Regional Magnetic Field Modelling: A Review

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Regional models of the earth's magnetic field have developed considerably since the days of hand contouring. They have incorporated varying levels of mathematical sophistication, from partial "mutual consistency" on a surface to full electromagnetic consistency in three-dimensional space. Each method has its advantages and its limitations. Some of the methods allow for radial variation so that data acquired at different altitudes can be analyzed directly, and so that fields from the resulting models can be calculated at any altitude. For other methods, altitude continuation is theoretically possible but numerically difficult and statistically unreliable. Some methods are limited, for numerical reasons, to small areas or are able to represent only fairly long wavelengths. The different methods also differ greatly with respect to their mathematical and computational complexity, the less complex methods also usually being more subjective.

1. Introduction

The purpose of regional geomagnetic field modelling is to represent or describe the geomagnetic field over a portion of the earth's surface, given data over this portion. The data may consist of either component or vector observations, and may include shipborne measurements at sea level, ground observations, aeromagnetic measurements at altitudes up to 4 or 5 km, and satellite measurements such as those of Pogo or Magsat at altitudes of 300 km and higher.

There are requirements for both main field and residual field information. The main field element for which there is the greatest demand is declination, because of its continued use in navigation. However, there is also a large demand—for *all* main field elements and also for residual field information—from petroleum and mineral exploration companies, land surveyors, research scientists and private individuals.

Regional models are usually based on denser data than are global models and are therefore more accurate over their regions than the global models, and their release is hopefully more timely. In addition, they are able to represent wavelengths shorter than those in the world models, and therefore represent not only sources in the core but also some sources in the crust.

The various ways of describing the field may be classified very generally as graphical and analytical. The graphical methods comprise both hand drawn and numerically processed, computer contoured charts while the analytical (or functional) methods comprise surface polynomials, equivalent dipole source methods, and mathematical expansions such as spherical harmonic analysis, rectangular harmonic analysis and spherical cap harmonic analysis. Frequently, even when an analytical model is produced, a secondary model is produced in the form of a contour chart for distribution to

interested users. In fact, this secondary model may be the intended final result, the analytical model simply being an intermediate mechanism for producing the contour chart. Occasionally, when secondary models are produced, a mixture of the graphical and analytical methods may be used. For example, two areas analytically analysed separately may be merged by graphically adjusting contours at the conjoining boundary. Similarly, areas around the geographic and magnetic poles have to be contoured manually, even when there is a single analytical model. This is because there are singularities in X , Y , $\dot{X}$, and $\dot{Y}$ at a geographical pole, in $\dot{H}$ and $\dot{I}$ at a magnetic pole, and in D and $\dot{D}$ at both poles, and these singularities cannot be handled by the usual computer contouring programs.

The earliest "models" of the magnetic field over a region of the earth's surface were simply hand drawn charts. Although their construction was fairly subjective and there were no "consistency" constraints between the components, very small wavelengths could be displayed. Some subjectivity disappeared with numerical processing, although it was still necessary to make subjective choices concerning weighting functions with respect to gridding and contouring. Mathematical or analytical modelling became easy with the advent of the computer, and fitting polynomials to surface data became straightforward. These polynomials usually contained no radial terms, and therefore no allowance could be made for data obtained at different altitudes nor could the final model be upward or downward continued to altitudes other than the mean altitude of the data. These problems became particularly acute with the coming of satellite data, and with the accompanying need to reduce the data to a common altitude and to merge for purposes of analysis the satellite data with aeromagnetic, shipborne and ground data. Techniques such as the equivalent dipole source technique and mathematical expansions designed to satisfy the electromagnetic constraints were developed to serve these purposes.

Previous discussions and reviews of regional modelling have been given by MCFARLAND (1930), WEINBERG (1935), LJUNGDAHL (1937), BERNSTEIN (1944), MALIN (1983), and ALLDREDGE (1987). A comprehensive bibliography of regional charts and methods was given by BARRACLOUGH (1981). Discussions on charting methods of the United Kingdom, the United States, and the U.S.S.R. have been given by WORLD MAGNETIC SURVEY BOARD (1971, pp. 104–110). See also WORLD MAGNETIC SURVEY BOARD (1967, pp. 527–530, 532–536).

2. Graphical Methods

Graphical methods are those that provide an interpolation of the data, either manually or by computer and possibly based on some weighting scheme. They comprise both hand contouring and computer contouring of data that in each case may have previously been processed or filtered numerically. Originally, observations were simply plotted on a map and contoured by hand. Later, particularly as computers began to come into use, some numerical processing began to be done (such as secular variation updating, and data editing and averaging) but the charts were still contoured by hand. Finally, various computer algorithms were developed to grid non-uniformly spaced data and to derive contours from these gridded data. By this time, the numerical processing techniques had also become more sophisticated.

The power of graphical methods lies primarily in their ability to represent short wavelength features. (Of course, these features may deliberately be suppressed, either by

processing procedures or by contouring procedures, if desired.) However, any determination of the field between contours must be estimated from the chart itself or else from an extensive look-up table in the computer by means of some interpolation algorithm. Furthermore, there is no means of correcting data to the altitude of the chart: this must be done by means of some global reference field or perhaps by means of a centered dipole field approximation. Similarly, the final chart applies to only one altitude; it is simply not applicable to other altitudes without some external adjustment. Finally, graphical charts are not geometrically consistent. For example, if charts were independently drawn of the horizontal field H , the vertical field Z , and the total field F , the geometrical constraint $H^2 + Z^2 = F^2$ would clearly not apply.

