The Limitations of Numerical Models of the Main Geomagnetic Field

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This paper points out that IGRF-type models of the main geomagnetic field (that coming from electric currents in the Earth's core) are only approximations to the actual field in that they involve low-pass filtering in space and in time, and are inevitably numerically inaccurate. It then discusses the magnetic fields from other sources, in particular from the magnetization of crustal rocks, from electric currents in the ionosphere, from the drift of the charged particles in the radiation belts, and from various magnetospheric processes, attempting to give an indication of typical magnitudes of the various fields.

1. Introduction

This paper is intended to indicate to the wide variety of users some of the problems involved in using a model such as the IGRF. The main message is that users must have some knowledge of the limitations of the model, and must consider carefully exactly what it is they wish to do if they are to avoid making possibly serious mistakes.

The paper starts by pointing out the inherent inability of real models to specify the actual main field, and subsequent sections discuss the origin and magnitude of the other magnetic fields present in the vicinity of the Earth.

2. Main Field Models

First let me try to make clear just what such a model is trying to do (although even this is not completely unambiguous). Essentially it is trying to produce a good approximation to the "main" field of the Earth, which is very nearly (but not quite) the same as saying the field produced by electric currents in the core of the Earth.

The model is an approximation in three respects.

2.1 The effect of truncation

The model does not try to represent that part of the field which has high spatial frequency, i.e. small wavelength. For an IGRF model, which is truncated at harmonic degree $n=10$, this means that it ignores wavelengths less than 4000 km at the Earth's surface; this corresponds to ignoring about 10 nT of the field coming from the core. The model can be thought of as a "(spatially) low-pass filtered" version of the field.

2.2 The effect of discreteness in time

Similarly, the model is (effectively) low-pass filtered in time. At the Earth's surface the core field itself probably does not vary on time scales much shorter than a year. For
simplicity IGRF models are specified at 5 year intervals; for times in the past we use linear interpolation between successive models, and for times after the last model we specify a linear extrapolation into the future. For most of the time, and in most places, these 5 year straight-line segments are not too bad an approximation to what we can think of as a more general power series representation. (The secular variation—the first time-derivative of the main field—is typically 80 nT/yr. For a typical secular acceleration of 3 nT/yr² the maximum difference between the chord and the parabola would be about 9 nT; LANGEL et al. (1988) estimated the actual global rms difference to be about 12 nT.) However, there are strong indications that on occasion this power series representation is itself not valid; during the so-called "geomagnetic jerk" of 1970 the second derivative appeared to change, effectively instantaneously, by about 2 nT/yr². Extrapolating forward for 5 years from just before such a jerk would give a field in error by 25 nT. (Note that in MALIN and HODDER (1982), and MALIN et al. (1983), the jerk coefficients need to be divided by a factor of 4.) Certainly, trying to extrapolate forward in time (from the last model) is about as safe as trying to predict prices on the Stock Exchange!

(Note that this linear interpolation is meant only for estimation of the main field; the slope of the chord might well not be a very good approximation to the instantaneous time derivative. With presently available data we cannot obtain a good model for the secular variation, particularly for the high (spatial) harmonics.)

2.3 The effect of inaccuracies in the coefficients

Even within the above limitations the model is inevitably still inaccurate. In the accompanying paper BARRACLOUGH (1990) discusses this inaccuracy in more detail, so let me just give some indication of what is involved; Table 1 summarises some typical values.

Our best knowledge of the field was for 1980, when we had 6 months of data from the polar orbiting satellite MAGSAT. As a result we were able to produce a Definitive Geomagnetic Reference Field for 1980.0 which was accurate to about ±20 nT; the field itself is typically of magnitude 45,000 nT. However the field varies in time by typically 80 nT/year, and at present we know that variation only to about 20 nT/year, and in some places only to about 100 nT/year! So our knowledge of the field is rapidly getting worse, and will continue to do so until someone flies another satellite!

