Frequency Response of the Transfer Functions of the Current Channelling between Two Oceans

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Several current channelling models between two oceans are presented for studying the frequency responses of the transfer functions and the anomalous horizontal magnetic field in the 'E-polarization' case (i.e., the inducing magnetic field is perpendicular to the direction of the channel). The results obtained from this numerical investigation give support to the idea that at longer periods, the lack of frequency dependence of the transfer functions and the anomalous horizontal magnetic field is the characteristic consequence of the channelling effect of the oceanic induced currents. On the contrary, the corresponding 2-D local induction models show that the frequency responses of the transfer functions and the anomalous horizontal field are quite sensitive to the period of the inducing field at the longer periods. When there is no highly conducting layer lying under the surface structure, we have found another property which arises from the channelling effect of the oceanic induced currents; that is, the vertical magnetic component is nearly in-phase with the normal magnetic field. Likewise, the horizontal component of the anomalous magnetic field will also be nearly in-phase with the normal field. However, when a conducting layer exists below the ocean and its buried depth is not very deep, the in-phase properties stated above no longer hold. Furthermore, we investigate the behaviour of the oceanic current in a channelling model for the 'B-polarization' case (i.e., the inducing magnetic field is parallel to the channel) at one period, $T=10000$ s. The result also shows that the anomalous magnetic field is nearly in-phase with the normal magnetic field, provided that no highly conducting layer lies under the surface structure.

1. Introduction

In the past ten years, the current channelling effect has been a topic of controversy. A critical review on the problem of current channelling was given by JONES (1983), who pointed out that "too many workers have not sufficiently considered the effects of 'local' induction, and have, perhaps, postulated 'currents' induced 'elsewhere' flowing ohmically through their region to describe their observations".

SUMMERS (1981, 1982) was the first worker to suggest that the concept of current channelling is not required to explain the property of the frequency independence of some inland anomalies and much of the data from differential geomagnetic soundings

(such as those observed by BABOUR and MOSNIER, 1979; VASSEUR *et al.*, 1977). He argued that these properties can be explained in terms of local induction models. To support his argument, he presented two numerical investigations of the near-surface and highly conductive anomaly (SUMMERS, 1982). Along the same lines, HEBERT (1983) and DUPIS and THÉRA (1982) proposed that the differential geomagnetic measurements obtained by BABOUR and MOSNIER (1980) can be also explained in terms of a 2-D local induction model.

Though the linearly-polarized anomalous magnetic fields can be satisfactorily explained by a 2-D induction model, we consider that Summers' main conclusion, that the frequency independence of the single station transfer functions can be explained in terms of 2-D local induction, is incorrect (CHEN and FUNG, 1986). As to the works of Hebert and Dupis and Théra, we think that their model calculation results cannot be compared satisfactorily with the observation results obtained by BABOUR and MOSNIER (1980) (CHEN and FUNG, 1987, submitted).

On the other hand, we incline, to a certain extent, to agree with McKirdy and Weaver's viewpoint. They pointed out that "the use of arguments based on strictly 2-dimensional induction is to beg the question of whether or not current channelling is actually taking place" (MCKIRDY and WEAVER, 1983). In fact, the possibility of the existence of the channelling effect has been studied in an earlier 3-D numerical investigation by WEIDELT (1977), who concluded that under suitable conditions the channelling effect definitely existed and could be more important than the local induction effect. MCKIRDY and WEAVER (1983) carried out another 3-D numerical modelling study. Their model comprises two oceans linked together by a conducting channel. One ocean occupies a half-plane and the other is confined to a quarter-plane. This asymmetry was deliberately designed in order to investigate the case when the regional inducing magnetic field is parallel to the channel. In such a situation the local induction effect should be absent, but the channelling effect of the oceanic induced currents could take place. We like to draw attention to two of their conclusions:

(i) The current channelling effect becomes more important with increasing period, but the local induction effect dominates at short periods.

(ii) They found that based on their model, currents are channelled and the quadrature current is significantly larger than the in-phase current component (Figs. 2-5 in their article).