2.1 *Hand drawn charts*

The earliest existing and probably most famous regional chart is the hand drawn chart produced by Edmond Halley in 1701 (see BAUER, 1896). The contour lines are very smooth, presumably because Halley recognized that the accuracy of the data, and of the data locations, did not warrant the inclusion of any short wavelength features in them.

In fact, it is this very possibility of describing short wavelength features, when warranted, that is the power of the hand contouring method. One of the problems with this method, however, is that it is very subjective. The chart wiggles should represent real features rather than data or position errors. Yet whether they do or not is very difficult for the chart compiler to determine, especially in areas where the data are sparse. A single data point can have too much influence when the contouring is done without prior filtering or smoothing.

Such short wavelength features can be clearly seen, for example, on the 1960 declination chart of the United States (NELSON *et al.*, 1962, Fig. 39). This was the last use of the hand drawn charts of the United States, some of the preceding ones having been described by KNAPP (1942), LUDY and HOWE (1945), and DEEL (1946). Most countries have, at one time or another, produced hand drawn charts (see, e.g., VELDKAMP (1951) for charts of The Netherlands). Instructions for hand contouring have been given by SUCKSDORFF (1981).

Numerical processing methods that have preceded hand contouring have included averaging within equal-area quadrangles (DAWSON and DALGETTY, 1966), smoothing by taking gliding means of 4 values each (SUCKSDORFF *et al.*, 1968), and two-dimensional splining (FINLAYSON, 1973).

2.2 *Computer contoured charts*

The next step in automation, after the numerical processing prior to manually contouring charts, was computer contouring. Wavelengths nearly as small as those of hand drawn charts can be portrayed in this type of chart, the minimum wavelength depending on the data density. The numerically contoured chart is not so subjective as the hand drawn chart, although it is still necessary to make some subjective decisions concerning gridding and contouring. However, computer algorithms can often put in "false" contours giving an appearance of short wavelengths, particularly in regions of sparse data, and it does suffer from the other problems of the hand drawn method.

An example of the use of computer contouring procedures is given by the 1970 charts of Finland (SUCKSDORFF *et al.*, 1971). These charts were produced by a numerical procedure known as "the correction method" used in contouring meteorological param-

eters. Another numerical procedure, known in meteorology as “the method of optimum interpolation”, was used to produce the Finnish charts for 1985 (SUCKSDORFF *et al.*, 1986).

There is another numerical procedure that is used frequently to analyse satellite data, which vary over a wide range of altitudes (from 300 or 400 km to many hundreds of kilometers). In this procedure, one averages over altitude all satellite residual data in given latitude-longitude bins, then numerically filters them if desired, and finally contours the filtered data by computer. The averaging in this case is done on residual data, i.e., data from which some global field has been subtracted. The data correction to a common altitude is generally quite small although still significant relative to the anomaly amplitudes.

An example of the application of this latter procedure is the map of F residuals north of 50°N from Pogo data by LANGE (1976, Fig. 1), a slightly modified version of which has been given by LANGE *et al.* (1981, Fig. 63). Other examples of the use of this procedure include COLES *et al.* (1982), RITZWOLLER and BENTLEY (1982), WON and SON (1982), YANAGISAWA *et al.* (1982), ARKANI-HAMED *et al.* (1985), COLES (1985), and many others.

3. Surface Polynomials

The first analytical method used to produce regional models was that of surface polynomials, by which is meant polynomials that are functions only of colatitude θ and longitude λ , not of radial distance r . For example, the declination D , the horizontal intensity H , and the total field F would be expressed as:

$$\begin{aligned} D &= a_0 + a_1\theta + a_2\lambda + a_3\theta^2 + a_4\theta\lambda + a_5\lambda^2 + \dots, \\ H &= b_0 + b_1\theta + b_2\lambda + b_3\theta^2 + b_4\theta\lambda + b_5\lambda^2 + \dots, \\ F &= c_0 + c_1\theta + c_2\lambda + c_3\theta^2 + c_4\theta\lambda + c_5\lambda^2 + \dots. \end{aligned}$$

Fitting surface polynomials in colatitude (or latitude) and longitude to various components is quite easy and eliminates one of the objections to graphical charts: that of not being able to compute easily the field at a given location. Of course, any of the components can be expressed as polynomials. The inclination I or the vertical component Z could be used in place of or in addition to the polynomials in D , H , and F . Similarly, other surface coordinates could be used in place of θ and λ as the polynomial variables.

The polynomial degree is usually chosen to be very low (3 or less), and so the minimum wavelengths depicted in the models are very long. This may or may not be desirable, depending on the application.

