Influences of Ions in the Ionosphere*

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Abstract

It is examined that how the propagation of radio wave in the ionized region is influenced by the positive and negative ions in its region, and how a record is altered for the wave reflected from the ionized region for these ions. Especially, these problems are discussed for the wave in and from the lower ionized region where the ionic density is considered to be great. The equation of motion and the Maxwell's equation are solved in the presence of the earth's magnetic field, ionic and electronic collisional frequencies, including the Lorentz polarization term. Results show that (1) influences by the two kinds of ions on the propagation become greater as the frequency of the wave decreases; (2) energy of the reflection increases; (3) the deviation of energy and the virtual height of reflection for wave of certain frequency become smaller than those in the regions consisted of only electron; (4) the height of reflection thus approaches to the true height (no retardation), but the magnitude of the electron density, which is given by the relation $N = Af^2$, appears to be doubtful in the case of the measurement by low frequency radio wave.

§ 1. Introduction

The role which ions play in the ionized region has been neglected by many workers in the study of the ionized region. This is because (1) it is considered that for the wave used in the observation of ionized region the wave is reflected from the *E*, *F*1 and *F*2 regions where the ionic density is small, therefore the wave is affected from ions and (2) an ionic density is unknown in the stage of a present knowledge. According to the study by Bates and Massey,⁽¹⁾ λ , which is used in the estimation of the order of ionic density, is considered to be 6 at 70 km, 0.4 at 90 km and 10 in the *F*2 region in the case of best estimation. These values are very small in comparison with those deduced by other workers. Especially, Booker and Berkner⁽²⁾ suggest that this ratio as 2×10^4 in the lower part of the *E* region from a result of an experiment and adopted the Lorentz polarization term based on this suggestion. Scott⁽³⁾ deduces 4×10^3 for the *E* region based on a calculation of the gyro-frequency in the arctic region. For these great differences of λ we have not any evidence that ionic density is

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small or great in the lower ionized region, hence we cannot easily determine.

Recently observations on ionized region are often held by the waves of low and very low frequencies. We know that the heights where these waves are reflected are considerably low, though it is not known whether they are the tail of the E region. The difference of the reflection height of these waves between day and night is abnormally large. It is, needless to say, admitted that the disappearance of electron at night is large for the great recombination coefficient at these levels. But besides, it is considered for us that the effect of ions may be introduced in the explanation for these results of measurements.

In this paper we only discuss the role of ions on the propagation and reflection of wave in the ion-electron mixture. Its application to the phenomena in a real ionized region will be discussed in another paper.

§ 2. Calculation of Complex Refractive Index

The propagation of wave in a ionized region, consisted of positive and negative ion and electron, was firstly discussed by Goubau.⁽¹⁾ He assumed that the positive and negative ion have all equal collisional frequency with other molecules and equal mass and density. Lorentz polarization term l was omitted without any explanation. In our study, we use same assumptions for ionic mass and density as Goubau did, but the Lorentz polarization term is not omitted in the equation of motion because we consider it is correct. This reason is explained in Appendix.

We take z -axis in the vertical direction and x , y -axes in the east and north directions respectively. Vector of the earth's magnetic field lies in the zy plane and makes angle α with z -axis. Direction of propagation of the wave lies along the z -axis and so the angle of incidence is zero.

The equation of the motion for a charged particle is given by

$$\ddot{\mathbf{P}} + \nu \dot{\mathbf{P}} = \frac{Ne^2}{m}(\mathbf{E} + 4\pi l\mathbf{P}) + \frac{e}{mc}[\mathbf{P} \times \mathbf{H}], \quad (1)$$

where $\mathbf{P}$, $\mathbf{E}$, and $\mathbf{H}$ are vectors of polarization, electric field and earth's magnetic field respectively; ν is the collisional frequency of particle; N is the number density of particles per c.c.; m , e are the mass and charge of a particle; c is the velocity of light and, $\dot{}$ represents the differentiation with the time. Maxwell's equation is given for one kind of particle as follows;

$$\text{rot } \mathbf{H} = \frac{1}{c}(\dot{\mathbf{E}} + 4\pi \mathbf{r} \dot{N}e), \quad (2)$$

$$\text{rot } \mathbf{E} = -\frac{1}{c} \dot{\mathbf{H}}, \quad (3)$$

where $\mathbf{r}$ is the coordinate vector which has components x , y , z . For the mixture of electron and two sorts of ions $4\pi eN\dot{\mathbf{r}}$ is replaced by $4\pi \sum eN\dot{\mathbf{r}}$.

From (1).

$$\left. \begin{aligned} \ddot{x} + \dot{x}\nu &= \frac{e}{m}(E_x + 4\pi lNe x) + \frac{e}{mc}(\dot{y}H_z - \dot{z}H_y), \\ \ddot{y} + \dot{y}\nu &= \frac{e}{m}(E_y + 4\pi lNe y) + \frac{e}{mc}(-\dot{x}H_z), \end{aligned} \right\} \quad (4)$$

$$\ddot{z} + \dot{z}\nu = \frac{e}{m}(E_y + 4\pi l N e z) + \frac{e}{mc}(\dot{x}H_y). \quad)$$

