Reexamination of Fluid Motion in the Earth's Core
Derived from Geomagnetic Field Data
—Is the ω -Effect Really Strong in the Core?—

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Fluid motion in the Earth's outer core has been derived from geomagnetic field data on the presumption that a strong ω -effect is operative in the core. In this case, it is important to properly estimate differential rotation which is expected to generate strong zonal toroidal magnetic fields. So far, we have relied on a physically simplified model in terms of virtual displacement of fluid particles. We here attempt to solve both the induction and the Navier-Stokes equations. The basic standpoint is that the non-axisymmetric poloidal magnetic field is maintained by the interaction between the strong zonal toroidal magnetic fields and a large-scale fluid motion within the outer core. The radial dependence of poloidal velocity field is unknown, but once a rather simple functional form is prescribed, the magnitude is determined so as to fit the magnetic field data. Then the toroidal velocity field can be derived by solving the Navier-Stokes equation. For simplicity, we assume that the steady state is realized at various epochs.

In order to check the validity of solutions, time-dependent behavior of the magnetic field is examined with the derived velocity field fixed. The time integration is performed using the fourth-order Runge-Kutta scheme. Unless the derived velocity field is appropriate, the magnetic field would vary rapidly contrary to the assumption of the steady state.

It turned out that the estimation of differential rotation based on virtual displacement of fluid particles is not valid. As long as a large-scale fluid motion and a large-scale magnetic field are considered, the ω -effect is not so effective as presumed and strong zonal toroidal magnetic fields are not generated. In order to derive fluid motion within the core, we should take all the interaction terms into consideration.

1. Introduction

One conventional approach to the derivation of fluid motion in the core is based on the frozen-flux approximation (ROBERTS and SCOTT, 1965); that is, the magnetic diffusion term in the induction equation is neglected for a time scale of some tens of years because of the high electrical conductivity of core fluid. Then the core behaves like a perfect conductor so that magnetic lines of force move along with the fluid. According to BACKUS (1968), however, this approximation alone warrants no unique velocity field, and an additional constraint is required; for example, steady flow (VOORHIES, 1986; WHALER and CLARKE, 1988; WHALER, 1990), geostrophic flow (LE MOUËL *et al.*, 1985; GIRE and LE MOUËL, 1990; HULOT *et al.*, 1990), or toroidal flow

(WHALER, 1980; GUBBINS, 1982; BLOXHAM, 1989; LLOYD and GUBBINS, 1990) has been considered as a constraint.

Meanwhile, BLOXHAM and GUBBINS (1986) presented a line of evidence showing that the frozen-flux approximation is invalid in the South Atlantic region where magnetic flux diffusion seems to be significant. BLOXHAM (1986) also pointed out that a strong toroidal magnetic field and upwelling motions existing near the core-mantle boundary (CMB) would invalidate the frozen-flux approximation. It should be noted, however, that the result of BLOXHAM and GUBBINS (1986) depends on their error estimates and BLOXHAM (1988) himself admits that their work does not necessarily mean that the frozen-flux approximation should be abandoned.

Another approach to the derivation of fluid motion in the core is based on the presumption that a strong toroidal magnetic field must be generated by a differential rotation in the core (RIKITAKE, 1967; HONKURA and RIKITAKE, 1972; HONKURA and MATSUSHIMA, 1988b; MATSUSHIMA and HONKURA, 1989a). The induction process is then simplified; the non-axisymmetric poloidal magnetic fields observed at the Earth's surface are generated only by the interaction between poloidal velocity fields and such a strong zonal toroidal magnetic field. Then the derivation of fluid motion in the core is rather straightforward.