Therefore, it is evident that if we want to distinguish which effect, current channelling or local induction, is responsible for producing the observed anomalous field, we should study the behaviour of these two effects at the longer period range. Following the works of McKirdy and Weaver, we proceed in this paper the numerical study of the frequency responses of the transfer functions and other relevant quantities in the case that the channelling effect of the oceanic induced currents dominates. The frequency responses of the same quantities for the strictly 2-D local induction models are also calculated for comparison.

2. The 3-D Finite Difference Method

The algorithm for solving the 3-D induction problem used here is the finite difference method generalized from the 2-D finite difference method of BREWITT-TAYLOR and WEAVER (1976) with a slight modification. In this method we did not use the concept of sharp boundaries and assumed the conductivity σ be a smoothly-varying function of position. The basic equation describing the spatial variation of the magnetic field $\mathbf{B}$ (in SI units) is

$$\nabla^2 \mathbf{B} + \frac{1}{\sigma} (\nabla \sigma) \times (\nabla \times \mathbf{B}) = i\omega\mu_0\sigma\mathbf{B}, \quad (1)$$

where ω is the angular frequency of the inducing field, μ_0 is the magnetic permeability in free space and the field is assumed to vary in time as $e^{i\omega t}$. Equation (1) may be written as three scalar equations in a rectangular Cartesian coordinate system:

$$\nabla^2 B_x + \frac{1}{\sigma} \left[\frac{\partial \sigma}{\partial y} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) - \frac{\partial \sigma}{\partial z} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) \right] = i\omega\mu_0\sigma B_x, \quad (2)$$

$$\nabla^2 B_y + \frac{1}{\sigma} \left[\frac{\partial \sigma}{\partial z} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) - \frac{\partial \sigma}{\partial x} \left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y} \right) \right] = i\omega\mu_0\sigma B_y, \quad (3)$$

$$\nabla^2 B_z + \frac{1}{\sigma} \left[\frac{\partial \sigma}{\partial x} \left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x} \right) - \frac{\partial \sigma}{\partial y} \left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z} \right) \right] = i\omega\mu_0\sigma B_z. \quad (4)$$

The first derivative of the magnetic component is expressed by the central-difference approximation, which is the weighted average of the forward-derivative and the backward derivative at the node (l, m, n) . The second derivative is defined as the linear gradient between the forward derivative and the backward derivative.

Since we do not use the sharp boundary concept, we now define the conductivity $\sigma_{l,m,n}$ at the node (l, m, n) to be the weighted average value of the eight neighbouring conductivities, $\sigma_{l+1/2,m+1/2,n+1/2}$, $\sigma_{l+1/2,m+1/2,n-1/2}$, $\sigma_{l+1/2,m-1/2,n+1/2}$, ...:

$$\begin{aligned} \sigma_{l,m,n} = & (R_{l,m,n} + R_{l-1,m,n} + R_{l,m-1,n} + R_{l-1,m-1,n} \\ & + R_{l,m,n-1} + R_{l-1,m,n-1} + R_{l,m-1,n-1} \\ & + R_{l-1,m-1,n-1}) / (p_{l-1} + p_l)(h_{m-1} + h_m)(k_{n-1} + k_n), \end{aligned} \quad (5)$$

where p_{l-1} , p_l , h_{m-1} , h_m , k_{n-1} and k_n denote the six adjacent grid spacings, and

$$\left. \begin{aligned} R_{l,m,n} &= p_l h_m k_n \sigma_{l+1/2,m+1/2,n+1/2}, \\ R_{l-1,m,n} &= p_{l-1} h_m k_n \sigma_{l-1/2,m+1/2,n+1/2}, \\ &\vdots \end{aligned} \right\} \quad (6)$$