If $\mathbf{E}$, x and y , z are proportional to $e^{i(\omega t + \frac{2\pi}{\lambda}\mu z)}$, then

$$\left. \begin{aligned} x &= \frac{e}{m}[AE_x + B \cos \alpha E_y - \sin \alpha \cdot E_z], \\ y &= \frac{e}{m}[-B \cos \alpha E_x + (A - C \sin^2 \alpha)E_y - C \sin \alpha \cdot \cos \alpha E_z], \\ z &= \frac{e}{m}[B \sin \alpha E_y - C \sin \alpha \cdot \cos \alpha E_y + (A - C \cos^2 \alpha)E_z]. \end{aligned} \right\} \quad (5)$$

with

$$\begin{aligned} A &= \frac{\omega\nu - \nu^2 - 4\pi Ne^2 l/m}{(i\omega\nu - \omega^2 - 4\pi Ne^2 l/m)^2 - \frac{e^2 H^2}{m^2 c^2} \omega^2}, \quad C = \frac{\omega^2 \frac{e^2 H^2}{m^2 c^2}}{\{i\omega\nu - \omega^2 - 4\pi Ne^2 l/m\} \left\{ (i\omega\nu - \omega^2 - 4\pi Ne^2 l/m) - \frac{e^2 H^2}{m^2 c^2} \omega^2 \right\}}, \\ B &= \frac{i\omega e H/mc}{(i\omega\nu - \omega^2 - 4\pi Ne^2 l/m)^2 - \frac{e^2 H^2}{m^2 c^2} \omega^2}, \end{aligned} \quad (6)$$

where ω is the frequency of wave, λ is the wave-length, and μ is a refractive index. From (2) and (3) it is given in the ion-electron mixture

$$\left. \begin{aligned} \dot{H}_x &= c \frac{\partial E_y}{\partial z} & -\frac{\partial H_y}{\partial z} &= \frac{1}{c}[\dot{E}_x + 4\pi \Sigma N e \dot{x}], \\ \dot{H}_y &= -c \frac{\partial E_x}{\partial z} & \frac{\partial H_x}{\partial z} &= \frac{1}{c}[\dot{E}_y + 4\pi \Sigma N e \dot{y}], \\ \dot{H}_z &= 0, & 0 &= \frac{1}{c}[\dot{E}_z + 4\pi \Sigma N e \dot{z}]. \end{aligned} \right\} \quad (7)$$

(7) leads to

$$\left. \begin{aligned} E_x(1 - \mu^2) &= -4\pi \Sigma N e x, \\ E_y(1 - \mu^2) &= -4\pi \Sigma N e y, \\ E_z &= -4\pi \Sigma N e z. \end{aligned} \right\} \quad (8)$$

Substituting (5) into (7) and considering the gyro-frequency ω_{H_i} ($= \frac{eH}{m_i c}$; suffix refers to ion) for and ion is neglected in comparison with that for electron, then

$$\left. \begin{aligned} (1 - \mu^2 + \Sigma A \delta)E_x + B \delta \cos \alpha E_y - B \delta \sin \alpha E_z &= 0, \\ -B \delta \cos \alpha E_x + (1 - \mu^2 + \Sigma A \delta - C \delta \sin^2 \alpha)E_y - C \delta \sin \alpha \cos \alpha E_z &= 0, \\ B \delta \sin \alpha E_x - C \delta \sin \alpha \cos \alpha E_y + (1 - \Sigma A \delta - C \sin^2 \alpha)E_z &= 0. \end{aligned} \right\} \quad (9)$$

where $\delta = \frac{4\pi N e^2}{m}$. If (9) has roots the determinant of coefficients for E_x , E_y , and E_z equals zero. When we solve this equation the refractive index is given by

$$\frac{\mu^2}{\mu_i^2} = 1 - \frac{x_e/\mu_i^2}{\left(1 + l \frac{x_e}{\mu_i^2} - iZ_e\right) - \frac{\frac{1}{2}y_T^4}{1 - (1-l)\frac{x_e}{\mu_i^2} - iZ_e} \pm \sqrt{\frac{\frac{1}{4}y_T^4}{\left\{1 - (1-l)\frac{x_e}{\mu_i^2} - iZ_e\right\}^3} + y_L^2}} \quad (10)$$

with

$$\mu_i^2 = 1 - \frac{x_i}{1 + l x_i - Z_i},$$

where

$$x_e = \frac{4\pi N e^2}{m_e \omega^2}, \quad x_i = \frac{4\pi N_i e_i^2}{m_i \omega^2}, \quad y_L = \frac{e H_L}{m_e c \omega}, \quad y_L = \frac{e H_T}{m_e c \omega}, \quad Z = \frac{\nu_e}{\omega}.$$

(suffixes e and i refer to ion and electron respectively)

In the case of electron alone μ_e^2 is given by putting in (10) as $\mu_i^2=1$.

§ 3. Reflection of Wave

(a) Examples of dispersion curves in the propagation in the region of electron have been already given by Taylor.⁽⁵⁾ In the case of propagation in the ion-electron mixture, as can be seen from the expression (10), it is convenient firstly to draw the dispersion curve for propagation in the electron region and secondly to add the influence of ions. It is clear that dispersion curves are similar to those in the electron alone. If Z_e is not neglected the real of the refractive index does not come to zero. This tendency becomes greater as the collisional frequency increases for a certain wave. $\frac{\mu^2}{\mu_i^2}$ becomes zero when

$$\left. \begin{aligned} (1-l)\frac{x_e}{\mu_i^2} &= 1 - iZ_e, & (A) \\ (1-l)\frac{x_e}{\mu_i^2} &= 1 + y - iZ_e, & (B) \\ (1-l)\frac{x_e}{\mu_i^2} &= 1 - y - iZ_e. & (C) \end{aligned} \right\} \quad (11)$$

(A) corresponds to the ordinary wave and (B) and (C) correspond to the extra-ordinary waves. If $\mu_i^2=1$, then these refer to the electron region. Further when $Z_e=0$, (11) is expressed as follows: (Z is neglected)

$$\left. \begin{aligned} x_e &= 1.5 \left(1 - \frac{x_i}{1+l x_i} \right), & (A)' \\ x_e &= 1.5 \left(1 - \frac{x_i}{1+l x_i} \right) (1+y), & (B)' \\ x_e &= 1.5 \left(1 - \frac{x_i}{1+l x_i} \right) (1-y). & (C)' \end{aligned} \right\} \quad (11)'$$

If we consider that a reflection of the wave occurs when the refractive index is zero, even if $x_e=0$, the reflection is possible when $\frac{x_i}{1+l x_i}=1$.