As in the case of graphical methods, there is a possibility of geometrical inconsistency. For the elements H , Z , I , and F , there are geometrical constraints that must be satisfied:

$$\begin{aligned} H^2 + Z^2 &= F^2, \\ \tan I &= Z/H. \end{aligned}$$

When there are observations of no more than three independent components, of course, there is no problem: those three components are modelled, either graphically or analytically, and the other components can be derived from them with no inconsistency. The problem occurs when more than three components are measured, individually: for example, D , I , H , and F . A polynomial can be fit to each of these four components but then there is a geometrical inconsistency between them. And if polynomials are fit to only three of them, the data on the fourth component are being ignored.

The geometrical constraints can be satisfied by imposing conditions on the coefficients, but then the fitting functions are non-linear in their coefficients and what was supposed to be a simple method turns out to be fairly complicated (FUJITA, 1968; SHOJI, 1978). It may be pointed out that this problem of geometrical inconsistency persists even in the more sophisticated models; it is not restricted to polynomials.

There is one further type of consistency constraint: that of electromagnetic theory which requires both the curl and the divergence of the vector magnetic field to be zero in source free regions. This imposes constraints on the polynomials, although for surface polynomials this can only be a partial constraint and only affects the horizontal components. Polynomials that are not so constrained will be discussed first.

3.1 *Unconstrained polynomials*

An early example of simple, or uncoupled, surface polynomials was given by LJUNGDAHL (1934), who computed linear polynomials in D , I , H , X , Y , and Z over Sweden. He also included hand drawn contours for comparison.

An example of combining the polynomial and graphical approaches was given by CULLINGTON (1954), who calculated polynomials in D , H , and I over New Zealand for 1950.5. The country was divided into 10 overlapping zones, or districts. Linear polynomials from each district were then plotted and smoothed graphically between districts (the effect of which can be seen clearly, for example, in the bend of the 18°E isogon on the declination chart). BURROWS (1967) used the same technique over New Zealand for 1960.5. FABIANO *et al.* (1979) used this overlapping polynomial approach for the 1975 charts of the United States, but they devised a numerical weighting scheme so that smoothing between their five zones could be done automatically.

The electromagnetic consistency of the horizontal components of the geomagnetic field over a given region can be checked numerically from the charts themselves (CHAPMAN, 1942; DAVIDS and BERNSTEIN, 1945). In the case of polynomials, the checking can be done analytically, either by taking line integrals and arriving at an average inconsistency over the region (CULLINGTON, 1954) or by taking derivatives and calculating the inconsistency at any location within the region (TSUBOKAWA, 1952; HAINES, 1968). When estimates of the variances and covariances of the unconstrained polynomial coefficients are available, the statistical significance of the inconsistency can also be checked (HAINES, 1968). This "inconsistency" will be discussed more fully in the next subsection.

3.2 *Constrained polynomials*

Electromagnetic theory stipulates that the divergence of the magnetic field, $\nabla \cdot \mathbf{B}$, is zero, and that in source free regions the curl of the field, $\nabla \times \mathbf{B}$, is also zero. The divergence is the microscopic limit of flux per unit volume, and equating it to zero expresses the fact that there is no such thing as a magnetic monopole from which the field

can emanate. The curl, in a given direction, is the microscopic limit of the line integral of the field around a closed loop in a plane perpendicular to this direction, divided by the area of the loop. Being a vector, it has three components. Putting each of these components to zero expresses the fact that the current density is zero in each of these directions. This gives three constraints on the geographic north field component X , the geographic east field component Y , and the vertical downward field component Z :

$$\frac{\partial}{\partial \theta} (rY \sin \theta) = - \frac{\partial}{\partial \lambda} (rX),$$

$$\frac{\partial}{\partial r} (rX) = \frac{\partial}{\partial \theta} (Z),$$

$$\frac{\partial}{\partial r} (rY \sin \theta) = - \frac{\partial}{\partial \lambda} (Z).$$

The scalar constraint from the zero divergence gives:

$$\begin{aligned} \frac{\partial}{\partial r} (Z) = & - \frac{\partial}{r^2 \partial \theta} (rX) - \frac{\cos \theta}{r^2 \sin \theta} (rX) \\ & + \frac{\partial}{r^2 \sin^2 \theta \partial \lambda} (rY \sin \theta) - \frac{2}{r} (Z). \end{aligned}$$

For the surface polynomials being discussed here, there is no radial variation and the only relevant constraint is the first one: that the radial, or vertical, component of the curl is zero (that is, that there are no radial, or vertical, electric currents). This has been called the "mutual consistency" constraint (CHAPMAN, 1942). To utilize this constraint, X and $Y \sin \theta$ are expressed as polynomials in θ and λ , rather than simply X and Y . The constraint is then quite easy to apply, since simple differentiations with respect to θ and λ allow coefficients of $Y \sin \theta$ to be expressed in terms of coefficients of X .

This of course is only a partial constraint, the other equations being ignored. However, once there is a model for $Y \sin \theta$, X , and Z (the first two being coupled through $(\nabla \times \mathbf{B})_r = 0$) the derivatives with respect to r can in theory be calculated from the other equations, and the field can be continued upward or downward in the vertical direction (ZMUDA and MCCLUNG, 1955; ZMUDA, 1956). In practice, however, this does not work well, the errors in the derivatives being too large to give meaningful results. (It is interesting to note, incidentally, that if radial variation were included in polynomials of $rY \sin \theta$, rX , and Z , the curl $\mathbf{B}$ constraints would give reasonably straightforward conditions on the coefficients, being simple differentiations with respect to r , θ , and λ but the div $\mathbf{B}$ does not. At any rate, to the author's knowledge, regional geomagnetic fields have never been modelled with polynomials in r , θ , and λ , even without applying such constraints.)