RIKITAKE (1967), HONKURA and RIKITAKE (1972), and HONKURA and MATSUSHIMA (1988b) further assumed that a strong zonal toroidal magnetic field of degree two and order zero in spherical harmonics (hereafter it is expressed as $\mathbf{B}[T_2^0]$) with an arbitrarily given magnitude exists only in a narrow shell within the core, and derived poloidal velocity fields near the CMB. MATSUSHIMA and HONKURA (1989a) improved the method by estimating differential rotation, which is expected to generate a strong toroidal magnetic field, and derived the magnitude and the radial dependence of the toroidal magnetic field $\mathbf{B}[T_2^0]$ through the interaction between the axial dipole magnetic field $\mathbf{B}[S_1^0]$ and the differential rotation $\mathbf{V}[T_1^0]$; this is the so-called ω -effect. In order to obtain the differential rotation $\mathbf{V}[T_1^0]$, they divided the outer core into spherical shells of equal volume, following the method of WATANABE and YUKUTAKE (1975). An underlying assumption is that each spherical shell is subject to rigid steady rotation as a result of the balance between the electromagnetic torque due to the Lorentz force and the effective torque due to the angular momentum transfer expected from virtual displacement of fluid particles. The effective torque was calculated as follows. It is possible to evaluate the volume of fluid which comes into or out of a spherical shell during a time interval, $\delta\tau$, if the mean radial flow velocity is known. The conservation of angular momentum enables one to calculate a change in angular momentum of the shell, δM . The effective torque was then derived from the estimation of the rate of change in the angular momentum of the shell, $\delta M/\delta\tau$. Thus we can calculate the zonal toroidal velocity field and derive the zonal toroidal magnetic field from the ω -effect.

We can easily realize that it is insufficient to represent differential rotation in terms of the zonal toroidal motion $\mathbf{V}[T_1^0]$ only. More realistically, differential rotation should depend not only on the r -direction but also on the θ -direction, as is the case for the convection zone in the Sun (e.g. BABCOCK, 1961). MATSUSHIMA and HONKURA (1989b; unpublished work), therefore, divided the outer core in the r - and θ -directions and derived differential rotation based on virtual displacement of fluid particles, assuming that each annulus is in the state of steady rotation, again. They regarded zonal toroidal magnetic fields $\mathbf{B}[T_l^0]$ ($l \leq 6$), thus obtained, as responsible for the generation

of the non-axisymmetric poloidal magnetic fields and derived fluid motion in the outer core.

It should be noted that plausible estimation of differential rotation is crucially important when we attempt to derive fluid motion in the core on the basis of strong zonal toroidal magnetic fields. The previous results have relied on a physically very simplified model; that is, virtual displacement of fluid particles in estimating differential rotation. Until this concept is justified, the method is by no means reliable and improvements in this respect are obviously required in order to derive more reliable fluid motion in the core. The best way to estimate toroidal velocity fields is to solve the Navier-Stokes equation.

The line along which we proceed is now clear; fluid motion in the Earth's outer core is to be derived from geomagnetic field data by solving both the induction and the Navier-Stokes equations. It is too difficult, however, to take into consideration the energy source in the Navier-Stokes equation; in this case we must also solve the energy equation. Furthermore, the energy source for the geodynamo is a problem under debate; which type of convection is more likely, a thermal or a compositional one (e.g. GUBBINS and ROBERTS, 1987)? Nevertheless, once the radial dependence of poloidal velocity field is prescribed yet the magnitude is considered as an unknown parameter, it is possible to derive fluid motion in the core by solving the Navier-Stokes equation for the toroidal constituent and the induction equation for the toroidal and the poloidal magnetic fields; the unknown parameters can be estimated in a consistent manner from the observed magnetic field. In this sense, the present approach may be considered as an intermediate model between kinematic and magnetohydrodynamic (MHD) dynamos. The point which is essentially different from the kinematic dynamo approach is that the Navier-Stokes equation including the Coriolis and the Lorentz force terms is solved. The energy source is implicitly incorporated into the prescribed radial dependence of poloidal velocity field.

In the actual computation, we consider a large-scale fluid motion and a large-scale magnetic field, and express them in terms of toroidal and poloidal fields, each of which is expanded into series of spherical harmonics of up to degree four. We then reexamine the work of MATSUSHIMA and HONKURA (1989b) based on the same presumption that the non-axisymmetric poloidal magnetic fields are generated simply by the interaction between poloidal velocity fields and zonal toroidal magnetic fields generated by the ω -effect.

