A method to estimate geomagnetically-induced currents (GICs) in a power grid based directly on magnetic recordings is presented. To determine the electric field on the earth’s surface for the calculation of GICs based on available magnetic data, a model is used in which the geomagnetic variation field propagates as a vertical plane wave in the earth and the earth is a half-space with a constant conductivity. To get statistically reliable results, the earth’s conductivity is allowed to be a function of the geomagnetic K-index. The reason for this unphysical behaviour, which means that the conductivity is, of course, not real but some kind of an effective conductivity, is explained.

1. Introduction

The electric field connected with geomagnetic variations causes ohmic quasi-dc currents, so-called geomagnetically-induced currents (GICs), in power transmission systems, pipelines, railways and telecommunication networks. GICs, which may be harmful to the system in question (e.g. LANZEROTTI, 1979), are especially large at auroral latitudes, where geomagnetic disturbances or storms are most intense. In addition to the location at auroral latitudes, the low conductivity of the earth increases the horizontal electric field and GICs in Scandinavia. That is why the study of the phenomenon is of great practical interest there (PERSSON, 1979).

Geomagnetic induction in the Finnish 400 kV power system has been studied for several years as collaboration between the Imatran Voima Oy power company and the Finnish Meteorological Institute. GICs have been measured directly in earthing leads of transformer neutrals (PIRJOLA, 1983). Their values have also been computed theoretically. PIRJOLA and LEHTINEN (1985) assumed the electric field on the earth’s surface to be either 1 V/km to the east or 1 V/km to the north. No calculations concerning real geomagnetic events were made. PIRJOLA (1985) considered only one magnetic storm, and the magnetic data he used were incomplete; also, the earth’s conductivity was obviously too large ($10^{-2} \Omega^{-1} m^{-1}$). In another investigation by Pirjola, the choice of the conductivity was made by finding the best least-square fit between the measured and calculated GICs in a four-hour event.
In this paper, which is based on Viljanen (1987, in Finnish), theoretical calculations and fittings of the data are made in a more careful and thorough way, avoiding the shortcomings of previous works. In addition, more new physical interpretation is included while the method of the computation of the electric field from magnetic data is essentially the same as that used by Pirjola (1985). The method of fitting the conductivity is new.

GIC measurements have been carried out mainly at one station, Huutokoski in central Finland (see Fig. 1). Hence, reliable statistics concerning the occurrence of GICs in the whole grid cannot be based on these measurements only. It should also be noted that a sunspot cycle (ca. 11 years) is the minimum time for statistical conclusions in connection with geomagnetic phenomena. Because the Finnish power grid has changed significantly during the Huutokoski measurements since 1977, no definite statistics based on GIC recordings can be made yet.

However, the existence of continuous geomagnetic data of a much longer time gives a possibility of estimating the occurrence of GICs by “converting” geomagnetic statistics to GIC statistics using a suitable model. As Lehtinen and Pirjola (1985) have shown, the calculation of GICs is straightforward using the circuit theory after the electric field connected with a geomagnetic variation is known. Thus, the main problem in the calculation of GICs is the determination of the electric field, which is a geophysical task. In this paper we use the simplest model, based on the vertical plane wave assumption about the geomagnetic variation and the electric field in the earth, and on the description of the earth by a homogeneous half-space. Geomagnetic data collected at the Nurmijärvi Geophysical Observatory, situated in southern Finland, is used to determine the horizontal electric field.

As is well known, the plane wave model used in this paper is somewhat questionable near auroral latitudes. We, however, believe that definite qualitative, and at least partly quantitative, conclusions of the occurrence of GICs in the Finnish grid can be obtained even with the present model, as will be seen in Section 3. It should also be kept in mind that the plane wave model evidently overestimates GICs (Albertson and Van Baelen, 1970).

An essential parameter in the model is the conductivity of the earth. To obtain reliable statistics, the conductivity is fitted in this paper to give the best agreement between the theoretical and measured GIC data at Huutokoski, and then used in the determination of GICs in the whole Finnish 400 kV power system. It appears that the conductivity has to be a function of the intensity of the geomagnetic disturbance, i.e., of the geomagnetic K-index (Lincoln, 1967). This is, of course, unphysical and states that the model is too rough and the conductivity has to be regarded as effective.