Older models will be less good, but it is more difficult to be quantitative. The adjective "definitive" means only that we think it is about the best we will ever be able to do using all the available data, not that the model is correct; note that the DGRF models

<table>
<thead>
<tr>
<th>Year</th>
<th>Main field</th>
<th>Secular variation</th>
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<tbody>
<tr>
<td>1945</td>
<td>45,000±200 nT</td>
<td>80±15 nT/yr</td>
</tr>
<tr>
<td>1980</td>
<td>±20</td>
<td>±20**</td>
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<tr>
<td>(1985 by extrapolation ±100?)</td>
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<td>(1990 by extrapolation ±200?)</td>
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*The figures given are average values; the maximum values (at particular locations) will be several times larger.

**The uncertainty of the secular variation for 1980 will be reduced when more recent data is incorporated in the next model.
replace, in retrospect, the IGRF models, and that there can be considerable differences. Note also that the uncertainties I quote are global root mean square averages, the older models being considerably worse than this for regions where there were fewer data. It is quite clear from retrospective studies that the coefficient standard deviations given by many workers were far too small; it is now realised that the simple statistical models used could grossly underestimate the effect of the truncation, and of the crustal and other fields (discussed later), on the standard deviations (see e.g., Lowes, 1990). For 1980 I have used a more realistic estimate by Langel et al. (1989), and for 1945 that of Langel et al. (1988).

Let me remind you that the 1980 figure I quoted of 45,000±20 nT was at the Earth's surface. Figure 1(b) shows a, simplified, picture of the radial component at $R=R_e$, the Earth's surface. If we move out to larger radii we introduce an extra geometrical low-pass filter, so the field becomes relatively simpler as well as absolutely smaller; as a result our model of the core field becomes both absolutely and relatively more accurate. (But some of the other contributions discussed below will become more important.) Figure 1(a) shows the field at $R=2R_e$ corresponding to that of Fig. 1(b); note the change of contour interval. Conversely, going into the Earth, towards the core, the field becomes relatively more complicated as well as absolutely larger, and our model becomes worse both in absolute and in relative terms. Figure 1(c) shows the corresponding field at $R=0.55R_e$, the core-mantle boundary; again note the change of contour interval. (Also, this downward extrapolation ignores the fact that there might be significant electric currents flowing in the lower part of the mantle.)

2.4 Other contributions to the magnetic field

So much for what our model is trying to do; a major problem is that the core field it tries to describe is only one contribution, if the major one, to the total magnetic field. Even if our main field model were perfect, which it is not, the field actually measured will consist not only of this main field, but also of smaller contributions from many other sources, these sources having a wide spread of typical length and time scales.

As well as electric currents in the core there are four other main geophysical sources of field: the magnetisation of crustal rocks, electric currents in the ionosphere, the ring current of the radiation belts, and a variety of sources linked with the magnetosphere. All of these fields are vectors, and vary with position; to allow me to make meaningful quantitative comparisons I will specify typical magnitudes usually in terms of the RMS average over the Earth's surface. I will now discuss each of these sources in more detail. (In addition to these geophysical sources there are of course many man-made sources.)

3. Crustal Magnetization

A thin layer, typically 10–20 km thick, of surface rocks is cool enough that the rocks can be magnetised, giving "crustal" magnetic fields. While at the Earth's surface the main field is typically of magnitude 45,000 nT, the crustal field is typically of magnitude 150 nT. This crustal magnetization is partly "permanent" (produced when igneous rock cooled down, as in the magnetic striping on either side of the mid-ocean ridges), and partly (probably mostly), induced because the rock is sitting in the present main field. Permanent magnetization tends to be of quite small length scales, of the order of 10 km; induced magnetization is also mostly at short length scales, because of the complexity of
Fig. 1. The radial component of the main geomagnetic field in 1980, as given by the DGRF 1980, but with fields of wavelength less than 45° omitted. Full contours for fields directed inwards, dashed contours for fields outwards. (a) Field at radius $R=2R_e$. Contour interval 1 $\mu$T. (b) Field at Earth's surface, $R=R_e$. Contour interval 10 $\mu$T. (c) Field at the core-mantle boundary, $R=0.55R_e$. Contour interval 100 $\mu$T.
the geology.