TSUBOKAWA (1952) modelled the field over Japan with both constrained and unconstrained quadratic polynomials, and showed contours of H and D from both analyses (his Figs. 4 and 5). The differences were less than 7' in D and less than 20 nT in H . FUJITA (1968) and REILLY and BURROWS (1973) also used constrained quadratic

polynomials to model the regional fields over Japan and New Zealand, respectively. DAWSON and NEWITT (1977) constrained sixth degree polynomials, in four overlapping zones, to model the field over Canada.

If the usual spherical components X and Y , rather than X and $Y\sin\theta$, are expanded as polynomials, the mutual consistency constraint can only be applied by approximate or iterative methods, and then only to the order of the polynomial (ZMUDA and MCCLAY, 1956; LE MOUEL, 1970; SHOJI, 1978). Similar remarks apply when fitting orthogonal components other than X and Y or using coordinates other than θ and λ (HAINES, 1967).

4. Dipoles

Another way of analytically modelling the geomagnetic field is by locating dipoles within the earth and solving for their moments, and sometimes for their orientations and locations. Here, curl and divergence constraints of source free regions are automatically satisfied by deriving a potential V and expressing the magnetic field as the negative gradient of that potential. If $\mathbf{m}_i$ is the vector moment of the i -th dipole and $\mathbf{r}_i$ is the vector distance from the dipole to the point P where the field is being calculated:

$$\begin{aligned} V &= - \sum \mathbf{m}_i \cdot \nabla \left(\frac{1}{r_i} \right) \\ &= \sum \frac{\mathbf{m}_i \cdot \hat{\mathbf{r}}_i}{r_i^2}, \end{aligned}$$

where $\hat{\mathbf{r}}_i$ is the unit vector in the direction $\mathbf{r}_i$, $r_i = |\mathbf{r}_i|$, and

$$\mathbf{B} = - \nabla V.$$

When the directions and locations of the $\mathbf{m}_i$ are known, or assumed, the equations for field components are linear in the unknowns $|\mathbf{m}_i|$, and so these can be solved for readily by linear least squares. The anomaly in total field can be approximated by the component of the anomaly field in the direction of the main (or normal) field, and in this case is also linear in $|\mathbf{m}_i|$. Solving for dipole directions or locations must be done iteratively.

This technique has so far been restricted, in regional applications, to modelling of total field residual satellite data. For this purpose an array of dipoles is located on a latitude-longitude or equal-area grid, at or close to the earth's surface, and usually oriented in the direction of the earth's magnetic field as determined by the global model used in calculating the residual field.

Regional residual field models using the dipole technique have been derived from Pogo data by MAYHEW (1979, 1982), MAYHEW *et al.* (1980), and LANGE and THORNING (1982), and from Magsat data by MAYHEW and GALLIHER (1982). MAYHEW (1982) located the dipoles at a depth of 40 km; all the others located them at the surface. Minimum dipole spacing giving a stable solution is about 300 km for Pogo data (whose altitudes go down to 450 km) and about 220 km for Magsat data (with altitudes down to 300 km).

Magnetic features that can be resolved by this method are of medium wavelength: shorter than those of surface polynomials but not as short as in the graphical methods.

The method is usually used as an intermediate step in determining a magnetization distribution for geophysical interpretation (and in this respect differs from the other analytical methods discussed here, where “sources”—often artificial—are an intermediate step in determining the field). The equivalent dipole source method is non-unique with respect to source distribution, but as long as the solution is not pushed to the point of instability the field description is quite adequate.

5. Global Expansions

When surface models (Section 3) were being utilized, applying the partial potential constraint resulted in a more complicated model. This was accepted since it was believed that the higher accuracy of the resulting model would justify its greater complexity. With three-dimensional models, on the other hand, the potential constraints allow simpler models to be derived than would otherwise be possible.

This section and the next will discuss three-dimensional mathematical expansions of the field which utilize these potential constraints. In modelling with mathematical expansions, the field is treated purely as a mathematical function of the spatial coordinates (e.g., spherical coordinates r , θ , and λ or cartesian coordinates x , y , and z). The familiar technique of spherical harmonic analysis (SHA), for example, comes from the mathematical solution of Laplace's Equation, subject to the appropriate global boundary conditions. This solution is in the form of a potential V :

$$V = a \sum_{n=0}^N \sum_{m=0}^n \left(\frac{a}{r} \right)^{n+1} P_n^m(\cos\theta) \{g_n^m \cos(m\lambda) + h_n^m \sin(m\lambda)\},$$

the negative gradient of which gives the magnetic field $\mathbf{B}$:

$$\mathbf{B} = -\nabla V.$$

The $P_n^m(\cos\theta)$ is the associated Legendre function of the first kind, with order m and degree n . Two ways of utilizing this global solution for regional modelling will be discussed in this section.