In addition, we define $\partial\sigma/\partial x$ as a linear gradient in the x -direction between the appropriate weighted average values of conductivity in y and z directions; for example:

$$\left(\frac{\partial\sigma}{\partial x}\right)_{l,m,n} = \frac{2}{(p_{l-1} + p_l)(h_{m-1} + h_m)(k_{n-1} + k_n)} \times \left(\frac{R_{l,m,n} + R_{l,m,n-1} + R_{l,m-1,n} + R_{l,m-1,n-1}}{P_l} - \frac{R_{l-1,m,n} + R_{l-1,m,n-1} + R_{l-1,m-1,n} + R_{l-1,m-1,n-1}}{p_{l-1}} \right). \quad (7)$$

Using the definition stated above, Eqs. (2)–(4) can be expressed in their finite difference forms. Here we only write one equation; for example, Eq. (2) becomes

$$\begin{aligned} & 2a_l^p(B_x)_{l+1,m,n} + 2b_l^p(B_x)_{l-1,m,n} + (2 - h_{m-1}\eta_{l,m,n})a_m^h(B_x)_{l,m+1,n} \\ & + (2 + h_m\eta_{l,m,n})b_m^h(B_x)_{l,m-1,n} + (2 - k_{n-1}\zeta_{l,m,n})a_n^k(B_x)_{l,m,n+1} \\ & + (2 + k_n\zeta_{l,m,n})b_n^k(B_x)_{l,m,n-1} + p_{l-1}a_l^p\eta_{l,m,n}(B_y)_{l+1,m,n} \\ & - p_lb_l^p\eta_{l,m,n}(B_y)_{l-1,m,n} + p_{l-1}a_l^p\zeta_{l,m,n}(B_z)_{l+1,m,n} \\ & - p_lb_l^p\zeta_{l,m,n}(B_z)_{l-1,m,n} \\ & = (d_{l,m,n} + c_m^h\eta_{l,m,n} + c_n^k\zeta_{l,m,n} + i\omega\mu_0\sigma_{l,m,n})(B_x)_{l,m,n} \\ & - c_l^p\eta_{l,m,n}(B_y)_{l,m,n} - c_l^p\zeta_{l,m,n}(B_z)_{l,m,n}, \end{aligned} \quad (8)$$

where

$$\left. \begin{aligned} \xi_{l,m,n} &= \frac{1}{\sigma_{l,m,n}} \left(\frac{\partial\sigma}{\partial x} \right)_{l,m,n}, \\ \eta_{l,m,n} &= \frac{1}{\sigma_{l,m,n}} \left(\frac{\partial\sigma}{\partial y} \right)_{l,m,n}, \\ \zeta_{l,m,n} &= \frac{1}{\sigma_{l,m,n}} \left(\frac{\partial\sigma}{\partial z} \right)_{l,m,n}, \end{aligned} \right\} \quad (9)$$

and the coefficients $a_l^p, b_l^p, c_l^p, \dots, d_{l,m,n}$ are the functions of the grid spacings around the node (l, m, n) ; the detailed relations between $a_l^p, b_l^p, c_l^p, \dots$ and $p_l, h_m, k_n, \dots$ can be found in the paper of CHEN (1985).

The advantage of solving the equation expressed by B , which is used in this paper, is that the boundary values of the magnetic field in the air region can simply be taken to be constant. Besides, at the ground surface ($n=q$, say) all the $\sigma_{l,m,q}$'s are not equal to zero, hence the equations can be solved without any difficulty. For solving

the finite difference equations, we do not use the matrix inversion. The well known Gauß-Seidel iterative method is used in this study.

The accuracy obtained by our computer program has been tested by making calculations for Weidelt's model (WEIDELT, 1975) and Ting and Hohmann's model (TING and HOHMANN, 1981). The comparisons of our numerical results and their numerical results are described in the Appendix. Generally speaking, those results obtained by different methods agree well for the most part of the curves shown in Fig. 11 and the patterns shown in Fig. 12.