Generally, it is considered that the collisional frequency of electron cannot be neglected at the level where the ionic density is remarkably large (though the collisional frequency of an ion is small in comparison with that of electron and completely neglected), and so (11)' cannot be, as it is, used at the level of E region or below the E region, but it is used in the $F1$ and $F2$ regions. For the magnitude of x_i in latter case the concerning wave frequency is very large and x_i does not easily exceeds 1 for the wave penetrated to these regions under the normal condition. For example, in

Table 1

y	N_i
0.5	6.32×10^8
1.0	1.51×10^8
10	1.51×10^6
100	1.51×10^4

Table 1, orders of N_i which makes $x_i=0.1$ are shown. The wave that attains to the $F1$ and $F2$ regions is about 4 mc/s ($y=0.37$) and so we need 1.09×10^9 /c.c. as the order of N_i . If the electron density is 1.09×10^5 /c.c., λ is about 10^4 and this is too great to accept. Even the wave reflected from the E region, N_i should be about 1.5×10^8 and x_i does not exceeds 1 unless we adopt Booker-

Berkner's and Scott's suggestion for λ .

When y exceeds 10, order of N_i which makes x_i great abrupt decreases and therefore the effect of ions becomes greater for the lower frequency wave, which is reflected at the level far below the E region. Consequently, we mainly need to study the effect of ions on the wave of low frequency.

(b) When $Z_e \neq 0$, real part of refractive index has not the value of zero or infinity. These examples are shown Fig. 1, 2, and for $y=10, 100, Z_e=1$, and 10 with imaginary parts without the influence of ions. Fig. 1 shows also curves including the

Fig. 1 Curves showing the real and imaginary part of the complex refractive index, including the influence of ions for the wave $y=10$. (Extraordinary wave). Hair lines and hair dotted lines show values of $R_e(\mu)$ and $I_m(\mu)$ in ion-electron mixture.

Fig. 2 Curves showing the real and imaginary part of the complex refractive index, including the influence of ions for the wave $y=10$. (Ordinary wave) Hair lines and hair dotted lines show values of $R_e(\mu)$ and $I_m(\mu)$ respectively in the ion-electron mixture.

Fig. 3 Curves showing the real and imaginary part of the complex refractive index for the wave $y=100$ in the case of large collisional frequency of electron.

influence of ions which has a uniform distribution through the region. Hair lines and dotted hair lines show real part of μ ($R_e(\mu)$) and imaginary part of μ ($I_m(\mu)$). Full lines and dotted full lines show $R_e(\mu_e)$ and $I_m(\mu_e)$. Owing to the unreasonable assumption $R_e(\mu)$ is far below unity as soon as the wave reaches to the ionosphere. It is natural that dispersion curves, when Z_e is small, approach to those of $Z_e=0$. Ordinary wave for the low frequency appears to be reflected at higher

electron density than it is really reflected, especially, for $y=100$ the ordinary wave appears to be reflected from $x_e=50$, though its presumed electron density necessary to reflect is 1.5. But this will be immediately broken when distributions of the reflected energy are computed as shown in § 4.

As the ionic density increases the reflection point of two components of waves

descends gradually to the point of the lower electron density than those in the case of electron alone. This implies that in the normal distribution of electron the height of reflection decreases. Further the absorption coefficient for a certain electron density increases because there the effect of ions adds. But considering the passage of the wave through the region it follows that the total absorption of the received wave will be remarkably diminished. As shown in Fig. 6, 7 and 8, the reflected energy in the propagation through the ionized region will increase. Variation of the reflection height of the wave when $Z_e \neq 0$, is estimated from the following approximate expression as

$$x_e = (1 \pm ky) \frac{3}{2} \cdot \frac{3 - 2x_i}{3 + x_i} \quad \begin{matrix} k=0 & \text{for ordinary wave} \\ k=1 & \text{for extraordinary wave.} \end{matrix} \quad (12)$$

If the ionic region where the wave of low frequency is reflected has slow gradient for the electron density to the reflection level, the variation of reflection height reaches to the order of a few kilometer for extraordinary wave while for the ordinary wave this is neglected. For example, the extraordinary wave of $y=10$ is reflected at the level of 4500/c.c. of electron density and if this level lies at the maximum electron density and the region has the form of Chapman's distribution which has the scale height of 6 km, then the variation of the height reaches about 9 km in the case of $x_i=1$, but if this region has maximum electron density of 10^4 /c.c. the variation is about 2 km.

Table 2

x_e	x_i
0.2	0.812
0.4	0.647
0.6	0.500
0.8	0.368
1.0	0.250
1.2	0.142
1.4	0.045
1.5	0

Hence for the wave reflected from the tail of the ionized region of great electron density this variation is negligible.

Table 2 shows the ratio of enough electron density of reflect the incident wave in the presence of ions to the electron density in the absence of ions. As mentioned above, the influence of ions easily increases for the very long wave and so the electron deduced from the data of the observation of very long wave by the well-known relation $N = Af^2$ will probably be incorrect and rather may be greater than it is. Electron density assumed by Wilkes *et al.*,⁽⁶⁾ Bates and Seaton,⁽⁷⁾ Bremmer⁽⁸⁾ and Mitra⁽⁹⁾ must be reexamined.

(c) As the ionic density is really not uniform through the ionized region, dispersion curves have a more complicated form. If we assume the ionic density is expressed as $x_e \lambda$ and $\lambda = \lambda_0 e^{-\alpha z}$ ($\alpha=1, 2, \dots$) and the electron density is distributed in a form of Chapman, then x_i is given by

$$x_i = \frac{4\pi e^2 N_0 \lambda_0}{m_i \omega^2} e^{\frac{1}{2}(1 - (2\alpha + 2) - e^{-2})}. \quad (13)$$

For a certain λ_0 , N_0 , and α the dispersion curves are shown in Fig. 4.

