = [F_{i(n+1)} \cdots F_{im}],$$

$$\mathbf{D}_{iI} = [D_{i(n+1)} \cdots D_{im}].$$

Only those elements on Γ_s have contribution to F_{il} , D_{il} .

Substituting (3.8)–(3.11) into (3.5), we obtain

$$\frac{\alpha_i}{2\pi} U_i^{(1)} = F_{is} U_s^{(1)} + D_{is} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{il} U_l^{(1)} + D_{il} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_i. \quad (3.12)$$

For each node we can derive an equation as above. From all the nodes i ($1 \leq i \leq n$) on Γ_s , we derive an equation system:

$$\frac{\alpha_s}{2\pi} U_s^{(1)} = F_{ss} U_s^{(1)} + D_{ss} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{sl} U_l^{(1)} + D_{sl} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_s. \quad (3.13)$$

From all the nodes i ($n+1 \leq i \leq m$) on Γ_l we derive another equation system:

$$\frac{\alpha_l}{2\pi} U_l^{(1)} = F_{ls} U_s^{(1)} + D_{ls} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{ll} U_l^{(1)} + D_{ll} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_l, \quad (3.14)$$

where

$$\alpha_s = \text{diag} [\alpha_1 \cdots \alpha_i \cdots \alpha_n]$$

$$F_{ss} = [F_{1s} \cdots F_{is} \cdots F_{ns}]^T$$

$$D_{ss} = [D_{1s} \cdots D_{is} \cdots D_{ns}]^T$$

$$F_{sl} = [F_{1l} \cdots F_{il} \cdots F_{nl}]^T$$

$$D_{sl} = [D_{1l} \cdots D_{il} \cdots D_{nl}]^T$$

$$C_s = (C_1 \cdots C_i \cdots C_n)^T$$

$$\alpha_l = \text{diag} [\alpha_{n+1} \cdots \alpha_i \cdots \alpha_m]$$

$$F_{ls} = [F_{(n+1)s} \cdots F_{is} \cdots F_{ms}]^T$$

$$D_{ls} = [D_{(n+1)s} \cdots D_{is} \cdots D_{ms}]^T$$

$$F_{ll} = [F_{(n+1)l} \cdots F_{il} \cdots F_{ml}]^T$$

$$D_{ll} = [D_{(n+1)l} \cdots D_{il} \cdots D_{ml}]^T$$

$$C_l = (C_{n+1} \cdots C_i \cdots C_m)^T.$$

Only those elements on Γ_s have contribution to F_{il} , D_{il} .

Substituting (3.8)–(3.11) into (3.5), we obtain

$$\frac{\alpha_i}{2\pi} U_i^{(1)} = F_{is} U_s^{(1)} + D_{is} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{il} U_l^{(1)} + D_{il} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_i. \quad (3.12)$$

For each node we can derive an equation as above. From all the nodes i ($1 \leq i \leq n$) on Γ_s , we derive an equation system:

$$\frac{\alpha_s}{2\pi} U_s^{(1)} = F_{ss} U_s^{(1)} + D_{ss} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{sl} U_l^{(1)} + D_{sl} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_s. \quad (3.13)$$

From all the nodes i ($n+1 \leq i \leq m$) on Γ_l we derive another equation system:

$$\frac{\alpha_l}{2\pi} U_l^{(1)} = F_{ls} U_s^{(1)} + D_{ls} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_s + F_{ll} U_l^{(1)} + D_{ll} \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_l + C_l, \quad (3.14)$$