Since we use the finite difference method for the numerical calculation, there is no limitation on designing the channelling model, in contrast to a surface anomalous channel presented by MCKIRDY and WEAVER (1983) whose numerical method is based on the thin sheet approximation. Therefore, we present here several models: surface current channelling model with or without an underlying highly conducting layer and sub-surface channelling model. However, we should also consider certain limitations on model constructions. For solving induction problem in terms of $\mathbf{E}$, the finite-difference method of LINES and JONES (1973a, b), is suitable for models which are characterized by a 2-D configuration at large distances in the two directions parallel to the inducing electric field (i.e., the uniform source field). On the contrary, for solving induction problem in terms of $\mathbf{B}$, the finite-difference method used here, is suitable for models which correspond to B -polarization limit at the two side boundaries in the directions parallel to the magnetic source field. Therefore, when we design the models used in this paper (described in Sections 3 and 5), we must impose limitation on the side boundary stated above.

Different meshes were designed for different models. The largest mesh consists of 15525 ($27 \times 23 \times 25$) grid points, in order to ensure that the distances between the outside boundaries (both in x and y directions) and the channel be larger than 3 skin-depths of the underlying medium. The successive over-relaxation techniques (SOR) was used and the relaxation factor was chosen to be 1.4. The calculations for the shorter periods were stopped after 600 iterations, and the calculations for the longer periods were stopped after 520 iterations. The convergence process is too slow in the short period case, so that we cannot carry out the calculations for the periods shorter than 200 s.

3. The Frequency Response of the Transfer Functions

3.1 *A simple surface channelling model*

In this model the narrow channel connects two separate oceans. In order to reduce the direct induction of the channel itself, the conductivity of the channel is taken to be 0.1 S/m, the reasonable value of a sedimentary conductor. The length of the channel is 100 km, its width is 20 km and its thickness is 2 km. The conductivity and the thickness of the two oceans are taken to be 4 S/m and 2 km respectively. The conductivity of the land and the base medium is assumed to be 0.002 S/m. The top view of the configuration is shown in Fig. 1.

The solid line in Fig. 2 shows the R_u - T and R_v - T curves at the position P. R_u and

Fig. 1. Top view of the configuration of the first model—a surface channelling model. The conductivity of the base medium is taken to be 0.002 S/m, the same as that of the land region.

Fig. 2. The R_u - T and R_v - T plots (solid lines) at location P for the surface channelling model between two oceans shown in Fig. 1. The dashed lines represent the R_u - T and R_v - T plots for the corresponding 2-D local induction model.

R_u denote the values of the real and imaginary Parkinson arrows, respectively. P is located at the central point of the edge of the channel (see Fig. 1). The inducing period ranges from 200 s to 10000 s. The inducing magnetic field is in the x -direction, perpendicular to the direction of the channel (i.e., ' E -polarization' case). Due to symmetry, the eastward component of the anomalous magnetic field at P should be equal to zero. Therefore, the Parkinson arrows at P are perpendicular to the direction of the channel. Here we reverse the sign of the real transfer function to meet the convention of the real Parkinson arrow R_u , but do not reverse the sign of the imaginary transfer function for constructing the imaginary Parkinson arrow R_v (CHEN and FUNG, 1985).

Figure 2 also shows the R_u - T and R_v - T plots (dashed lines) for a strictly 2-D local induction model. This 2-D model for comparison is constructed by removing the two oceans (described in Fig. 1) and extending infinitely the length of the channel.

From Fig. 2, we can see that R_u and R_v for the channelling model between two oceans decrease more slowly than those for the strictly 2-D local induction model when the inducing period increases. R_u and R_v for this particular channelling model appear to be frequency independent when $T \geq 5000$ s (while the directly induced currents in the channel can be disregarded altogether). At the same period range (200–10000 s), R_u and R_v for the 2-D local induction model appear to be rather strong functions of the period, the magnitude of R_u or R_v is nearly proportional to the frequency of the inducing field. Besides, the real and imaginary transfer functions tend to have the same magnitude for the local induction model. This result implies that the phase difference between the vertical magnetic field B_z and the total horizontal magnetic field B'_x approaches the value of $\pi/4$ at longer period range. However, the real transfer function for the channelling model tends to a constant magnitude when the inducing period increases, but the imaginary transfer function approaches a rather small value. In other words, the induced vertical field is nearly in-phase with the normal field at longer periods.