Fig. 4 Variation of the real part of the refractive index for assumed λ_0 , α and electron density distribution for the wave $y=10$.

As can be seen in these figures, if x_i varies sharply and has the maximum value, $R_e(\mu)$ decreases abruptly at the lower electron density than at the normal where the reflection occurs. In these points the abnormal reflection may occur, and if the considerable energy after a partial reflection can penetrate to the point of normal reflection, a second reflection occurs. If the first reflection is remarkably great, the second reflection will be unable to occur. If a partial reflection is possible, three or four reflection wave referred to the ordinary and extraordinary wave can be recorded.

Fig. 5 The reflection point of the wave in the case of introduction of the ion.

In order to examine whether the double reflection occur or not for one kind of wave it is convenient to use the $x_e - x_i$ curves as shown in Fig. 5 in which dotted lines show the points of the normal reflection and full lines show the distribution of x_i . The reflection occurs at the neighbourhood of the intersection of two curves and even if two curves do not intersect the reflection occurs when the x_i curve approaches near to the dotted one. Therefore if a curve firstly approaches the dotted curve and secondly intersects with this, then the double reflection is possible. To establish this condition

it is necessary to examine the reflected energy.

§ 4. Energy of Wave, Energy of Partial Reflection

The energy of wave is expressed by the real part part of the complex poynting vector which is given by

$$\left. \begin{aligned} \mathbf{S} &= \frac{c}{4\pi} [\mathbf{E} \times \mathbf{H}] = \frac{c}{8\pi} [(E_y \tilde{H}_x - E_x \tilde{H}_y) \mathbf{i} + (E_z \tilde{H}_x - E_x \tilde{H}_z) \mathbf{j} + (E_x \tilde{H}_y - E_y \tilde{H}_x) \mathbf{k}] \\ &= \frac{c}{8\pi} [-E_z \tilde{H}_y \mathbf{i} + E_z \tilde{H}_x \mathbf{j} + (E_x \tilde{H}_y - E_y \tilde{H}_x) \mathbf{k}], \end{aligned} \right\} \quad (14)$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$, are unit vectors in the directions of x , y , z axis and $\tilde{A}$ represents conjugate of A . Using the relation (7)

$$\mathbf{S} = \frac{c}{8\pi} [-E_z \mu \tilde{E}_x \mathbf{i} + E_z (-\mu \tilde{E}_y) \mathbf{j} + \{E_x \mu \tilde{E}_x - E_y (-\mu \tilde{E}_y)\} \mathbf{k}], \quad (14)'$$

Substituting of (8) into (14), then we obtain

$$\left. \begin{aligned} S_x &= -\frac{c}{8\pi} \mu (1 - \mu) |E_x|^2 \frac{\sum N e z}{\sum N e x}, \\ S_y &= -\frac{c}{8\pi} \mu (1 - \mu^2) |E_y|^2 \frac{\sum N e z}{\sum N e y}, \\ S_z &= \frac{c}{8\pi} \mu [|E_x|^2 + |E_y|^2]. \end{aligned} \right\} \quad (15)$$

Real parts of $S_i (i = x, y, z)$ is the required energy which flows in three directions. In

order to simplify (15) we make the following operations. As $\frac{\sum Nez}{\sum Nex}$, $\frac{\sum Nez}{\sum Ney}$ are very complicated expressions and we must at first represent them in a simplified form. Before doing this it is convenient to start from the representation $\frac{\sum Nez}{\sum Ne(x,y)}$ for the electron alone. In this case

$$\frac{\sum Nez}{\sum Nex} = \frac{(z)_e}{(x)_e}, \quad \frac{\sum Nez}{\sum Ney} = \frac{(z)_e}{(y)_e}, \quad \mu = \mu_e,$$

where $(x)_e$, $(y)_e$ and $(z)_e$ represent components of $\mathbf{r}$ for electron.

From (5) and (8) we get

$$\left. \begin{aligned} (x)_e \left[iZ_e - 1 - lx_e - \frac{x_e}{1 - \mu_e} \right] - iy_L(y)_e + iy_T(z)_e &= 0, \\ (x)_e iy_L + \left(iZ_e - 1 - lx_e + \frac{x_e}{1 - \mu_e^2} \right) (y)_e &= 0, \\ -x_e iy_T + (iZ_e - 1 - lx_e + x_e)(z)_e &= 0. \end{aligned} \right\} \quad (16)$$

From (16) we get

$$\frac{(z)_e}{(x)_e} = \frac{iy_T}{iZ_e - 1 - lx_e + x_e}, \quad \frac{(z)_e}{(y)_e} = -\frac{y_T}{y_L} \frac{iZ_e - 1 - lx_e + \frac{x_e}{1 - \mu_e^2}}{iZ_e - 1 - lx_e + x_e}. \quad (17)$$

Substituting (17) into (15), we get

$$\left. \begin{aligned} (S_x)_e &= -\frac{c}{8\pi} \mu_e (1 - \mu_e) |E_x|^2 \frac{iy_T}{iZ_e - 1 - lx_e + x_e}, \\ (S_y)_e &= \frac{c}{8\pi} \mu_e (1 - \mu_e^2) |E_y|^2 \frac{iZ_e - 1 - lx_e + \frac{x_e}{1 - \mu_e^2}}{iZ_e - 1 - lx_e + x_e} \frac{y_T}{y_L}, \\ (S_z)_e &= \frac{c}{8\pi} \mu_e [|E_x|^2 + |E_y|^2]. \end{aligned} \right\} \quad (18)$$

Further we get following relations

$$\left. \begin{aligned} \frac{x_e}{1 - \mu_e^2} &= 1 + lx_e - iz_e - \frac{(1/2)y_T^2}{1 - (1-l)x_e - iz_e} \pm \sqrt{\frac{y_T^4/4}{(1 - (1-l)x_e - iz_e)^2} + y_L^2}, \\ \frac{|E_x|^2}{|E_y|^2} &= \left| \left\{ -\frac{1}{2}y_T^2 \pm \sqrt{\frac{1}{4}y_T^4 + y_L^2(1 - (1-l)x_e - iz_e)^2} \right\} / \left\{ iy_L(1 - (1-l)x_e - iz_e) \right\} \right|^2. \end{aligned} \right\} \quad (19)$$