5.1 Regional spherical harmonic analysis

First, the global solution (obtained by calculating a scalar potential) can be used directly as the regional solution. This can be done either by including only data over the region, or by adding, or “filling in”, data over the remainder of the earth. The difficulty with including only regional data is that the basis functions $P_n^m(\cos\theta)\cos(m\lambda)$ and $P_n^m(\cos\theta)\sin(m\lambda)$ are so non-orthogonal over the modelling region, and consequently the least-squares matrix so poorly conditioned, that only the first 20 or 30 terms of the potential can be determined (depending on computer word size). In this case the minimum wavelength of the regional model is even larger than that of global models. When the rest of the globe is filled in with data from another model, the matrix conditioning problem is solved, but this still restricts the regional model to wavelengths comparable to those of the world models.

BENKOVA and TYURMINA (1961) used SHA to derive a Z model for the Soviet

Union by the “filling-in” method. In this case, the “filling-in” was done by analytically extending the data beyond the modelled region by symmetric relationships which allowed a regional solution to be obtained.

PEDDIE (1987) also used the regional SHA technique, but utilized only data over the modelling region, in deriving magnetic charts for the United States for epoch 1985. The maximum degree and order of the expansion was 4, with 24 coefficients and a minimum wavelength of 10000 km.

5.2 Component spherical analysis

The second way of utilizing the global solution for a regional analysis relies on a theorem that says that almost any function of θ and λ can be expanded on the surface of the sphere with these same spherical basis functions (with $r=a$), regardless of whether it is actually a potential or not. Of course, if it is not a potential it cannot be continued in a radial direction. The technique of “component spherical analysis” expands components of the field in this way, although of course not all being harmonic (only the vertical component satisfies Laplace’s Equation in spherical coordinates) they will not be “mutually consistent” nor can they be continued upward or downward. The advantage of this technique is that when gridded data are used, an inverse transform can be utilized, bypassing the normal (non-diagonal) least-squares matrix solution. Data must be “filled-in” over the remainder of the sphere in order to fully utilize the orthogonality of the basis functions, although by using residuals the “filled-in” data are simply zeros and require no extra computation.

This technique was used by DAWSON *et al.* (1981) to model the declination over Canada for 1980. The horizontal residual fields ΔX and ΔY were each expanded to degree and order 180, with a resulting minimum wavelength of 222 km. The authors point out that the resulting charts are “almost” mutually consistent, since the global field to which the model residual field is added is certainly consistent and the residual field comprises less than 1% of the final global plus residual field.

6. Regional Expansions

Considering the difficulties associated with using, over a portion of the earth’s surface, basis functions that are designed to be used over the whole earth’s surface, it was natural that people who were modelling regional areas would try to design and use basis functions that were appropriate to the region itself.

6.1 Rectangular harmonic analysis

One such attempt was rectangular harmonic analysis (RHA), where the basis functions of the potential are those of a double Fourier series of sines and cosines multiplied by an exponential of z :

$$\begin{aligned}
 V = & Ax + By + Cz + \sum_{m=1}^{M_0} \{a_m^0 \cos(mx) + b_m^0 \sin(mx)\} \exp\{-k_x mz\} \\
 & + \sum_{n=1}^{N_0} \{a_n^0 \cos(ny) + c_n^0 \sin(ny)\} \exp\{-k_y nz\} \\
 & + \sum_{m=1}^M \sum_{n=1}^N \{a_n^m \cos(mx) \cos(ny) + b_n^m \sin(mx) \cos(ny) \\
 & + c_n^m \cos(mx) \sin(ny) + d_n^m \sin(mx) \sin(ny)\} \exp\{-k_z \sqrt{m^2 + n^2} z\}
 \end{aligned}$$

$$+ c_n^m \cos(mx) \sin(ny) + d_n^m \sin(mx) \sin(ny) \} \exp \left\{ - \sqrt{(k_x m)^2 + (k_y n)^2} z \right\},$$

$$\mathbf{B} = - \nabla V.$$

The a_0^0 term is omitted from the potential because it cannot be determined from, nor is it required for modelling, the magnetic field $\mathbf{B}$. The x , y , and z here are ordinary cartesian, or rectangular, coordinates, with z vertically upward at some chosen x , y value. The origin of the coordinate system is usually taken at the center of the region of data to be modelled. (These rectangular coordinates should not be confused with the north, east, and vertically downward spherical coordinates, also denoted in geomagnetic work as x , y , and z .) The coordinates are further scaled so that the x and y dimensions of the data region represent the appropriate fraction of the fundamental wavelengths required by the particular basis functions chosen.

To use this technique, then, the spherical coordinates r , θ , and λ are converted into the rectangular coordinates x , y , and z . In the way that RHA is normally used, the coordinates are scaled so that the x and y data dimensions are 2π . The k_x and k_y are functions of this scaling:

$$k_x = 2\pi / L_x; \quad k_y = 2\pi / L_y,$$

where L_x and L_y are the dimensions of the rectangular region in the x and y directions, respectively. Frequently, the individually appearing sine and cosine terms are omitted ($M_0 = N_0 = 0$), and usually the maximum wavenumber for x is the same as that for y ($M = N$). Similarly, the coefficients A , B , and C may be set to zero *a priori*, although they are included in all RHA models produced at present.