Figure 2 shows the observed “power” spectrum, the total mean square field given at the Earth’s surface by each spherical harmonic degree $n$. The steep initial part of the spectrum is due to the core field, while the much flatter later part (which continues to much higher degree) is due to field from the crust. The changeover in the spectrum occurs at about $n=13$, which is why main field modelling is not taken beyond $n=13$. (More recent work (Cain et al., 1989) suggests that the two contributions are equal at $n=14$.) But this means that (for a model with $n\leq13$) there is about 5 nT core field in the higher spatial frequencies (short wavelengths) not included in the model, and (less certainly) about 30 nT of the low frequency (long wavelength) crustal field that has been incorporated into the so-called “main” field. (For a model with $n\leq10$ these figures are about 25 nT and 25 nT respectively.) So if crustal anomalies are found by subtracting even a perfect IGRF type model from the actual observed field, this anomaly field will still have in it a small amount of higher frequency core field, and will not have in it some lower frequency crustal field; this latter means, for example, that most of the oceanic/crustal contrast is missing. (In practice, except at 1980, this effect is mostly masked by the fact that the main field is even more uncertain.)

Because of the small length scale of the crustal magnetisation, its field falls off very rapidly with height. (Apart from the difference in length scale, the geometry of the crustal magnetization and its resulting field closely resembles that of the magnetization of magnetic recording tape. In a Fourier series representation, the field falls off exponentially with height, the “half height” being proportional to the horizontal wavelength.) By 400 km, a typical satellite altitude, the crustal field has reduced to about 10 nT. If the aim is to model the main field then this reduction of “noise” is welcome; on the other hand if the interest is in the crustal field itself this reduction in “signal” is a great nuisance when using satellite data to give global coverage.

![Spatial power spectrum of the main geomagnetic field](image)

Fig. 2. Spatial power spectrum of the main geomagnetic field. The ordinate is the (logarithm of the) mean square field, over the Earth's surface, produced by all harmonics of degree $n$. (After Langel, 1987.)
4. Ionospheric Fields

I am now coming to topics to which whole symposia are devoted, so I can give only
the simplest outline of the sorts of fields produced.

The ionosphere is a region of the atmosphere, from about 100 to 1,000 km altitude,
made conducting during the day time by ionisation by near ultra-violet radiation from
the Sun. Large scale, daily, atmospheric motions move this conducting gas in the
presence of the main magnetic field, giving motionally-induced electric currents. These
currents are essentially horizontal, and are probably confined to the lower regions of the
ionosphere. As seen from the Sun the current system is roughly constant and stationary,
but an observer on the Earth's surface sees a daily variation, though with considerable
annual modulation (because of the inclination of the Earth's axis of rotation). For quiet
solar conditions typical amplitudes are 30-50 nT. Because of the large horizontal scale of
the currents the magnitude of the daily variation does not change much with height
(though the horizontal components reverse sign as we move up through the current
region itself).

Since the ionospheric field is varying with time, as seen by the (slightly) conducting
Earth, electric currents are induced in the main body of the Earth; near the Earth's
surface these induced currents contribute about two fifths of the observed daily variation.

Because of the anisotropy of the conductivity of the ionosphere, there is a narrow
region, about 10° wide and centred on the magnetic dip equator (where the main field is
horizontal), where the daily variation is significantly enhanced; this enhanced current is
called the equatorial electrojet.

Even in "quiet" conditions there can be significant day to day variability, and in solar
disturbed conditions the solar UV radiation can be considerably enhanced, as is then the
daily variation; there is therefore an 11 year modulation of the daily variation. (Because
of imperfections in our filtering of the data, our main-field model for a particular epoch
might contain a contribution from this modulation.)

As well as these motionally-induced currents, in the auroral and polar regions there
are larger, and more complicated, currents due to magnetospheric effects; see Section 6
below.