No solution may be unique, since we deal with non-linear algebraic simultaneous equations. We must somehow check the validity of solutions. One way would be to examine time-dependent behavior of the magnetic field with the derived velocity field fixed. If the assumption made in the derivation of fluid motion in the core is inappropriate, the magnetic field would vary considerably, for example, decay or grow within a short time, because of the lack of induction terms which should have been taken into consideration in deriving fluid motion in the core; that is, some other induction terms must be important in maintaining a steady magnetic field.

Finally, we discuss whether the ω -effect is really strong in the Earth's core, through the derivation of fluid motion in the core and the examination of its plausibility.

2. Method

2.1 Fundamental equations and mathematical formulation

The induction equation is derived from Maxwell's equations, ignoring the displacement current, as

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \nu_m \nabla^2 \mathbf{B}, \quad (1)$$

where $\mathbf{B}$ is the magnetic field, $\mathbf{V}$ the velocity field, and $\nu_m = (\mu_0 \sigma)^{-1}$ the magnetic diffusivity, with μ_0 the magnetic permeability of vacuum and σ the electrical conductivity of the core. For the sake of simplicity, σ is assumed to be constant throughout the inner and outer cores.

In the Boussinesq approximation (e.g. CHANDRASEKHAR, 1961) and in the frame of reference rotating with a constant angular velocity $\boldsymbol{\Omega}$, corresponding to the angular velocity of the mantle, the Navier-Stokes equation is written as

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla p - 2\boldsymbol{\Omega} \times \mathbf{V} + \frac{1}{\rho \mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + C\mathbf{g} + \nu \nabla^2 \mathbf{V}, \quad (2)$$

where $\mathbf{g}$ is the gravitational acceleration vector, and p , ν , ρ and C denote the pressure, the kinematic viscosity, the mean density of the core, and the buoyancy parameter, respectively.

Any solenoidal vector field can be decomposed into toroidal and poloidal vector fields (CHANDRASEKHAR, 1961; BACKUS, 1986). We introduce the toroidal-poloidal decomposition of the magnetic field $\mathbf{B}$ and the velocity field $\mathbf{V}$, assuming that fluid in the core is incompressible. The velocity and the magnetic fields are expressed in terms of toroidal and poloidal vectors as

$$\begin{aligned} \mathbf{V}(\mathbf{r}, t) &= \mathbf{V}[T](\mathbf{r}, t) + \mathbf{V}[S](\mathbf{r}, t) \\ &= \nabla \times (V[T](\mathbf{r}, t) \hat{\mathbf{r}}) + \nabla \times \nabla \times (V[S](\mathbf{r}, t) \hat{\mathbf{r}}) \end{aligned} \quad (3)$$

and

$$\begin{aligned} \mathbf{B}(\mathbf{r}, t) &= \mathbf{B}[T](\mathbf{r}, t) + \mathbf{B}[S](\mathbf{r}, t) \\ &= \nabla \times (B[T](\mathbf{r}, t) \hat{\mathbf{r}}) + \nabla \times \nabla \times (B[S](\mathbf{r}, t) \hat{\mathbf{r}}), \end{aligned} \quad (4)$$

where $\mathbf{r}$ is the position vector, $\hat{\mathbf{r}}$ the radial unit vector, and t the time. $V[T](\mathbf{r}, t)$ and $V[S](\mathbf{r}, t)$ are the toroidal and the poloidal scalar functions for the velocity field, respectively, defining the toroidal constituent $\mathbf{V}[T](\mathbf{r}, t)$ and the poloidal constituent $\mathbf{V}[S](\mathbf{r}, t)$ of the velocity field $\mathbf{V}(\mathbf{r}, t)$. $B[T](\mathbf{r}, t)$, $B[S](\mathbf{r}, t)$, $\mathbf{B}[T](\mathbf{r}, t)$ and $\mathbf{B}[S](\mathbf{r}, t)$ are those for the magnetic field $\mathbf{B}(\mathbf{r}, t)$.