3.2 A subsurface channelling model

The structure of the model is shown in Fig. 3. The plan view of the configuration is indicated in Fig. 3(a). Figure 3(b) is the vertical section view over the symmetric axis c - d . The central part in Fig. 3(a) represents a land area with a width 70 km, which divides the ocean into two separate parts. The depth of the ocean is assumed to be 2 km and its conductivity is taken to be 4 S/m. The subsurface channel is 30 km wide, 100 km long and 4 km thick; it is embedded at a depth of 2 km, just below the ocean and partly connects the two oceans at both sides. The conductivity of the channel is taken to be 0.1 S/m. The conductivity σ_0 of the land and the base medium are assumed to be 0.002 S/m.

Similar to Fig. 2, the solid line in Fig. 4 shows the R_u - T and R_v - T plots at the location P. The inducing magnetic field is also in the x -direction, perpendicular to the direction of the channel. For comparison, we also plot the R_u - T and R_v - T curves of a corresponding 2-D local induction model (see the dashed lines in Fig. 4). This 2-D model is constructed by removing the ocean and extending infinitely the length of the

Fig. 4. The R_u - T and R_v - T plots (solid lines) at location P for the sub-surface channelling model shown in Fig. 3. The dashed lines show the R_u - T and R_v - T plots for the corresponding 2-D local induction model.

the situation in the surface channelling model shown in Fig. 1. The main reason is that the channel brings in not only the surface currents in the ocean, but also part of the 'leakage' currents, which means that the vertical currents flow out or into the ocean sheet (using McKirdy and Weaver's terminology). In this subsurface channelling model, the length of the channel is shorter and the area of the cross section is larger; these two factors also enhance the channelling effect.

3.3 A Channelling model with a highly conducting layer below the surface structure

The top view of this model is the same as the first model, a surface channelling model. In this model, however, an underlying highly conducting layer exists below the surface structure. The conductivity of this layer is assumed to be 0.1 S/m and its buried depth is taken to be 50 km. The purpose of this numerical modelling is to examine the influence of the presence of the underlying highly conducting layer on the transfer functions at the ground surface.

Figure 5 shows the R_u - T curves and R_v - T curves for this model and the corresponding 2-D model at the position P. The R_u - T and R_v - T graphs for the channelling model appear to be frequency independent when $T \geq 5000$ s, similar to

Fig. 5. The R_u - T and R_v - T plots (solid lines) at location P for the model in which an underlying highly conducting layer exists. The conductivity of the underlying layer is assumed to be 0.1 S/m and its buried depth is 50 km. The dashed lines represent the R_u - T and R_v - T plots for the corresponding 2-D local induction model.

those shown in Fig. 2. However, the magnitudes of R_u and R_v in this model tend to have the same value 0.035 as the period increases, implying that the phase difference between B_z and B_x^t approaches $\pi/4$ at the longer periods.

As to the 2-D local induction model, the magnitude of R_v becomes greater than that of R_u ($|R_v| \approx 2|R_u|$ for this particular model), though the behaviour of the transfer functions is frequency dependent in general. $|R_v/R_u| \approx 2$ implies that the phase difference $\Delta\phi$ between B_z and B_x^t becomes greater than $\pi/4$; in this particular model the asymptotic value of $\Delta\phi$ is about 64° .

All of the results for the transfer functions in the three models described above indicate that the channelling effect dominates at the longer periods and the behavior of the transfer functions appears frequency independent (or less frequency dependent), as expected. However, the strictly 2-D local induction effect is characterized by a frequency dependent property for the transfer functions. At the longer period range, redistribution of the "DC-like" current can be used to interpret the channelling effect. Therefore, the phase difference $\Delta\phi$ between B_z and B_x^t in the channelling effect between two oceans reflects the phase property of the oceanic