2. Method of Computation

The earth is described as a half-space with a constant conductivity \( \sigma \) and the geomagnetic variation field propagates as a vertical plane wave in the earth. Let us
use a Cartesian coordinate system in which the \( x \)-axis is directed to the north, \( y \) to the east and \( z \) downwards. Taking into account that geomagnetic phenomena are slow the electric field on the surface of the earth is (see CAGNIARD, 1953; PIRJOLA, 1982, p. 23)

![Figure 1. Finnish 400 kV power grid in 1986.](image)
\[ E_y(t) = -\frac{1}{\sqrt{\pi \mu_0 \sigma}} \int_{-\infty}^{t} \frac{g_x(u)}{\sqrt{t-u}} \, du, \]  

(1)

where \( g_x \) denotes the time derivative of the magnetic component \( B_x \). The same relation, of course, holds between \( E_x \) and \( B_y \) if one only changes the sign in front of the integral. We shall later use the notation \( E = E_y, B = B_x \) and \( g = g_x \).

There are two problems in using (1):

(i) How to determine the time derivative of the magnetic field if we only know the average values of a time interval \( \Delta \) (e.g., one-minute values which we used)?

(ii) What is \(-\frac{1}{\pi \sigma} \) in practice?

We assume as a first approximation that a one-minute value of the magnetic field equals the instantaneous value in the middle of the one-minute interval and simply connect the successive values by a straight line. So the magnetic field is

\[ B(t) = B_{n-1} + (B_n - B_{n-1})(t - T_{n-1})/\Delta, \quad T_{n-1} \leq t \leq T_n, \]  

(2)

where \( B_n = B(T_n) = \text{average value of the interval } T_n - \Delta/2, \ldots, T_n + \Delta/2 \) and \( \Delta = T_n - T_{n-1} \) (\( n \) is an integer). The time derivative is then

\[ g(t) = (B_n - B_{n-1})/\Delta, \quad T_{n-1} < t < T_n. \]  

(3)

Now it is easy to show that (1) gives a practical formula

\[ E(T_N) = \frac{2}{\sqrt{\pi \mu_0 \sigma}} (R_{N-1} - R_N - M b_{N-M}), \]  

(4)

where \( b_n = B_n - B_{n-1} \) and

\[ R_N = \sum_{n=N-M+1}^{N} b_n \sqrt{N - n + 1}. \]  

(5)

In principle, \( M \) is infinite. However, analyses of several events showed that a sufficient accuracy (at least better than 5%) is usually achieved by setting \( M = 720 \). In other words, we take into account the magnetic variations in the 12 hour interval before the moment \( T_n \). Pirjola (1985) here used 29 hours, which is unnecessarily long in practice.

After the electric field (4) is known, GICs flowing in the Finnish 400 kV power system can be calculated. In our simple model GIC, which may be either the earthing current at a station or the current in a transmission line, is

\[ I = a E_x + b E_y, \]  

(6)

where \( a \) and \( b \) are constants characteristic to the particular part of the system and depending on the geometry and resistances of the network. Typical values for \( a \) and \( b \) in Finland are 20–70 Akm/V for transformers and about 30–100 Akm/V for
transmission lines (PIRJOLA and LEHTINEN, 1985).

3. Results

As mentioned in Section 1, magnetic data from the Nurmijärvi Geophysical Observatory and GIC data from the Huutokoski station were used. It must be stressed that measured GIC data were available only at Huutokoski. The data used mainly concerned the geomagnetically active years 1982 and 1983. GICs were calculated from (4), (5) and (6), and the results were compared to measurements for several events. We assumed that $I(T_N)$ from (6) equals the average value in $T_N - \Delta/2, \ldots, T_N + \Delta/2$ (cf. the interpretation of the values of the magnetic field).

Serious difficulties arose in choosing a value for the conductivity of the earth in (4), and it was necessary to let the value of the conductivity vary from event to event according to the Nurmijärvi magnetic $K$-index. The absolute values of GIC one-minute means were considered, and the requirement was made that for each $K$-value the numbers of these means exceeding 20 A equal both in theoretical results and measurements at Huutokoski.

It can be pointed out here that CAMPBELL (1978) used the $Ap$-index and LUNDBY et al. (1985) used the $Kp$-index when they made statistical analysis between geomagnetic indices and GICs. However, they did not use such an analytical method to calculate GICs from magnetic data as is presented in this paper. Also, they were not able to estimate the occurrence of GICs as exactly as in this paper.