Since this is a rectangular solution to a spherical problem, the technique can only be applied over a limited area: up to a few thousand kilometers for each side of the rectangle, although this also depends on the truncation levels M and N of the series expansion. What happens is that, at large m , n , and z , the exponential term becomes very large relative to what it is when m , n , and z are small, and the least-square matrix becomes very poorly conditioned.

There is a second problem. The basis functions, in the usual rectangular harmonic analyses, are periodic and give an expansion that does not converge uniformly for any function that itself is not periodic over that interval (for example, see HAINES, 1990b). That is, if the function being modelled (in this case, the potential) is such that its value at $y=0$ is not the same as its value at $y=2\pi$ for any x , or if its value at $x=0$ is not the same as its value at $x=2\pi$ for any y , then the expansion—or the model—will not be uniformly convergent and will exhibit “ringing” at the boundary. This is not only annoying when plotting contours of the model close to the boundary (in fact, most people include a margin around their data to hide this very effect) or when determining estimates of the residual mean square over the region, but it also has severe mathematical consequences.

Mathematically, when the potential being modelled is not periodic over the region, the derivative of the expansion with respect to x or y is *not* the series of the termwise derivatives. The differentiation operator cannot be taken inside the summation signs. And yet, in this technique, the magnetic field $\mathbf{B}$ comes from differentiating the potential. The errors in the resulting coefficients of course depend on the difference in potential at opposite boundaries, and if a good reference field is subtracted before the analysis, these

errors will presumably be small.

There is one other difficulty with the expansion as currently used by most workers: the Ax , By , and Cz terms do not tend to zero as z tends to ∞ , as a potential should if it is due only to “internal” sources. These terms may have been introduced to cut down on the ringing due to the non-uniform convergence of the expansion as well as to cut down on the boundary problems associated with the rectangular approximation of spherical geometry. Again, it may be that errors resulting from this procedure are small, since the technique is used simply to describe data over a region, but continuation outside this region could certainly be invalid.

It might also be pointed out that, although all the periodic basis functions (as well as the Ax , By , and Cz terms) are harmonic, the limit function (the limit as M , M_0 , N , and N_0 tend to ∞) is not harmonic. This is because the limit function is discontinuous at the boundary and therefore does not satisfy Laplace’s Equation. (Strictly speaking, it is therefore perhaps a misnomer even to refer to this technique as a harmonic analysis, when non-periodic functions are modelled.)

The method of RHA has been known for a long time. VESTINE and DAVIDS (1945, pp. 16–18) discussed its use for modelling residual fields, referring to it as the “well known solution of Laplace’s equation”. In fact, TSUBOI (1938a, 1938b) and TSUBOI and FUCHIDA (1938) had used RHA in attempting to determine subterranean mass distributions from gravity data. NAGATA (1938a, 1938b) attempted a similar thing for magnetization distributions from magnetic anomaly data. BHATTACHARYYA (1965) used RHA for modelling anomalous total field. None of these workers included the Ax , By , or Cz terms, and all seem to have chosen $M_0 = M$ and $N_0 = N$.

ALLDREDGE (1981) continued the use of periodic RHA in modelling residual fields but chose $M_0 = 0$, $N_0 = 0$, and $N = M - m + 1$, and also introduced the Ax , By , and Cz terms, possibly following the introduction of an A_0x term in a one-dimensional potential series by MCNISH (1938). NAKAGAWA and YUKUTAKE (1985) and NAKAGAWA *et al.* (1985) also used periodic RHA with the Ax , By , and Cz terms to model residual Magsat data over Japan, using $M_0 = M = 3$ and $N_0 = N = 3$. Periodic RHA with Ax , By , and Cz terms was used by BARTON *et al.* (1987) to model the magnetic field over Australia for the 1985 epoch; the authors here used $M_0 = N_0 = 1$ and $M = N = 4$.

It was pointed out at the beginning of this subsection that normally the x and y coordinates are scaled so that their dimensions are 2π . ALLDREDGE (1982, 1983) called the 2π an upper limit, and in fact used π as his scaling. In the first of these papers he kept $M_0 = N_0 = 0$, but in the second he set $M_0 = M$ and $N_0 = N$. The effect of this was to make the RHA expansion uniformly convergent (at least in the second paper), and thus to eliminate the undesirable ringing at the boundaries. However, the Ax , By , and Cz terms were retained in both papers.

HAINES (1990a) has shown how to derive basis functions for RHA depending on which expansion derivatives are required in the model. He has suggested dropping the Ax , By , and Cz terms in order to satisfy the boundary condition at infinite z , and suggested scaling the x and y dimensions as π in order to provide expansions that are uniformly convergent. The particular set of basis functions to be used depends on the particular set of magnetic field components chosen to be modelled. However, including the full (non-orthogonal) set will increase the speed of convergence and lower the truncation level of the model for a given truncation error (HAINES, 1990b).