where

$$\alpha_s = \text{diag} [\alpha_1 \cdots \alpha_i \cdots \alpha_n]$$

$$F_{ss} = [F_{1s} \cdots F_{is} \cdots F_{ns}]^T$$

$$D_{ss} = [D_{1s} \cdots D_{is} \cdots D_{ns}]^T$$

$$F_{sl} = [F_{1l} \cdots F_{il} \cdots F_{nl}]^T$$

$$D_{sl} = [D_{1l} \cdots D_{il} \cdots D_{nl}]^T$$

$$C_s = (C_1 \cdots C_i \cdots C_n)^T$$

$$\alpha_l = \text{diag} [\alpha_{n+1} \cdots \alpha_i \cdots \alpha_m]$$

$$F_{ls} = [F_{(n+1)s} \cdots F_{is} \cdots F_{ms}]^T$$

$$D_{ls} = [D_{(n+1)s} \cdots D_{is} \cdots D_{ms}]^T$$

$$F_{ll} = [F_{(n+1)l} \cdots F_{il} \cdots F_{ml}]^T$$

$$D_{ll} = [D_{(n+1)l} \cdots D_{il} \cdots D_{ml}]^T$$

$$C_l = (C_{n+1} \cdots C_i \cdots C_m)^T.$$

E_x in region $\Omega_1, \Omega_2, \Omega_3$, respectively. For region Ω_3 , $U^{(3)}(P)$ at point P on Γ_s is derived from (2.5) as

$$\begin{aligned} \frac{\alpha'_p}{2\pi} U^{(3)}(P) &= \oint \left(U^{(3)} \frac{\partial \phi}{\partial n} - \phi \frac{\partial U^{(3)}}{\partial n} \right) d\Gamma \\ &= \left(\int_{\Gamma_\infty} + \int_{\Gamma_s} \right) \left(U^{(3)} \frac{\partial \phi}{\partial n} - \phi \frac{\partial U^{(3)}}{\partial n} \right) d\Gamma, \end{aligned} \quad (4.1)$$

where α'_p is the angle subtended by Ω_3 at P . Because the normal vector is defined inwards (Fig. 3), there is no negative sign before the integral of (4.1). The fundamental solution in Ω_3 is $\phi = -(1/4)N_0(k_3r)$, where $k_3 = \sqrt{\omega^2 \mu \varepsilon}$. Differentiating ϕ partially with respect to n , we obtain

$$\frac{\partial \phi}{\partial n} = \frac{\partial \phi}{\partial r} \frac{\partial r}{\partial n} = \frac{k_3}{4} N_1(k_3r) \cos \beta.$$

The anomalous field on Γ'_∞ is considered to be zero. From (1.17),

$$\begin{aligned} U^{(3)}|_{\Gamma'_\infty} &= \cos(\omega \sqrt{\mu \varepsilon} z) - (1-i) \sqrt{\frac{\sigma_1}{2\omega \varepsilon}} \sin(\omega \sqrt{\mu \varepsilon} z) \\ &= \cos(k_3 z) + i \frac{k_1}{k_3} \sin(k_3 z), \end{aligned}$$

where

$$k_1 = \sqrt{i\omega\mu\sigma_1}$$

$$\frac{\partial U^{(3)}}{\partial n} \bigg|_{\Gamma'_\infty} = \left(\frac{\partial U^{(3)}}{\partial z} \frac{\partial z}{\partial n} \right) \bigg|_{\Gamma'_\infty} = -[k_3 \sin(k_3 z) + k_1 \cos(k_3 z)] \cos(n, z),$$

where (n, z) is the angle between vector $\mathbf{n}$ and the z -axis. So the integral on Γ'_∞ is given as

$$\begin{aligned} \int_{\Gamma'_\infty} \left(U^{(3)} \frac{\partial \phi}{\partial n} - \phi \frac{\partial U^{(3)}}{\partial n} \right) d\Gamma &= \int_{\Gamma'_\infty} \frac{1}{4} [(k_3 \cos(k_3 z) - k_1 \sin(k_3 z))] N_1(k_3 r) \cos \beta d\Gamma \\ &\quad - \int_{\Gamma'_\infty} \frac{1}{4} [k_3 \sin(k_3 z) + k_1 \cos(k_3 z)] N_0(k_3 r) \cos(n, z) d\Gamma. \end{aligned}$$

Dividing Γ'_∞ into elements, the above integral can be calculated with the Gaussian quadrature formula. It is a function of point P and denoted as A_p . Equation (4.1) can be written as

$$\frac{\alpha'_p}{2\pi} U^{(3)}(P) = \frac{1}{4} \int_{\Gamma_s} U^{(3)} k_3 N_1(k_3 r) \cos \beta \, d\Gamma + \frac{1}{4} \int_{\Gamma_s} \frac{\partial U^{(3)}}{\partial n} N_0(k_3 r) \, d\Gamma + A_p.$$