It is evident then that if we neglect the collisional frequency of electron the real part of $(S_x)_e$ in (18) is zero no energy is flowing in the x direction. If we do not neglect Z_e , the energy in the three directions exists except special cases which are discussed in § 5. If ions exist in the region $\frac{\sum Nez}{\sum Nex}$ and $\frac{\sum Nez}{\sum Ney}$ are different from those in electron alone, then (18) must be modified. Though terms $\frac{\sum Nez}{\sum Nex}$, $\frac{\sum Nez}{\sum Ney}$ in (15) are very complicate they are simplified by an approximate method. In an exact calculation $(x)_i$, $(y)_i$ and $(z)_i$ must be deduced from similar three equations as (15) for electron. But as is shown in (5) the differences in form of $(x, y, z)_i$ and $(x, y, z)_e$ are controlled by the collisional frequency ν_e , ν_i and the mass of particle m_e , m_i . Ratio of $\frac{\nu_i}{\nu_e}$ is about 1/40 and $\frac{m_i}{m_e}$ is about 5×10^4 , then if we neglect the difference between ν_i and ν_e , we can put

$$\left. \begin{aligned} (x)_i &= k(x)_e, \quad (y)_i = k(y)_e, \quad (z)_i = k(z)_e \\ \sum Nex &= (Nex)_i + (Nex)_e = (1+k)(Nex)_e \\ \sum Ney &= (1+k)(Ney)_e \end{aligned} \right\} \quad (20)$$

$$\Sigma Nez = (1+k)(Nez)_e$$

hold approximately. Consequently it is given that

$$\frac{\Sigma Nez}{\Sigma Nex} = \frac{(z)_e}{(x)_e}, \quad \frac{\Sigma Nez}{\Sigma Ney} = \frac{(z)_e}{(y)_e}. \quad (21)$$

This implies that influence of ions is approximately zero in the case of ratio. Therefore we can also use (17) in an ion-electron mixture. The influence of ions in the energy is introduced by replacing μ_e by μ . Then the energy in the ion-electron mixture is given by real part of

$$\left. \begin{aligned} S_x &= -\frac{c}{8\pi} \mu(1-\mu^2) |E_x|^2 \frac{iy}{iz_e - 1 - lx_e + x_e}, \\ S_y &= \frac{c}{8\pi} \mu(1-\mu^2) |E_y|^2 \frac{iZ_e - 1 - lx_e + \frac{x_e}{1-\mu_e^2} y_T}{iZ_e - 1 - lx_e - x_e} \frac{y_T}{y_L}, \\ S_z &= \frac{c}{8\pi} \mu [|E_x|^2 + |E_y|^2]. \end{aligned} \right\} \quad (18)$$

If we neglect the collisional frequency we get

$$\frac{(S_x)_e}{S_x} = \frac{\mu_e(1-\mu_e^2)}{\mu(1-\mu^2)}, \quad \text{etc.}, \quad (22)$$

and if we introduce the collisional frequency we get

$$\frac{R_e(S_x)_e}{R_e(S_x)} = \frac{R_e \left\{ \mu_e(1-\mu_e^2) \frac{(z)_e}{(x)_e} \right\}}{R_e \left\{ \mu(1-\mu^2) \frac{(z)_e}{(x)_e} \right\}} E_{0,x} e^{-2\int(\chi_e - \chi)dz}, \quad \text{etc.}, \quad (23)$$

where $E_{0,x}$ is the initial amplitude of the incident wave in the x direction. The χ_e and χ represents the absorption coefficient. The energy at any level in the ionized region is given by the addition of $R_e(S_i)$, that is

$$R_e(\mathbf{S}) = [\{ R_e(S_x) \}^2 + \{ R_e(S_y) \}^2 + \{ R_e(S_z) \}^2]^{1/2}. \quad (24)$$

In this case, the energy reflected in the ionized region on the way till the primary wave attains to a certain level, is excluded, hence the variation of principal energy is due to the absorption in the region of same level. Generally the ionized region is composed of the electron distributed continuously and no has the sharply bounded discontinuities. Therefore the reflection does not occur on the plane in the region, but it occurs through a region. But the principal energy is reflected mainly over a range of limited narrow regions. In order to estimate approximately we divide this region in a narrow region which has the smaller breadth than the semi-wave length of the wave and there the electron density is constant through out, and assume the reflection occurs at the lower boundary of this narrow region. The energy of the wave incident in each region is the energy given by (24). The reflection coefficient of the energy is given by

$$\frac{(n_2 - n_1)^2 + \left(\frac{c\chi_2}{\omega} - \frac{c\chi_1}{\omega} \right)^2}{(n_2 + n_1)^2 + \left(\frac{c\chi_2}{\omega} + \frac{c\chi_1}{\omega} \right)^2} (=R), \quad (25)$$

where n_2, n_1 is the real part of μ in a narrow region 2 and 1, and $\frac{c\chi_2}{\omega}, \frac{c\chi_1}{\omega}$ is the imaginary part of μ , χ_2, χ_1 is the absorption coefficient. The energy of the wave reflected from the upper boundary of the narrow region 1 is given by

$$2R_e(S) \cdot R = 2R_e(S) \frac{(n_2 - n_1)^2 + \left(\frac{c\chi_2}{\omega} - \frac{c\chi_1}{\omega} \right)}{(n_2 + n_1)^2 + \left(\frac{c\chi_2}{\omega} + \frac{c\chi_1}{\omega} \right)^2} \quad (26)$$

For simplicity, we assumed that the energy reflected is not absorbed on the way to the receiver and also it is not reflected upward from the point below the level of the first reflection. The computation of $R_e(S)$ is very labourious and so we take the special cases that there is no flow in x, y directions. These special places lie in the points of dip = 0° and 90° . Then the energy is given by

$$R_e(S) = R_e(S_z) = \frac{c}{8\pi} R_e(\mu) \{ |E_x|^2 + |E_y|^2 \} e^{-2f\chi dz} \quad (27)$$

Fig. 6 Reflection coefficient of the incident energy and reflected energy for ordinary and extraordinary waves of $y=10$.