6.2 Component rectangular harmonic analysis

There is one important advantage to switching from spherical to rectangular coordinates, and that is that all rectangular components of the field—as well as any component in a fixed direction—are harmonic (satisfy Laplace's Equation) and therefore can be expanded in the same way the potential can be. This means that any one rectangular component can be used to derive the total vector. This is not the case, for example, in spherical coordinates where a single horizontal spherical component, not being harmonic, cannot provide the other spherical components. (The horizontal components, however, can be derived from the spherical vertical component, which *is* harmonic.) When a single component is expanded for this purpose, of course, the full set of harmonic basis functions must be used. Coefficients of the unknown components are then expressed in terms of coefficients of the single known component by utilizing the potential constraints on the first partial derivatives (equating the three rectangular components of $\nabla \times \mathbf{B}$ to zero).

This in itself is a fairly nice result, but the real usefulness of the result comes from the fact that total field observations are much more accurate than component observations and usually there are many more of them because of the ease in taking this kind of measurement. Then, if the “normal” field $\mathbf{B}_n$, or the field unaffected by the anomalous crustal field $\Delta \mathbf{B}$, is approximately constant over an area, the component of $\Delta \mathbf{B}$ in the direction of $\mathbf{B}_n$ is approximately harmonic and the full vector anomaly $\Delta \mathbf{B}$, and therefore the full vector $\mathbf{B} = \mathbf{B}_n + \Delta \mathbf{B}$, can be determined. Now since the normal field $\mathbf{B}_n$ is usually very large compared to the anomaly field $\Delta \mathbf{B}$, the component of $\Delta \mathbf{B}$ in the direction of $\mathbf{B}_n$ is essentially the same as the difference between the scalar intensity of $\mathbf{B}$ and the scalar intensity of $\mathbf{B}_n$ (Fig. 1). If this is not so, one proceeds by iteration, or else breaks up the region into zones over which this approximation is valid. The full vector $\mathbf{B}$ is thus determined from a knowledge of $\mathbf{B}_n$ and the scalar total field—hopefully with a greater accuracy and resolution, even under the given approximations, than could be obtained

Fig. 1. If the vector anomaly field $\Delta \mathbf{B}$ is small relative to the vector “normal” field $\mathbf{B}_n$, then the component of $\Delta \mathbf{B}$ in the direction of $\mathbf{B}_n$ is given approximately by the scalar total field anomaly $|\mathbf{B}| - |\mathbf{B}_n|$. If, in addition, $\mathbf{B}_n$ is approximately constant over a region, the vector anomaly field $\Delta \mathbf{B}$ can be derived in rectangular coordinates since in that coordinate system any component in a fixed direction is harmonic.

by utilizing measurements of the rectangular components directly. The expressions for the coefficients of the rectangular components in terms of the coefficients of the component in the direction of $\mathbf{B}_n$ have been given by LOURENCO and MORRISON (1973).

To benefit from the greater density of total field observations, it is usual to grid the data (if they have already been corrected to a common surface) and utilize the Fast Fourier Transform. Also, it is customary to reduce the area of analysis to sliding windows of a few hundred kilometers square centered on each grid point, analogous to a filtering procedure (LE MOUEL, 1970) or to a convolution procedure directly (BHATTACHARYYA, 1977). It is estimated by these authors that this results in an error of only a few nanotesla. The variation of $\mathbf{B}_n$ over the window also seems to result in very little error in the determination of $\mathbf{B}$.

A further approximation usually made is to ignore the exponential term in the expansion—basically a “flat-earth” approximation. When the dimensions of the window are small, this also results in very little error.

The technique (with filtering and flat-earth approximations) was used by LE MOUEL (1970) to obtain a declination chart of France for 1965. The vector field was derived from scalar intensity data that were gridded onto a 10 km grid. Minimum wavelengths are thus of the order of a few tens of kilometers. The window used was 21×21 points, or 200 km square. The magnetic charts of France continue to be produced by this method (GALDEANO *et al.*, 1980).

BHATTACHARYYA (1977) utilized a convolution approach to obtain vector anomalies from total field satellite data, after devising a scheme to reduce the residual data to a common surface. He also used a window of 21×21 points.

The procedure of Le Mouel was followed by ALLDREDGE (1989) to produce a declination chart of the United States from the DENAG magnetic anomaly map of North America. However, the variance of observed declination values relative to this model increased by about 70% over those relative to previously produced models. The author attributed the failure of the method to the poor knowledge of the normal field used in the calculations.

6.3 Spherical cap harmonic analysis

The basis functions for spherical cap harmonic analysis (SCHA), like those of RHA, are designed to be used over a region rather than over the whole sphere (HAINES, 1985a). The technique gives an expansion for the potential V and for its negative gradient, the magnetic field, directly in terms of spherical coordinates:

$$V = a \sum_{k=0}^K \sum_{m=0}^k \left(\frac{a}{r} \right)^{n_k(m)+1} P_{n_k(m)}^m(\cos\theta) \{g_k^m \cos(m\lambda) + h_k^m \sin(m\lambda)\},$$

$$\mathbf{B} = -\nabla V,$$

where the degree of the associated Legendre function is real rather than integral as in the case of global spherical harmonic analysis. By rotating the geographical coordinate system, the center of any region on the sphere can be transformed to the pole of a spherical cap. The basis functions, unlike those of RHA, are designed to give uniformly convergent expansions for the magnetic field $\mathbf{B}$. This means that there is no ringing at the boundaries, and in fact the field can be continued, or extrapolated, for small distances

outside the data area without tremendous errors. The price one pays for this is that the basis functions are not entirely orthogonal. There are two sets of basis functions, denoted by $k-m=\text{even}$ and $k-m=\text{odd}$. Within each set, the basis functions are orthogonal; however, some functions in the one set are not orthogonal to some functions in the other. If one were not interested in differentiating with respect to θ —for example, in modelling the radial field only—one could use only the series with $k-m=\text{even}$, which is a uniformly convergent, orthogonal expansion.