We use the spherical coordinates (r, θ, ϕ) to express the velocity and the magnetic fields, and expand their scalar functions into series of spherical harmonics for θ - and ϕ -dependence, since we consider large-scale velocity and large-scale magnetic fields. We then have

$$V[T](\mathbf{r}, t) = \sum_{l=1}^L \sum_{m=0}^l \{V[T_l^{mc}](r, t) \cos m\phi + V[T_l^{ms}](r, t) \sin m\phi\} P_l^m(\cos \theta), \quad (5a)$$

$$V[S](\mathbf{r}, t) = \sum_{l=1}^L \sum_{m=0}^l \{V[S_l^{mc}](r, t) \cos m\phi + V[S_l^{ms}](r, t) \sin m\phi\} P_l^m(\cos \theta), \quad (5b)$$

$$B[T](\mathbf{r}, t) = \sum_{l=1}^L \sum_{m=0}^l \{B[T_l^{mc}](r, t) \cos m\phi + B[T_l^{ms}](r, t) \sin m\phi\} P_l^m(\cos \theta), \quad (5c)$$

and

$$B[S](\mathbf{r}, t) = \sum_{l=1}^L \sum_{m=0}^l \{B[S_l^{mc}](r, t) \cos m\phi + B[S_l^{ms}](r, t) \sin m\phi\} P_l^m(\cos \theta), \quad (5d)$$

where $P_l^m(\cos \theta)$ is an associated Legendre function with degree l and order m . L corresponds to the truncation level in the degree of spherical harmonics.

The orthogonality of toroidal and poloidal vectors and also of spherical surface harmonics enables one to separate the induction and the Navier-Stokes equations into equations for toroidal or poloidal scalar functions with the set of (l, m, z) , where $z = c$ or $z = s$ according to the cosine or sine term.

From the induction equation (1), as in BULLARD and GELLMAN (1954), we obtain

$$\begin{aligned} r^2 \frac{\partial B[T_\gamma]}{\partial t} = \nu_m \left\{ r^2 \frac{\partial^2 B[T_\gamma]}{\partial r^2} - \gamma(\gamma + 1) B[T_\gamma] \right\} \\ + \sum_{\alpha} \sum_{\beta} \{ (V[S_\alpha] B[S_\beta] T_\gamma) + (V[T_\alpha] B[S_\beta] T_\gamma) \\ + (V[S_\alpha] B[T_\beta] T_\gamma) + (V[T_\alpha] B[T_\beta] T_\gamma) \}, \end{aligned} \quad (6)$$

$$\begin{aligned} r^2 \frac{\partial B[S_\gamma]}{\partial t} = \nu_m \left\{ r^2 \frac{\partial^2 B[S_\gamma]}{\partial r^2} - \gamma(\gamma + 1) B[S_\gamma] \right\} \\ + \sum_{\alpha} \sum_{\beta} \{ (V[S_\alpha] B[S_\beta] S_\gamma) + (V[T_\alpha] B[S_\beta] S_\gamma) + (V[S_\alpha] B[T_\beta] S_\gamma) \}. \end{aligned} \quad (7)$$

From the Navier-Stokes equation (2), as in FRAZER (1974),

$$\begin{aligned} r^2 \frac{\partial V[T_\gamma]}{\partial t} = \nu \left\{ r^2 \frac{\partial^2 V[T_\gamma]}{\partial r^2} - \gamma(\gamma + 1) V[T_\gamma] \right\} \\ - 2\Omega \sum_{\alpha} \{ (V[S_\alpha] S_1^\Omega S_\gamma) + (V[T_\alpha] S_1^\Omega S_\gamma) \} \\ + \frac{1}{\rho\mu_0} \sum_{\alpha} \sum_{\beta} \{ [B[S_\alpha] B[S_\beta] S_\gamma] + [B[T_\alpha] B[S_\beta] S_\gamma] + [B[T_\alpha] B[T_\beta] S_\gamma] \} \\ - \sum_{\alpha} \sum_{\beta} \{ [V[S_\alpha] V[S_\beta] S_\gamma] + [V[T_\alpha] V[S_\beta] S_\gamma] + [V[T_\alpha] V[T_\beta] S_\gamma] \}. \end{aligned} \quad (8)$$