Fig. 7 Reflection coefficient of the incident energy and reflected energy for ordinary and extraordinary waves of $y=100$, when $Z_e=10$.

Fig. 8 Reflection coefficient of the incident energy and reflected energy for ordinary and extraordinary waves of $y=100$, when $Z_e=10$.

The ratio of the energy at a certain level to the energy of incident wave is shown in Fig. 6, 7 and 9 and at the same time the reflected energy is also shown. For a comparison, μ for two kinds of waves are shown in the hair line. In these figures we assume that the electron density is distributed linearly and the thickness of the ionized region is about 15 km which is considered to be greater than the expected thickness. From these computations it

is evident that the energy of the wave is reflected at a presumed electron density where the variation of the refractive index is large. For the ordinary wave in Fig. 8 the explanation mentioned before does not hold and we must take another explanation that the absorption at a far distance from the normal reflection point, increases sharply as the wave penetrates in the ionized region and the energy reached there is

very small. It is too clear that the effect of ions is to increase the energy of the reflection.

We can estimate by the same method the energy of the partial reflection for the then ordinary and extraordinary waves. The magnitude of the energy of reflection depends on the point of the abrupt variation of the refractive index and the frequency of the wave. For example, if μ decreases by 0.2 in the region of the breadth of semi-wave length as soon as the wave enter in the ionized region the sufficient energy to receive is given. This phenomenon may be possible in the practical observation and we may be able to receive the double reflection. But for the very long wave, as these reflection points approach each other it may be difficult to distinguish the double reflection from the image of the receiver.

§ 5. Deviation of Energy in the Ionized Region

In connection with the flow of the energy in three directions, the deviation of the energy in the ionized region is an interesting problem. This is deduced from (18) or (18'). If we take the ratio $R_e(S_x)$ to $R_e(S_z)$ and $R_e(S_y)$ to $R_e(S_z)$, the ratio of deviation of energy is given by

$$\left. \begin{aligned} \frac{R_e(S_x)}{R_e(S_z)} &= \frac{-R_e\{\mu(1-\mu^2)|E_x|^2 i y_T / (iZ_e - lx_e + x_e - 1)\}}{R_e\{\mu[|E_e|^2 + |E_y|^2]\}}, \\ \frac{R_e(S_y)}{R_e(S_z)} &= \frac{R_e\{\mu(1-\mu^2)|E_y|^2 y_T \left((i - Z_e - lx_e + x_e - \frac{x}{1-\mu_e^2}) / y_L (iZ_e - lx_e + x_e - 1) \right)\}}{R_e(\mu[|E_x|^2 + |E_y|^2])}. \end{aligned} \right\} \quad (28)$$

If Z_e is neglected the deviation in the x direction is zero and (28) is changed to

$$\left. \begin{aligned} \frac{R_e(S_x)}{R_e(S_z)} &= \frac{(1-\mu^2)|E_x|^2 i y_L / (x_e - lx_e - 1)}{|E_x|^2 + |E_y|^2} = \frac{(1-\mu^2) i y_L / \{x_e(1-l) - 1\}}{\frac{|E_y|^2}{|E_x|^2} + 1}, \\ \frac{R_e(S_y)}{R_e(S_z)} &= \frac{(1-\mu^2) y_T \left(x_e - lx_e - \frac{x_e}{1-\mu_e^2} \right) / (x_e - lx_e - 1)}{1 + \frac{|E_x|^2}{|E_y|^2}}. \end{aligned} \right\} \quad (29)$$

Further we take the following special cases as $\text{dip}=0^\circ$ and 90° , then

$$\text{For } \text{dip}=0 \quad \frac{S_y}{S_z} = \frac{S_x}{S_z} = 0 \quad \text{for ordinary and extraordinary waves,}$$

$$\text{For } \text{dip}=90^\circ \quad \frac{S_y}{S_z} = \frac{S_x}{S_z} = 0 \quad \text{for ordinary and extraordinary waves,}$$

These hold without omitting the collisional frequency of electron. The sign of the ratio is given in the northern hemisphere as follows

$$\begin{aligned} Z_e=0 \quad \frac{R_e(S_y)}{R_e(S_z)} &= \begin{array}{l} + \text{ for ordinary wave,} \\ - \text{ for extraordinary wave,} \end{array} \quad \frac{R_e(S_x)}{R_e(S_z)} = 0 \quad \text{for two waves,} \\ Z_e \neq 0 \quad \frac{R_e(S_y)}{R_e(S_z)} &= \begin{array}{l} + \text{ or } - \text{ for ordinary wave,} \\ - \text{ or } + \text{ for extraordinary wave,} \end{array} \quad \frac{R_e(S_x)}{R_e(S_z)} = \begin{array}{l} - \text{ or } + \text{ for two waves.} \end{array} \end{aligned} \quad (30)$$

It is evident that the ordinary wave deviates to the north and extraordinary wave to the south in the northern hemisphere and *vice versa* in the southern hemisphere in

the absence of the collisional frequency of electron. If the collisional frequency is not neglected this rule holds only soon after the incidence in the ionized region, but becomes invalid as the wave passes through the region and the deviation ratio changes the sign from + to - or from - to + depending on the magnitude of the electron collisional frequency, hence the energy flows often to the opposite direction. In Fig. 9 the deviation ratio when Z_e is zero is shown for $y=1/2$ and 10 in the region of electron. Curves for $y=10$ is different from those for $y=1/2$ and do not increase always as the electron density increases. Fig. 10 shows the deviation ratio in the presence of Z_e . Two waves change the sign and especially the extraordinary wave

Fig. 9 The magnitude of the deviation ratio and the direction of deviation of two components of the wave in the northern hemisphere in the absence of the collisional frequency of electron.