induced currents. If there is no highly conducting layer lying under the ocean, the induced currents in the ocean should be nearly in-phase with the inducing magnetic field (PRICE, 1967). Basically, the phase difference $\Delta\phi$ between B_z and B_x^i in the channel region can be regarded alternatively as the phase difference $\Delta\phi'$ between the normal electric field E_y^n and the normal magnetic field B_x^n outside the channel (i.e., in the ocean) at the longer periods. The latter phase difference $\Delta\phi'$ tends to zero when the underlying highly conductive layer is absent. Otherwise, the phase difference $\Delta\phi'$ of E_y^n and B_x^n should tend to other values, as shown in the case of the third model. This property can be found in magnetotelluric sounding (KAUFMAN and KELLER, 1981). When the model is a three-layer structure and belongs to K type (i.e., $\sigma_1 > \sigma_2 < \sigma_3$, where σ_1 , σ_2 and σ_3 denote the conductivities of the first, the second and the third layers respectively), such as at the ocean part of the third model, the phase difference $\Delta\phi'$ between E_y^n and B_x^n can be estimated by the formula (KAUFMAN and KELLER, 1981)

$$\Delta\phi' \approx \cot^{-1} \frac{2\pi \cdot 10^{-3} \sqrt{\rho_3 / 10T}}{(8\pi^2 \cdot 10^{-7} H / T) + (2\pi \cdot 10^{-3} \sqrt{\rho_3 / 10T})} \quad (10)$$

at longer period range; where H denotes the buried depth of the third layer and ρ_3 is the resistivity of the third layer.

Therefore, the in-phase behaviour of B_z and B_x^i may be destroyed by the existence of the underlying highly conducting layer if its buried depth is not deep enough (say, less than 100 km).

4. The Frequency Response of the Anomalous Horizontal Field

We plot the $(B_x^a / B_x^n) - T$ curves at the central point Q of the channel (see Figs. 1 and 3) for the first, second and third models in Figs. 6, 7 and 8 respectively. In each figure, the left part is the frequency response of the real anomalous horizontal field and the right part indicates the imaginary anomalous horizontal field. We can draw similar conclusions for B_x^a / B_x^n as those for the transfer functions discussed in the preceding section:

(1) B_x^a / B_x^n appears frequency independent at the longer periods for all channelling models. On the contrary, B_x^a / B_x^n is definitely frequency dependent for strictly 2-D local induction models.

(2) When there is no highly conducting layer lying under the ocean (say 50 km below the ocean surface), B_x^a is almost in-phase with B_x^n . However, in the case of the third model, the phase difference between B_x^a and B_x^n tends to 45° .

In fact, in the channelling models the behaviour of B_x^a should be inherently consistent with the behaviour of the transfer functions. The reason is that both of them arise from the same induced oceanic currents which merge in the channel. We think that these channelled currents should be called anomalous currents, though they are normal currents in the ocean.

Fig. 6. The $\text{Re}(B_x^a/B_x^n)$ - T and $\text{Im}(B_x^a/B_x^n)$ - T plots for the surface channelling model shown in Fig. 1. The dashed lines represent the $\text{Re}(B_x^a/B_x^n)$ - T and $\text{Im}(B_x^a/B_x^n)$ - T plots for the corresponding 2-D model.

Fig. 7. The $\text{Re}(B_x^a/B_x^n)$ - T and $\text{Im}(B_x^a/B_x^n)$ - T plots (solid lines) at location Q for the sub-surface channelling model shown in Fig. 3. The dashed lines represent the $\text{Re}(B_x^a/B_x^n)$ - T and $\text{Im}(B_x^a/B_x^n)$ - T plots for the corresponding 2-D induction model.

Fig. 8. The $\text{Re}(B_x^a/B_x^0)$ - T and $\text{Im}(B_x^a/B_x^0)$ - T plots at location Q for the surface channelling model in which an underlying highly conducting layer exists. The dashed lines represent the $\text{Re}(B_x^a/B_x^0)$ - T and $\text{Im}(B_x^a/B_x^0)$ - T plots for the corresponding 2-D induction model.