In these equations, Greek letters appearing as a subscript denote the set of (l, m, z) or simply the degree l of the spherical harmonics, as in BULLARD and GELLMAN (1954). The interaction terms in parentheses are given by BULLARD and GELLMAN (1954), and those in brackets by FRAZER (1974). All the interaction terms are expressed in

terms of the Gaunt or Elsasser integrals and the scalar functions, represented by α , β , γ , with their partial derivatives with respect to r . The conditions imposed on α , β and γ are known as the selection rules and shown in BULLARD and GELLMAN (1954).

The scalar function $S_1^\Omega(r)$, which arises from the angular velocity Ω , is given as $\frac{1}{2}r^2$, since Ω can be expressed as

$$\Omega = \Omega \hat{\mathbf{z}} = \Omega \nabla \times \nabla \times (S_1^\Omega(r) P_1(\cos \theta) \hat{\mathbf{r}}), \quad (9)$$

where $\hat{\mathbf{z}}$ is the unit vector along the rotation axis.

In numerically solving (6), (7) and (8) to derive fluid motion in the core, the toroidal and poloidal scalar functions and their partial derivatives with respect to r must be expressed in terms of finite differences or series of orthogonal functions. In this paper, we rely on the latter, since derivatives can be accurately evaluated and a possible numerical instability arising from the finite difference scheme can be avoided. We expand the scalar functions in Chebyshev polynomials to describe their radial dependence, as discussed in GLATZMAIER (1984). For example, the scalar function for a toroidal velocity field with degree l and order m is expressed as

$$V[T_l^{mz}](r, t) = \sum_{n=0}^N {}'V[{}_nT_l^{mz}](t) T_n(x), \quad (10)$$

where $T_n(x)$ is a Chebyshev polynomial of order n defined as

$$T_n(x) = \cos(n \cos^{-1} x). \quad (11)$$

N is the truncation level in the order of Chebyshev polynomials, and the summation $\sum'$ means that the $n = 0$ and $n = N$ terms are multiplied by $\frac{1}{2}$. The radial coordinate r in the outer core is transformed into the coordinate x by

$$x = \frac{2r - r_{oc} - r_{ic}}{r_{oc} - r_{ic}}, \quad (12)$$

where r_{oc} and r_{ic} denote the radii of the outer and inner cores, respectively.

For simulation of global convection and magnetic field generation with a Chebyshev collocation method, GLATZMAIER (1984) used the Chebyshev mesh-points defined as

$$x_k = \cos\left(\frac{k\pi}{N}\right) \quad k = 0, 1, \dots, N. \quad (13)$$

In this paper, however, we do not use the Chebyshev mesh-points, since the values of the first-order derivative of the Chebyshev polynomial with maximum order N are always zero except for $k = 0$ and $k = N$.

In the present method of deriving fluid motion in the core, the terms associated with the first-order derivative of Chebyshev polynomials appear in the coefficient matrix of simultaneous equations to be solved. For example, the term $[B[T_\alpha]B[S_\beta]S_\gamma]$ in (8) is expressed as

$$\begin{aligned} [B[T_\alpha]B[S_\beta]S_\gamma] &= \frac{K_{\alpha\beta\gamma}}{2N_\gamma} \left[\alpha(\alpha+1)\{\alpha(\alpha+1) - \beta(\beta+1) - \gamma(\gamma+1)\} B[T_\alpha] \frac{\partial B[S_\beta]}{\partial r} \right. \\ &\quad \left. + \beta(\beta+1)\{\alpha(\alpha+1) - \beta(\beta+1) + \gamma(\gamma+1)\} \frac{\partial B[T_\alpha]}{\partial r} B[S_\beta] \right], \quad (14) \end{aligned}$$