Considering the boundary condition (1.17) and using $U^{(1)}$ and $\partial U^{(1)}/\partial n$ to substitute $U^{(3)}$ and $\partial U^{(3)}/\partial n$, the formula can be written as

$$\frac{\alpha'_p}{2\pi} U^{(1)}(P) = \frac{1}{4} \int_{\Gamma_s} U^{(1)} k_3 N_1(k_3 r) \cos \beta \, d\Gamma + \frac{1}{4} \int_{\Gamma_s} \frac{\partial U^{(1)}}{\partial n} N_0(k_3 r) \, d\Gamma + A_p.$$

We decompose the above integral as a sum of integrals over each element. For node i , the above formula can be rewritten as

$$\frac{\alpha'_i}{2\pi} U_i^{(1)} = \sum_{\Gamma_s} \frac{1}{4} \int_{\Gamma_E} U^{(1)} k_3 N_1(k_3 r) \cos \beta \, d\Gamma + \sum_{\Gamma_s} \frac{1}{4} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial n} N_0(k_3 r) \, d\Gamma + A_i. \quad (4.2)$$

In a manner similar to (3.6) and (3.7), the integral on an element is derived as

$$\begin{aligned} & \int_{\Gamma_E} U^{(1)} \frac{k_3}{4} N_1(k_3 r) \cos \beta \, d\Gamma \\ &= \int_{\Gamma_E} [\xi_j \quad \xi_k] \frac{k_3}{4} N_1(k_3 r) \cos \beta \, d\Gamma \cdot [U_j^{(1)} \quad U_k^{(1)}]^T = [g_{ij} \quad g_{ik}] [U_j^{(1)} \quad U_k^{(1)}]^T \\ & \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial n} \frac{1}{4} N_0(k_3 r) \, d\Gamma = \int_{\Gamma_E} [\xi_j \quad \xi_k] \frac{1}{4} N_0(k_3 r) \, d\Gamma \cdot \left[\left(\frac{\partial U^{(1)}}{\partial n} \right)_j \left(\frac{\partial U^{(1)}}{\partial n} \right)_k \right]^T \\ &= [e_{ij} \quad e_{ik}] \left[\left(\frac{\partial U^{(1)}}{\partial n} \right)_j \left(\frac{\partial U^{(1)}}{\partial n} \right)_k \right]^T, \end{aligned}$$

where

$$g_{ij} = \int_{\Gamma_E} \xi_j \frac{k_3}{4} N_1(k_3 r) \cos \beta \, d\Gamma,$$

$$g_{ik} = \int_{\Gamma_E} \xi_k \frac{k_3}{4} N_1(k_3 r) \cos \beta \, d\Gamma,$$

$$e_{ij} = \frac{1}{4} \int_{\Gamma_E} \xi_j N_0(k_3 r) \, d\Gamma,$$

$$e_{ik} = \frac{1}{4} \int_{\Gamma_E} \xi_k N_0(k_3 r) \, d\Gamma.$$

These four integrals can be calculated with the Gaussian quadrature formula. The sum of integrals over elements on Γ_S is

$$\sum_{\Gamma_S} \frac{1}{4} \int_{\Gamma_E} U^{(1)} k_3 N_1(k_3 r) \cos \beta \, d\Gamma = [G_{i1} \dots G_{ij} \dots G_{in}] [U_1^{(1)} \dots U_j^{(1)} \dots U_n^{(1)}]^T = \mathbf{G}_{is} \mathbf{U}_S^{(1)}, \quad (4.3)$$

$$\begin{aligned} \sum_{\Gamma_S} \frac{1}{4} \int_{\Gamma_E} \frac{\partial U^{(1)}}{\partial \mathbf{n}} N_0(k_3 r) \, d\Gamma &= [E_{i1} \dots E_{ij} \dots E_{in}] \left[\left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_1 \dots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_j \dots \left(\frac{\partial U^{(1)}}{\partial \mathbf{n}} \right)_n \right]^T \\ &= \mathbf{E}_{is} \left(\frac{\partial \mathbf{U}^{(1)}}{\partial \mathbf{n}} \right)_s \end{aligned} \quad (4.4)$$