5. The Channelling Model in Which the Inducing Magnetic Field is Parallel to the Channel

In order to examine the behaviour of the anomalous currents in the case where the inducing magnetic field is parallel to the channel, we present another model, the top view of which is shown in Fig. 9. The thickness of the ocean is taken to be 2 km, and the conductivity of the underlying half space is taken to be 0.001 S/m. The channel which connects the two oceans is a sea-water path. No deep conductive layer exists in this model.

Figure 10 shows the pattern of the anomalous horizontal magnetic fields B_h^a in the channel and its vicinity. The magnitude of B_h^a is relative to that of the normal field B^n . B^n is the field at the centre of the land when y approaches the right (or left) side boundary. The period of the inducing field is taken to be 10000 s. The real and imaginary parts of B_h^a are represented by the pointed arrow and the flat-headed arrow respectively. We can clearly see that the oceanic induced currents are definitely channelled through the channel (i.e., parallel to the direction of the inducing magnetic field). B_h^a at the entrance of the channel points to the south direction and has a significant value, being 0.08. This fact implies that the anomalous currents flow eastwards into the channel from one ocean to another ocean. Since B_h^a is nearly in-phase with B^n , the anomalous current is also in-phase with the normal field B^n . It has been reported recently, by the direct observation at the Cook Strait between the

Fig. 9. Top view of the configuration of the surface channelling model for the ' B -polarization' case, while the inducing magnetic field is parallel to the direction of the channel. The conductivity of the base medium is taken to be 0.001 S/m.

Fig. 10. The pattern of the anomalous horizontal component B_h^a at the grid points of the channel region. The pointed arrows represent the in-phase fields and the flat-headed arrows represent the imaginary fields. The normal field B^n is parallel to the channel (y -direction), and the period of the inducing field is 10000 s.

two main islands of New Zealand (BOTELER *et al.*, 1987), that the anomalous current is in-phase with the normal field.

Strictly speaking, there should be no locally induced currents parallel to the direction of the channel, since the inducing magnetic field is parallel to the channel. However, the local induction phenomenon should still exist in the x -direction. This local induction mechanism will drive a cross-channel current parallel to the x -direction and partly leak into the land regions. Since the arrows of B_h^a in the mid-way of the channel are pointing south-west, as shown in Fig. 10, we consider that there are two current components, one due east and the other due south, so that the superimposed current is pointing south-east. The east-ward current is simply the channelling which flows from the ocean in the "left" to that in the "right". The south-ward current is the local induction current.

Unfortunately, we cannot obtain the frequency response of B_h^a and the transfer functions for this model in the case that the inducing field is parallel to the channel. Even for the period of 10000 s, the convergence speed of the iteration process is so slow that we cannot obtain accurate values of the magnetic fields after 600 iteration times. The magnetic fields B_h^a shown in Fig. 10 can only be treated qualitatively. However, we think that the conclusion, that B_h^a is nearly in-phase with B^n , is right.

6. Conclusions

The numerical calculations for the channelling models between two oceans clearly indicate that the current channelling effect dominates over the local induction effect at the longer periods, while the response of transfer functions or anomalous horizontal magnetic fields appears to be frequency independent. On the contrary, the corresponding 2-D local induction models show that the responses of transfer functions and anomalous horizontal magnetic fields are quite strong functions of the period of the inducing field.

Moreover, if there is no highly conducting layer lying under the surface structure, the vertical magnetic component will be nearly in-phase with the normal magnetic field, i.e., the magnitude of the imaginary Parkinson arrow will be much smaller than that of the real Parkinson arrow. The horizontal component of the anomalous magnetic field will also be in-phase with the normal magnetic field. However, when a conductive layer exists as the bottom layer under the ocean and its buried depth is not very deep (being < 100 km say), the in-phase properties stated above no longer hold.