5. Radiation Belt and Ring Current

Because of the dipole nature of the magnetic field, charged particles arriving near the
Earth with appropriate velocities are trapped, travelling backwards and forwards in
latitude along the field lines. However, the charged particles also drift in longitude,
positives ones to the West, and negative ones to the East, giving an East-to-West "ring-
current" at a few Earth radii. In the region of the Earth this gives an almost uniform
North-to-South field (parallel to the dipole axis) of typically 30 nT in quiet conditions.
However, particularly when the Sun is active, the ring current is quite variable, and can
give 1000 nT or more. (It is quite difficult to separate this (external) uniform ring-current
field from the (internal) dipole field, so any one main field model might contain a small,
but significant error in its dipole term from this effect.) The time variation of the ring
current also produces induced electric currents in the Earth which give a (genuine) dipole
field. LANGEL and ESTES (1985) have proposed a way of relating the ring-current
external field, and induced internal field, to the \( D_{st} \) index (a retrospective index of the ring-current field), and their recent models specify the \( D_{st} \) level for which they are valid.

6. Magnetospheric Fields

Here even more severe simplifications are necessary! The Earth’s magnetic field is in fact confined to the “magnetosphere”, a cavity carved out of the (conducting) solar wind. The confinement is given by currents flowing in the surface of the cavity, and in the “neutral sheet”—the “downstream” equatorial plane. These currents give significant distortion of the magnetic field round the Earth; the lines of force are compressed on the daylight side, and dragged out on the night side. The cavity is roughly stationary as seen from the Sun, so an observer moving with the Earth sees a diurnal variation, though this is only a few nanoteslas at the surface and not easily separated from the ionospheric variations.

Out to about 60° geomagnetic latitude the lines of force, though somewhat distorted, are still “closed”, i.e. return to the Earth at both ends. However the lines of force from most of the polar regions are “open”, being swept downstream into the magnetotail.

Like the solar UV, the solar wind can be quite variable, again particularly during the maxima of the solar 11-year cycle. The amount and detail of the field confinement can show considerable variations on quite short time scales, giving what we call magnetic disturbance, or in severe cases magnetic storms. (If the observer is interested in other aspects of the field, then during such disturbed conditions he curses, and, if possible, waits for quieter conditions. But of course magnetospheric scientists might appreciate the extra data—as in so much of geomagnetism, one man's signal is another man's noise!)

Variations in the solar wind give a wide variety of effects, with time scales ranging from fractions of a second, and amplitude fractions of a nanotesla, to magnetic storms lasting several hours and of amplitude up to thousands of nanoteslas. At any one time the larger disturbances of the solar wind tend to be triggered by certain regions on the Sun's surface; therefore the 27-day rotation period of the sun tends to give a 27-day recurrence pattern, and of course the whole is modulated by the 11-year sunspot cycle.

The magnetosphere is defined on the outside by the surface of the cavity in the solar wind; its inner boundary is the conducting ionosphere. Between the two “surfaces” there is a very complicated situation; charged particles tend to flow along magnetic lines of force—“field aligned currents”—particularly along the “open” lines of force in the polar regions; these field-aligned currents give local field perturbations of the order of hundreds of nanoteslas, varying quite rapidly in time. Different regions of field-aligned currents are joined by current flow over the polar caps of the ionosphere, and around the auroral electrojet, giving considerable fields at the Earth’s surface. Also the lines of force themselves can be vibrated like plucked strings, giving “pulsations” having periods of the order of seconds to minutes; again the corresponding currents in the ionosphere give fields at the Earth’s surface, of the order of 1 to 100 nT.

TSYGANENKO and USMANOV (1982) and TSYGANENKO (1988) give a numerical model for the average magnetic field in the magnetospheric cavity.

7. Discussion

For convenience, I have considered only the post-1940 IGRF-type models, though
what I have said will also apply (qualitatively) to the models going back to 1700 which have been produced by Gubbins and Bloxham and their co-workers.

It is clear that a good model of the main geomagnetic field, say to ±20 nT, can only be obtained for epochs for which there is good global coverage by a satellite vector magnetometer. On the assumption that the time variation of the field is fairly smooth, this satellite data will also help to constrain the field for nearby epochs, but there is no avoiding the fact that without adequate good data our models soon deteriorate.

However good (or bad) such a model is in representing the “main” field, there are many other sources of field, varying significantly with position (in and near the Earth) and in time, which might well be relevant in a particular application; it is important that users should appreciate the consequent limitations of the model. Perhaps the models should be labelled with a government (=IAGA) “health warning” about their abuse; even before any possible commercialization by the IGRF we should warn users caveat emptor!

REFERENCES