as shown in FRAZER (1974). $K_{\alpha\beta\gamma}$ and N_γ are defined as

$$K_{\alpha\beta\gamma} = \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} Y_\alpha(\theta, \phi) Y_\beta(\theta, \phi) Y_\gamma(\theta, \phi) \sin \theta d\theta d\phi$$

and

$$N_\gamma = N_l^m = \frac{2\pi l(l+1)}{2l+1} \frac{(l+m)!}{(l-m)!} (1 + \delta_{m0}),$$

where $Y_\alpha(\theta, \phi) = P_l^m(\cos \theta) \cos m\phi$ or $P_l^m(\cos \theta) \sin m\phi$ and δ_{ij} is the Kronecker's delta. No information, however, is obtained from the terms containing the first-derivative of maximum order N of Chebyshev polynomials. It is therefore inappropriate to use the Chebyshev mesh-points. Instead, we use the equi-interval mesh-points between the radii of the inner and outer cores; that is,

$$x_k = \frac{2k}{N} - 1. \quad (15)$$

Hence we do not directly use the orthogonality of orthogonal functions; we just approximate the scalar functions using the values at all the mesh-points.

Now, Eqs. (6), (7) and (8) are satisfied at the mesh-points for $k = 1$ to $k = N - 1$. At the mesh-points for $k = 0$ and $k = N$, corresponding to the inner core boundary (ICB) and the CMB, boundary conditions are required to specify the problem and solve the second-order differential equations. We then have $N + 1$ equations to determine $N + 1$ unknowns for each set of (l, m, z) .

2.2 Boundary conditions

Boundary conditions for magnetic fields are such that the normal component of $\mathbf{B}$ and the tangential components of $\mathbf{H} = \mathbf{B}/\mu$ must be continuous at boundaries.

The magnetic field outside the Earth is expressed in terms of the magnetic potential W as

$$\mathbf{B}(r, \theta, \phi, t) = -\nabla W(r, \theta, \phi, t) \quad (16)$$

and

$$W(r, \theta, \phi, t) = r_e \sum_{l=1}^L \left(\frac{r_e}{r} \right)^{l+1} \sum_{m=0}^l \{g_l^m(t) \cos m\phi + h_l^m(t) \sin m\phi\} \varepsilon_l^m P_l^m(\cos \theta), \quad (17)$$

where g_l^m 's and h_l^m 's are the Gauss coefficients and r_e denotes the Earth's mean radius. The Schmidt spherical functions have been used in spherical harmonic analysis of the Earth's magnetic field. We, however, use associated Legendre functions and require, in (17), ε_l^m defined as

$$\varepsilon_l^m = \left\{ \frac{(l-m)!}{(l+m)!} (2 - \delta_{m0}) \right\}^{1/2} \quad (18)$$

The magnetic field just above the core surface is obtained through downward continuation provided that the mantle is an insulator. This procedure is reasonable, since we deal with a magnetic field of long length-scale, expressed in terms of spherical

harmonics of low degrees, which varies with a rather long time-scale. The boundary conditions for $B[S_l^{mc}]$ at $r = r_{oc}$, for example, lead to

$$\frac{l(l+1)}{r_{oc}^2} B[S_l^{mc}](r_{oc}, t) = (l+1) \left(\frac{r_e}{r_{oc}} \right)^{l+2} g_l^m(t) \varepsilon_l^m \quad (19)$$

and

$$\frac{1}{r_{oc}} \frac{\partial B[S_l^{mc}](r_{oc}, t)}{\partial r} = - \left(\frac{r_e}{r_{oc}} \right)^{l+2} g_l^m(t) \varepsilon_l^m. \quad (20)$$

From (19) and (20), we obtain

$$r_{oc} \frac{\partial B[S_l^{mc}](r_{oc}, t)}{\partial r} + l B[S_l^{mc}](r_{oc}, t) = 0. \quad (21)$$