The behaviour stated above on the transfer functions and the anomalous horizontal magnetic fields are obtained from the 'E-polarization' case (i.e., the inducing magnetic field is perpendicular to the direction of the channel). In the 'B-polarization' case (i.e., the inducing magnetic field is parallel to the channel), the situation is more complicated. Figure 10 shows that in the middle region of the channel the local induction mechanism drives a cross-channel current perpendicular to the channel direction. However, the current which flows along the channel can be attributed entirely to the channelling oceanic current. The channelling oceanic currents as well as the south-ward components of the anomalous fields B_h^a (in the particular model shown in Fig. 10) appear to be nearly in-phase with the normal field B^n when $T = 10000$ s, if there is no highly conducting layer lying under the ocean. The frequency independence property of the transfer functions and B_x^a may also be expected at the longer periods for the channelling effect in the 'B-polarization' case; however, we cannot carry out the calculations for $T < 10000$ s at the moment, because our numerical method is not powerful enough and the convergence speed of the iterations is too slow.

In brief, the frequency independent behaviour of the transfer functions at longer periods can help us to discriminate the channelling effect of the oceanic induced currents from the directly local induction effect. Moreover, if the observation data reveal that B_z and B_x^i (or B_x^a and B_z^n) are about in-phase at longer periods, then it can be deduced that this property is due to the channelling effect of the oceanic induced

current; in addition, we can infer that no highly conducting layer exists below the surface structure in this situation.

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Appendix 1. Comparison of the Results of B_z/B_n Obtained by WEIDELT (1975) and our modelling results

WEIDELT (1975) carried out the numerical calculations for a set of outcropping anomaly models using the integral equation method. The model configurations are shown in the upper part of Fig. 11. The widths in these models are assumed to be 15 km, 25 km, 50 km, 125 km and ∞ . The lower part of Fig. 11 shows the ratio B_z/B_n along the central profile for different models. The lower left part of Fig. 11 is Weidelt's

Fig. 11. The comparison of our numerical results and Weidelt's results. The upper part shows the model configurations (redrawn from Fig. 8 of Weidelt's article, 1975). The lower left part shows the ratio B_z/B_n along the central profile for different models obtained by WEIDELT (1975), and the lower right part shows our numerical results for B_z/B_n . The period is 120 s.

results (taken from Fig. 8 of his article) and the lower right part of Fig. 11 is ours. Clearly, the two results are well consistent when the width L_x of the anomaly is not very large. However, our result for model 4 ($L_x = 125$ km) appears to be somewhat inconsistent with Weidelt's result. The possible reason could be that too elongated cells were designed for model 4 in our numerical modelling.

Appendix 2. Comparison of the Results of 'Tipper' Value Obtained by TING and HOHMANN (1981) and Our Modelling Results

TING and HOHMANN (1981) define the 'tipper' magnitude to be

Fig. 12. The comparison of our numerical results and Ting and Hohmann's results for the 'tipper' magnitude (the definition of 'tipper' is explained in the text). The upper part shows the plan view and the vertical section view of the model presented by Ting and Hohmann. The lower left part shows the spatial distribution of 'tipper' magnitude obtained by TING and HOHMANN (reproduced from Fig. 17 of their article, 1981). The lower right part shows that obtained by our computer program. The period is 0.1 s, and the contour interval is 0.02.

$$|T_p| = [|a|^2 + |b|^2]^{1/2},$$

where a and b are the single station transfer functions, which satisfy

$$B_z = a B_x + b B_y.$$

The model designed by TING and HOHMANN is now shown in the upper part of Fig. 12, which is duplicated from Fig. 10 of their article (1981). Their numerical results calculated by the integral equation method are shown in the lower left part of Fig. 12 (reproduced from Fig. 17 of their article). Our numerical results are shown in the lower right part of Fig. 12. One can see that these two plots are very similar. Mostly, the values of $|T_p|$ obtained by us are little smaller than those obtained by them, but we think that the results obtained from integral equation method may be slightly overestimated for a large anomalous conductivity body. In this particular model, the size of the anomaly ($=1 \times 2 \times 2 \text{ km}^3$) is very large compared to the skin depth ($=0.35 \text{ km}$) for the anomalous conductivity when the frequency is taken to be 10 Hz.

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