The SCHA technique has been used by HAINES and NEWITT (1986) to derive the magnetic charts of Canada for 1985. The authors believe that the model includes wavelengths down to almost 800 km, equivalent roughly to an SHA with $n=50$. Such an SHA expansion would require about 2700 coefficients rather than the 289 coefficients required by the SCHA model.

A chart of the main field over a large area like Canada looks much smoother than its 800 km wavelength content would indicate. This is because the spectral content at the short wavelengths is small relative to those of the longer wavelengths. The shorter wavelengths are clearly visible in the residual field actually modelled by SCHA (NEWITT and HAINES, 1989, Fig. 3), which is the “correction” to the global IGRF model subtracted from the data prior to analysis. These features are visually masked out when the IGRF is added back on, but the differences in certain places are as high as 100 or 150 nT.

A procedure was discussed earlier (Subsection 2.2) of averaging residual satellite data in latitude-longitude bins over altitude. HAINES (1985b) produced an analytical model of vertical field residuals north of 40°N by the technique of SCHA, and gave maps of the residual field at both the upper limit (535 km altitude) and lower limit (335 km) of Magsat altitudes. The anomaly amplitudes at 335 km are approximately double those at 535 km. Two of the larger anomalies—the Alpha Ridge anomaly and the Kirsk anomaly—have amplitudes of 16 and 10 nT, respectively, at 535 km altitude but have amplitudes of 32 and 22 nT at 335 km. This demonstrates the limitations inherent in the numerical procedure of averaging over altitude. Of course, the analytical model can be computed at any altitude between the 335 and 535 km limits. In fact, it can even be extrapolated outside these limits somewhat, without incurring an unduly large extrapolation error.

7. Summary

Several types of regional models have been discussed. The major characteristics of these models are summarized in Table 1. Characteristics that are not, in theory, essential to the particular technique, but apply to the technique as normally used, are put in parentheses.

One of the principal advantages of the graphical models—hand drawn or numerically processed and contoured—is their short wavelength content. However, they are quite subjective and so it is not always easy to evaluate how significant this short wavelength content is. They do not vary with altitude nor do they satisfy the electromagnetic constraints of zero divergence and curl. Field values cannot be computed directly, since the “model” is the chart itself, or some large table of values.

Field values from analytical models like polynomials can be computed directly. Surface polynomials may or may not satisfy the partial “mutual consistency” requirement

Table 1. Characteristics of regional models.

Technique	Electro-magnetic constraints	Radial variation	Minimum wave-length	Remarks
Graphical	No	No	Short	No geometrical constraints. Subjective
Unconstrained polynomials	No	(No)	Long	Altitude continuation difficult
Constrained polynomials	Partial	(No)	Long	
Dipoles	Yes	Yes	Med	Non-unique
SHA	Yes	Yes	Long	Non-orthogonal
Component SA	No	No	Short	Computer time
RHA	Yes	Yes	Med	Areal limitation (Ringing at boundary)
Component RHA	Yes	Yes	Short	Same as for RHA
SCHA	Yes	Yes	Med	(Non-orthogonal)

that the vertical component of the curl be zero. Also, polynomial models in practice do not vary with altitude, although in principle one could include a radial variation or infer it from the divergence and curl equations. Minimum wavelengths are usually quite long.

Dipoles have been used in order to produce magnetization models for geophysical interpretation, but the computational effort is much higher than in the spatial harmonic expansion methods.

SHA can sometimes be applied without modification to a region, but the minimum wavelength is very long. Component SA can give short wavelengths, but its electro-magnetic constraints and radial variation are satisfied only insofar as they are satisfied by the global reference field utilized in the residual analysis.

RHA is limited in area because it uses a rectangular approximation to spherical geometry, and the periodic basis functions used currently give ringing at the boundary and errors in the coefficients because of improper termwise differentiation. Furthermore, many RHA expansions include terms in the potential that do not satisfy the boundary conditions at infinite z . Component RHA, because it is able to take advantage of the high accuracy and density of total field observations, can provide very short minimum wavelengths, but is limited by the requirement that the normal field must be approximately constant over the area of analysis.

Spherical cap harmonic analysis gives uniformly convergent expansions for both the potential and the magnetic field, so that there is no ringing at the boundary. The expansion is directly in terms of spherical coordinates and so takes advantage of the spherical geometry. Some basis functions are not orthogonal to some of the others, which means that in modelling known analytical functions, coefficients cannot be expressed algebraically as simple integrals or sums. However, in practice regional models are determined from numerical data and are solved by an inverse procedure like least squares.

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