Based on the method mentioned above, the following values for the (effective) conductivity in Finland are obtained (the subscript refers to $K$):

$$
\sigma_5 = 1.2 \times 10^{-4} \ \Omega^{-1} \text{m}^{-1}, \quad \sigma_6 = 2.4 \times 10^{-4},
$$

$$
\sigma_7 = 3.1 \times 10^{-4}, \quad \sigma_8 = 8.0 \times 10^{-4},
$$

$$
\sigma_9 = 1.0 \times 10^{-3},
$$

and a rough estimate for the smaller $K$-values is $1 \times 10^{-4}$. The accuracy of these values is about $\pm 0.3 \times 10^{-4} \ \Omega^{-1} \text{m}^{-1}$. (The method is explained in details in VILJANEN, 1987.) The conductivities are in the same order as JONES et al. (1983) and HJELT et al. (1986) have reported. Figure 2 is a typical example of computed and measured values. The forms of the curves are qualitatively similar, but quantitative deviations are large at times. Consequently, the given conductivities are not necessarily very good if we consider a single event. Instead, we must have many events under statistical analysis.

Now after the earth’s effective conductivity is known, GICs in any part of the Finnish 400 kV power grid may be estimated based on equations given in Section 2. In Table 1, theoretical GIC results are compared to measurements at Huutokoski in January 1, 1982, to October 31, 1983. We took here about 15 events for every $K$. We did not use exactly the same events in both columns, but this cannot cause a considerable error in practice. As expected, the possibility of high GICs increases
Fig. 2. Comparison of the measured and calculated GIC at the Huutokoski power station on September 22, 1982. The K-index was 7 and the earth's conductivity $2.5 \times 10^{-4} \Omega^{-1} \text{m}^{-1}$.

Table 1. Occurrence of GICs at the Huutokoski power station in January 1, 1982, to October 31, 1983. For each K-index, the left column gives the average time in minutes during which the value of GIC belongs to the given range in a 3 hour K period, and was obtained from measurements. The right column is similar and concerns theoretical results. In the "year" column, the durations (in minutes) of GIC in given ranges for a year are presented.

<table>
<thead>
<tr>
<th>$K$</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>year</th>
</tr>
</thead>
<tbody>
<tr>
<td>GIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20–30</td>
<td>0.2</td>
<td>0.2</td>
<td>1.5</td>
<td>1.6</td>
<td>4.5</td>
<td>5.3</td>
</tr>
<tr>
<td>30–40</td>
<td>0.1</td>
<td>0.0</td>
<td>0.2</td>
<td>0.1</td>
<td>1.6</td>
<td>1.8</td>
</tr>
<tr>
<td>40–50</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.3</td>
<td>0.2</td>
</tr>
<tr>
<td>50–60</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>60–70</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>70–80</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>80–90</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.1</td>
<td>0.0</td>
</tr>
<tr>
<td>Sum</td>
<td>0.3</td>
<td>0.2</td>
<td>1.9</td>
<td>1.7</td>
<td>6.8</td>
<td>7.5</td>
</tr>
</tbody>
</table>
Fig. 3. The vertical axis gives the number of events (event: the absolute GIC value in the given range) per year when GIC has the value given on the horizontal axis at the Pirttikoski power station based on theoretical calculations. The numbers above the columns are average durations of the events in minutes. For example a GIC event 30–40 A occurs about 700 times per year. Each event lasts for one minute on average. Thus, GIC is 30–40 A for 700 minutes per year.

with $K$. The annual duration of a particular GIC value also shown in Table 1 is easy to calculate after the average number of $K$-indices per year is known.

Theoretical values for $K=9$ are overestimated on purpose in Table 1. This is due to the fact that the GIC data of $K=9$ events is insufficient. Especially, we have no measurements of the largest magnetic storm on July 13–14, 1982. According to theoretical calculations, the GIC may have been as high as 400 A at Huutokoski then, corresponding to a horizontal electric field of 7.6 V/km. Theoretical values of this storm are not included in Table 1. Since the determination of the earth's conductivity is based on the same GIC data that is also included in Table 1, the table in fact only gives an idea of the accuracy of this determination.