Fig. 10 The magnitude of the deviation ratio and the direction of deviation of two components of the wave in the northern hemisphere in the presence of the collisional frequency of electron.

twice. In the presence of ions this ratio for a certain electron density increases, but does not change the sign. Though the ratio of the deviation is large, the total deviation distance through the ionized region is rather small because the reflection height decreases. If the electron density is distributed linearly the area below the curve equals to the total deviation distance, and it is clear that the deviation becomes smaller.

When there is no collisional frequency it is given that

$$\frac{\{R_e(S_y)/R_e(S_z)\}_{i \rightarrow e}}{\{R_e(S_y)/R_e(S_z)\}_e} = \frac{1-\mu^2}{1-\mu_e^2}. \quad (31)$$

§ 7. Virtual Height

As the reflection height descends by the effect of ions, the virtual height may become smaller than that under the absence of ions. This problem is dealt with by the same method as the case of the electron. The group velocity of the wave is

$$u = \frac{1}{\frac{d}{d\omega} \left(\frac{\omega}{v} \right)} = \frac{c}{\frac{d}{d\omega} (\omega\mu)}, \quad (32)$$

where v is the phase velocity. Then the height from the base of the ionized region is given by

$$ct = \int \left(\mu + \omega \frac{d\mu}{d\omega} \right) dh = \int p dh, \quad (33)$$

where h is the coordinate of height. From (10) we obtain

$$\mu = \mu_i^2 \left\{ 1 - \frac{\delta / \mu_i^2}{\varphi} \right\}^{1/2}, \quad (34)$$

where φ is given by

$$\varphi = (\omega^2 + l\delta / \mu_i^2 - i\omega\nu_e) - \frac{\frac{1}{2}\omega^2\omega_H^2\sin^2\alpha}{\omega^2 - (1-l)\frac{\delta}{\mu_i^2} - i\omega\nu_e} \pm \sqrt{\frac{\frac{1}{4}\omega^4\omega_H^4\sin^4\alpha}{\left(\omega^2 - (1-l)\frac{\delta}{\mu_i^2} - i\omega\nu_e\right)} + 4\omega^2\omega_H^2\sin^2\alpha}. \quad (35)$$

If we differentiate μ with ω we obtain

$$\frac{d\mu}{d\omega} = \frac{1}{2} \cdot \mu_i^2 \frac{\frac{1}{\varphi^2} \left(\frac{\delta}{\mu_i^2} \right) \frac{d\varphi}{d\omega}}{\mu}. \quad (36)$$

Further

$$\frac{1}{\varphi} = \frac{\mu_i^2 - \mu^2}{\delta},$$

$$\frac{1}{\varphi} \left(\frac{\delta}{\mu_i^2} \right) = \frac{\mu_i^2 - \mu^2}{\mu_i^2}.$$

Thus

$$\frac{d\mu}{d\omega} = \frac{(\mu_i^2 - \mu^2)^2 \frac{1}{\delta} \cdot \frac{d\varphi}{d\omega}}{2\mu}. \quad (37)$$

$\frac{d\varphi}{d\omega}$ is given by

$$\begin{aligned} \frac{d\varphi}{d\omega} = \omega \left[2 - iZ_e - \frac{y_T^2 [1 - (1-l)x_e - iZ_e] - \frac{1}{2} y_T^2 (2 - iZ_e)}{1 - (1-l) \frac{x_e}{\mu_i^2} - iZ_e} \right. \\ \left. \pm \frac{1}{2} \frac{y_L^2 + \frac{y_T^4 \left\{ 1 - (1-l) \frac{x_e}{\mu_i^2} - iZ_e \right\} - \frac{1}{2} y_T^4 (2 - iZ_e)}{(1 - (1-l) \delta / \mu_i^2 - iZ_e)^3}}{\left\{ \frac{1}{4} y_T^4 + \left\{ 1 - (1-l) \frac{x_e}{\mu_i^2} - iZ_e \right\}^2 + y_L^2 \right\}^{1/2}} \right]. \quad (38) \end{aligned}$$

The height from the base is given in (33), (37) and (38) by the numerical integration.

Fig. 11 The magnitude of $\mu + \frac{d(\mu, \omega)}{d\omega}$ for extraordinary wave for $y=10$.

Fig. 12 The magnitude of $\mu + \frac{d(\mu, \omega)}{d\omega}$ for ordinary wave for $y=10$.

$p = \left(\mu + \omega \frac{d\mu}{d\omega} \right)$ is shown in Fig. 11, 12 where it is known that the collisional frequency of electron influences rather than ions. Though p is greater a little for a certain x_e in ion-electron mixture than that in electron alone, the difference of the virtual height is rather smaller. Especially, this tendency increases for large ionic density. Hence we regard that the virtual height of reflection of low frequency wave is in great measure correct. In the cases of $\text{dip}=0^\circ$ and 90° (37) represented by

$$\begin{aligned}
 \text{dip}=0^\circ \quad R_e \frac{d(\mu\omega)}{d\omega} &= R_e \left[\mu + \frac{(\mu_i^2 - \mu^2)(2 - iZ_e)}{2\mu x_e} \right] \quad \text{ordinary wave} \\
 &= R_e \left[\mu + \frac{(\mu_i^2 - \mu^2)2 - iZ_e - \frac{y_T^2 \left\{ -\frac{4}{3} x_e / \mu_i^2 - iZ_e \right\}}{\{1 - (1-l)x_e / \mu_i^2 - iZ_e\}^2}}{2\mu x_e} \right] \quad \text{extraordinary wave} \\
 \text{dip}=90^\circ \quad R_e \frac{d(\mu\omega)}{d\omega} &= R_e \left[\mu + \frac{1}{2} \frac{(\mu_i^2 - \mu^2)(2 - iZ_e \pm 1)}{\mu x_e} \right] \quad \begin{array}{l} \text{for upper sign} \\ \text{ordinary wave} \\ \text{for lower sign} \quad \text{extraordinary wave} \end{array}
 \end{aligned} \quad (39)$$

§ 7. Conclusion

It is known that influences of ions become greater as the frequency of the wave decreases. By the existence of ions the absorption of wave decreases and the energy of the reflection become larger considerably than that in the case of electron alone. In the region of great number of ions, the estimation of the electron density must be reexamined. Deviation of the incident energy in the ionized region and the virtual height of reflection level is reduced. Especially, for a very long wave virtual height approaches closely to the true height.