where G_{ij} and E_{ij} are the sum of g_{ij} , e_{ij} over elements around node j :

$$\mathbf{G}_{is} = [G_{i1} \dots G_{in}],$$

$$\mathbf{E}_{is} = [E_{i1} \dots E_{in}].$$

Substituting (4.3) and (4.4) into (4.2), we obtain

$$\frac{\alpha'_i}{2\pi} U_i^{(1)} = \mathbf{G}_{is} \mathbf{U}_S^{(1)} + \mathbf{E}_{is} \left(\frac{\partial \mathbf{U}^{(1)}}{\partial \mathbf{n}} \right)_s + A_i.$$

For each node i on Γ_S , we can derive an equation as above. We now have the equation system for all nodes:

$$\frac{\alpha'_s}{2\pi} \mathbf{U}_S^{(1)} = \mathbf{G}_{ss} \mathbf{U}_S^{(1)} + \mathbf{E}_{ss} \left(\frac{\partial \mathbf{U}^{(1)}}{\partial \mathbf{n}} \right)_s + \mathbf{A}_s, \quad (4.5)$$

where

$$\alpha'_s = \text{diag} [\alpha'_1 \dots \alpha'_n],$$

$$\mathbf{G}_{ss} = [G_{1s} \dots G_{ns}]^T,$$

$$\mathbf{E}_{ss} = [E_{1s} \dots E_{ns}]^T,$$

$$\mathbf{A}_s = (A_1 \dots A_n)^T.$$

The equations in Ω_1 and Ω_2 are almost the same as in the H_x polarization case. In Ω_1 , we have the same two equation systems as (3.13) and (3.14). Considering the boundary condition of E_x polarization $\partial U^{(1)} / \partial \mathbf{n} = \partial U^{(2)} / \partial \mathbf{n}$, we obtain an equation system corresponding to (3.15):

$$\mathbf{U}_I^{(1)} = -\mathbf{F}'_{II} \mathbf{U}_I^{(1)} - \mathbf{D}'_{II} \left(\frac{\partial \mathbf{U}^{(1)}}{\partial \mathbf{n}} \right)_I. \quad (4.6)$$

Combining (3.13), (3.14), (4.5) and (4.6), we derive a linear equation system:

$$\begin{pmatrix} F_{SS} - \frac{\alpha_S}{2\pi} & F_{SI} & D_{SS} & D_{SI} \\ F_{IS} & F_{II} - \frac{\alpha_I}{2\pi} & D_{IS} & D_{II} \\ G_{SS} - \frac{\alpha'_S}{2\pi} & 0 & E_{SS} & 0 \\ 0 & F'_{II} + I - \frac{\alpha'_I}{2\pi} & 0 & D'_{II} \end{pmatrix} \begin{pmatrix} U_S^{(1)} \\ U_I^{(1)} \\ \left(\frac{\partial U^{(1)}}{\partial n}\right)_S \\ \left(\frac{\partial U^{(1)}}{\partial n}\right)_I \end{pmatrix} = \begin{pmatrix} -C_S \\ -C_I \\ -A_S \\ 0 \end{pmatrix}. \quad (4.7)$$

We have $2m$ equations in total. The number of unknowns is also $2m$. We resolve this equation to derive the values of U (that is: E_x) and $\partial U/\partial n$ on the ground surface. Substituting $E_x = U$, $H_y = (1/i\omega\mu)\partial E_x/\partial z = -(1/i\omega\mu)\partial U/\partial n$ into (1.2), the apparent resistivity can be found.