Complete GIC statistics concerning the whole Finnish 400 kV grid are given by VILJANEN (1987). “Huutokoski in 1982–83” is quite a typical station of the Finnish grid from the GIC point of view. GICs in transformers are normally lower than 100 A, but during magnetic storms ($K=7,...,9$) they can reach 100 A for some minutes at some stations, especially at Pirttikoski (Fig. 3). In a very great storm like that in July, 1982, GIC can even reach 700 A at Pirttikoski and some hundreds of amperes in many other transformers. Such an event statistically occurs at least once during each sunspot cycle. Currents in transmission lines are higher on average than the earthing currents, but in practice they are not harmful.

4. Conclusions

Theoretical estimation of geomagnetically-induced currents in a given network of conductors using geomagnetic recordings requires some model by which the
An electric field connected with a geomagnetic variation can be calculated. In this paper, which deals with the Finnish 400 kV power grid, the simplest model is used, in which the geomagnetic variation and the associated electric field constitute a vertical plane wave in the earth and the earth is a half-space with a constant conductivity. Shortcomings of this simple model are compensated by letting the earth's conductivity change with the intensity of the geomagnetic variation, i.e., with the K-index. This is, of course, unphysical but certainly makes the results much more reliable than those obtained using only one fixed value. Consequently, we have at least correct estimates of the orders of GIC magnitudes that are useful and important in the practical estimation of possible GIC inconveniences.

The exact dependence of the earth's conductivity on the K-index is obtained by comparing calculated GIC results to measured data, and it is found that the conductivity, which has to be regarded as effective "in a GIC sense", increases with K.

A geophysical interpretation of the increase of the conductivity as a function of the K-index is possibly the following: The plane wave model used in this paper gives an upper limit for the electric field and the line current model a lower limit when we study one Fourier component (Albertson and Van BaeLEN, 1970, see also Fig. 4). Since the higher the conductivity the lower the electric field (cf. (1)), it seems that at large K-values the increase of the conductivity compensates an overestimation of the electric field caused by the use of the plane wave model.

It can thus be concluded that the largest K-indices are associated with well-localized auroral currents and smaller K-values with broader current sheets. This conclusion is surprising, but it is, at least qualitatively, supported by figures presented by GraFE (1983) who has studied several magnetic bay disturbances at auroral latitudes. In the figures the width of the electrojet current really seems to become smaller in many events when the disturbance becomes larger, although Grafe does not state this explicitly in his paper. On the other hand, the purpose of our paper is not to make definite conclusions about the complex auroral electrojet current system, and before they can be made much more data and studies would be needed.

Another physical reason for the growth of the conductivity with K is the complexity of the earth's conductivity: it varies both vertically and horizontally (Jones et al., 1983; HjELT et al., 1986). If we consider only one Fourier component of the electromagnetic field, we can describe the earth with one value of the conductivity which depends on the frequency (apparent conductivity). However, a detailed investigation of the geomagnetic spectrum for different K-values would be needed for definite conclusions.

As a result, our estimates are rough due to five main reasons:
(a) The vertical plane wave assumption is not exactly valid near the auroral current system.
(b) The model for the conductivity of the earth in Finland is too simple.
(c) We can compare theoretical results to measurements only at Huutokoski, because of the lack of other GIC data.
Fig. 4. Curve $|E_0|$, UPPER LIMIT represents the amplitude of the electric field on the earth's surface as a function of the time period $T$, when the primary field is a down plane wave, the earth has eight conductivity layers and the amplitude of the magnetic field on the earth's surface is $|B_0|$. Curve $|E_0|$, LOWER LIMIT and the notation $x=0$ METRE belong to a model in which the primary source is a line current. The dashed curve corresponds to $|E_0|$, UPPER LIMIT, but the earth is assumed to be homogeneous with a conductivity $10^{-4} \Omega^{-1} m^{-1}$ (ALBERTSON and VAN BAELEN, 1970; PIRJOLA, 1982, Fig. 2).

(d) Our statistical method is very simple.
(e) We use magnetic data from Nurmijärvi and GIC data from Huutokoski, about 250 km away.

Of course, a more complicated geophysical model can be used to determine the electric field on the earth’s surface. After that, the method presented in this paper for fitting the conductivity can be used. If the dependence of the conductivity on $K$-index vanishes then, one has obviously found rather a good geophysical model.

We would like to thank the Imatran Voima Oy power company for great support in our studies on GICs in the Finnish power system, and for many discussions and useful advice on the topic of this paper.

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