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APPENDIX

On the Lorentz Polarization Term

1. On the studies of Several Workers

It is studied by several workers whether the Lorentz polarization term must be introduced or not in the equation of motion. Martyn and Munro⁽¹⁰⁾⁽¹¹⁾ (M and $\bar{M}$) assert the introduction of this term and Booker and Berkner⁽²⁾ (B and $\bar{B}$) assert the introduction. Eckersley⁽¹²⁾ suggests that the phenomenon of existence of the critical frequency near 1.4mc/s is explained by the consideration that this occurs $y_L=1$ and it is not important to persist the polarization term. Appleton, Farmer and Ratcliffe (A , F and R)⁽¹³⁾ also discussed this problem. Their conclusion are as follows;

(a) sort of wave causing a retardation

M and $\bar{M}$ =ordinary wave by a second reflection

B and $\bar{B}$ =extraordinary wave

2. Mechanism of retardation Phenomenon

In the explanation of the retardation phenomenon it is necessary to note five defects mentioned above and to know the behaviour of the extraordinary wave near $(y_L \text{ or } y_T)=1$. Some examples are shown in Fig. 13 and 14.

It is easily found that the extraordinary wave behaves abnormally in a small range near $(y_L \text{ or } y_T)=1$. Collisional frequency of electron also plays an important role there, and as the collisional frequency increases the maximum of $R_e(\mu)$ becomes smaller as can be seen in Fig. 13.

If we neglect Z_e the refractive index is given by

$$\mu^2 = 1 - \frac{x_e}{1 + lx_e - \frac{1}{1 - (1-l)x_e} \left[\frac{1}{2} y_T^2 \pm \left\{ \frac{1}{4} y_T^4 + y_L^2 (1 - (1-l)x_e)^2 \right\}^{\frac{1}{2}} \right]} \quad (40)$$

For the infinity of the refractive index the expression is given by

$$\left(1 + \frac{1}{3} x_e\right)^2 \left(1 - \frac{2}{3} x_e\right) - y_T^2 \left(1 + \frac{1}{3} x_e\right) - y_L^2 \left(1 - \frac{2}{3} x_e\right) = 0. \quad (41)$$

Relation between y_L and y_T is easily determined by knowing the inclination and so from (41) the relation between y_T or y_L and x_e (consequently y and x_e) to make the refractive index infinity, is determined. This relation in the presence of l differs from that in the absence of l , that is, in the former the value of y exceeds generally 1 in the low or moderate electron density. On the other hand, in the latter, the value of y does never exceeds 1. In other words, in the former the wave frequency to cause the retardation is greater or smaller than the gyro-frequency in the retarding region, and in the latter the wave frequency is greater than the gyro frequency. It is evident that the wave frequency of retardation in two cases depends on the electron density of the retarding region. Details are shown in Fig. 15.

From the results mentioned above we explain the phenomenon of critical frequency near 1.4 mc/s as follows (we consider the case of $\text{dip} = 60^\circ$);

(i) When the frequency of the wave is lower than about 1.4mc/s, the extraordinary part of the wave does not cause the retardation in the E region for the reason that the wave frequency is far below the gyro-frequency in the E region and it probably penetrates the E region though the penetration of this wave depends on the magnitude of the electron density of the E region.

Fig. 15 Curves showing the relation between y and x_e which makes the refractive index infinity. Dotted lines show the case $l=1/3$ and full lines show the case $l=0$.

(ii) When the frequency of the wave is 1.38mc/s (Washington) or 1.4mc/s (Sydney), the wave reaches to the lower part of the *F2* region (probably below 300 km) with the electron density 5×10^4 /c.c. and there the wave is reflected in the absence of the special condition. But at this height the magnitude of the gyro-frequency is about 1.43mc/s or more, hence from Fig. 20 the retardation occurs at height with the electron density less than 5×10^4 /c.c. when we use $l=1/3$ (dip= 60°) but does not occur when $l=1$.

(iii) If the frequency of the wave increases to about 1.48mc/s the retardation of the wave in the *E* region also occurs. But there the absorption of the wave is very strong so that the wave is not clearly received.

(iv) As mentioned above (ii) we are forced to accept the case $l=1/3$ in order to accord with the observation. This leads to the result that the disappearance of the retardation in lower latitudes (about below dip= 45°) is due to the fact that there the condition is similar to that of $l=0$, that is, the retardation occurs when $y < 1$.

(v) As a result the frequency of the wave causing the retardation is greater as the inclination increases.

These retardation mechanism is different from that of other workers is the following point while similar points exist.

(i) The retardation is caused in the lower part of the *F2* region under a certain condition derived from a relation between y and x_e . This varies depending on the situation of the electron density, then the frequency of wave causing the retardation slightly varies.

(ii) The magnitude of the frequency of wave causing the retardation is different in each latitude for the reason that y varies considerably for different latitudes.

(iii) The reason that the retardation of wave near 1.4mc/s is not observed is such that the value of y causing the retardation is smaller than unity in the lower latitude below 45° of the inclination, and this corresponds to the case of $l=0$.

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