6. Examples

6.1 H_x polarization

Figure 5 shows a syncline model. The resistivity of surrounding rock is $\rho_1 = 20 \Omega\text{m}$, and of syncline is $\rho_2 = 1 \Omega\text{m}$. We calculated the magnetotelluric profile for a

Fig. 5. The magnetotelluric profile for H_x polarization for $T=10$ sec. The curve indicates the result of the boundary element method, while dots indicate the result of the finite element method. Model parameters are $a=140$ m, $b=70$ m, $c=200$ m, $d=800$ m, $\alpha=19.3^\circ$.

period $T=10$ s. The wavelength of the electromagnetic wave in surrounding rock is $\lambda_1=\sqrt{10\rho_1 T}=44.7$ km, and in syncline is $\lambda_2=10$ km. The locations of nodes are shown in Fig. 5. The number of nodes on the ground surface is 29 (8 nodes located beyond ± 1.5 km are not shown in Fig. 5) and on the conductivity boundary is 16. The length of element is less than $\lambda_1/8$ and $\lambda_2/8$. The curve in the upper half of Fig. 5 is the magnetotelluric profile calculated by the boundary element method and the points near the curve are the result by the finite element method. The results by both methods are in good agreement.

6.2 E_x polarization

Figure 6 is an erect thick plate model. The resistivity of surrounding rock is $\rho_1=10\ \Omega\text{m}$, and of plate is $\rho_2=1\ \Omega\text{m}$. We calculated the magnetotelluric profile for a period $T=1$ s. The wavelength of the electromagnetic wave in surrounding rock is $\lambda_1=\sqrt{10\rho_1 T}=10$ km, and in plate is $\lambda_2=3.16$ km. The locations of nodes are shown in Fig. 6. The number of nodes on the ground surface is 17 (6 nodes located beyond ± 2 km are not shown in Fig. 6) and on the conductivity boundary is 12. The length of element is less than $\lambda_1/8$ and $\lambda_2/8$. The curve in the upper half of Fig. 6 is the magnetotelluric profile calculated by the boundary element method and the points near the curve are the result by the finite element method.

Fig. 6. The magnetotelluric profile of E_x polarization for $T=1$ sec (—: the result of the boundary element method, $\cdot$: the result of the finite element method).

Figure 7 is the magnetotelluric response curve at point A in Fig. 6. The curve is the result of the boundary element method and the points represent the result of the finite element method. From Figs. 5, 6 and 7 it is obvious that the results of the boundary element method are identical to those of the finite element method. This shows that the boundary element method is effective.

Fig. 7. The magnetotelluric sounding curve at point A (see Fig. 6) (—: the result of boundary element method, ·: the result of the finite element method).

7. Conclusion

In the finite element method, element division is made over the whole domain. When the anomalous body is of complicated shape, this element division and initial data preparation are tedious. In the boundary element method, only the boundary needs to be divided, so the division is easier. It is also possible to use more elements to approximate the boundary more precisely. In addition, the boundary element method needs far less memory space, so it can run even on a microcomputer. The examples given above show that the results obtained by both methods are very similar. In particular, the boundary element method is suitable for a model with one anomalous body. When there are several anomalous bodies, derivation of the formula and programing are complicated.

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Appendix: The Integration of δ Function on an Area

Let p be a fixed point on a plain, q a moving point and ε a small circle enclosing p . We define

$$\delta_{\varepsilon}(q, p) = \begin{cases} \frac{1}{\varepsilon}, & q \in \varepsilon; \\ 0, & q \notin \varepsilon. \end{cases}$$

and

$$\delta(p) = \lim_{\varepsilon \rightarrow 0} \delta_{\varepsilon}(q, p) = \begin{cases} \infty, & q = p; \\ 0, & q \neq p. \end{cases}$$

Then $\delta(p)$ is a 2-D δ function centered at p . Let Ω represent a 2-D domain and Γ be the boundary of Ω . When p is on Γ (Fig. 2), ε is divided into two parts. Let ε' represent the part within Ω . Then

$$\int_{\Omega} \delta(p) d\Omega = \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon'} \delta(p) d\Omega = \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon'} \frac{d\Omega}{\varepsilon} = \lim_{\varepsilon \rightarrow 0} \frac{\varepsilon'}{\varepsilon} = \frac{\alpha_p}{2\pi},$$

where α_p is the angle subtended by Ω at p . If U is a continuous function of p , then

$$\int_{\Omega} U \delta(p) d\Omega = \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon'} U \delta(p) d\Omega = U(p) \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon'} \delta(p) d\Omega = \frac{\alpha_p}{2\pi} U(p).$$

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