For $B[S_l^{ms}](r, t)$, the same condition is derived at $r = r_{oc}$. On the other hand, toroidal magnetic fields must vanish at the CMB because the mantle is insulating;

$$B[T_\gamma](r_{oc}, t) = 0. \quad (22)$$

Boundary conditions for poloidal magnetic fields at the ICB yield

$$B[S_\gamma](r_{ic}, t) = B^{(i)}[S_\gamma](r_{ic}, t), \quad (23)$$

$$\frac{\partial B[S_\gamma](r_{ic}, t)}{\partial r} = \frac{\partial B^{(i)}[S_\gamma](r_{ic}, t)}{\partial r}, \quad (24)$$

where the superscript (i) denotes the magnetic fields in the inner core. Boundary conditions for toroidal magnetic fields are given as

$$B[T_\gamma](r_{ic}, t) = B^{(i)}[T_\gamma](r_{ic}, t), \quad (25)$$

$$\frac{\partial B[T_\gamma](r_{ic}, t)}{\partial r} = \frac{\partial B^{(i)}[T_\gamma](r_{ic}, t)}{\partial r}, \quad (26)$$

where the latter is derived from the specific conditions; that is, the electrical conductivity σ is assumed to be constant throughout the outer and inner cores, the inner core rotates identically with the mantle, and no slip is allowed at the ICB.

When we consider the steady state, the induction equation in the solid inner core is written as

$$\nabla^2 B^{(i)} = 0. \quad (27)$$

From the conditions (23), (24), (25), (26) and the solution of (27), we can express the boundary conditions for steady magnetic fields at the ICB as

$$r_{ic} \frac{dB[T_\gamma](r_{ic})}{dr} - (\gamma + 1) B[T_\gamma](r_{ic}) = 0, \quad (28)$$

and

$$r_{ic} \frac{dB[S_\gamma](r_{ic})}{dr} - (\gamma + 1) B[S_\gamma](r_{ic}) = 0. \quad (29)$$

One boundary condition for velocity fields is that the normal component of the velocity must vanish at boundary surfaces. Two further boundary conditions depend

on the nature of the surface. On a rigid surface, no slip is allowed; that is, not only the radial but also the tangential components must vanish there. On a free surface, tangential viscous stresses must vanish (e.g. CHANDRASEKHAR, 1961). Hence the boundary conditions for velocity fields are

$$V[S_\gamma] = \frac{\partial V[S_\gamma]}{\partial r} = V[T_\gamma] = 0 \quad \text{for a rigid surface,} \quad (30)$$

$$V[S_\gamma] = \frac{\partial^2}{\partial r^2} \left(\frac{V[S_\gamma]}{r} \right) = \frac{\partial}{\partial r} \left(\frac{V[T_\gamma]}{r^2} \right) = 0 \quad \text{for a free surface.} \quad (31)$$

2.3 Procedure of the derivation of fluid motion in the core

Basically we solve the non-linear simultaneous equations (6) for toroidal magnetic fields, (7) for poloidal magnetic fields, and (8) for toroidal velocity fields. If we take into consideration all the spherical harmonic modes up to order and degree L and Chebyshev polynomials up to order N , the total number of equations is $L(L+2)(N+1)$ for a toroidal or a poloidal field. In this paper, we assume the quasi-steady state at various epochs in the sense that the steady state is realized at various epochs, even if magnetic fields vary slowly from one epoch to another.

With regard to poloidal velocity fields, radial dependence of the scalar function is unknown unless the Navier-Stokes and the energy equations are fully solved. It is extremely difficult, however, and beyond the scope of the present study to solve all the equations which describe the physical state in the Earth's core. In view of this difficulty, we prescribe a scalar function with one of the simplest radial forms which satisfy boundary conditions given in (30) or (31). Now we express the scalar function for the poloidal velocity field as

$$\begin{aligned} V[S_l^{mz}](r, t) &= U[S_l^{mz}](t) \xi_l^{mz}(r) \\ &= U[S_l^{mz}](t) \sum_{n=0}^N V[nS_l^{mz}] T_n(x), \end{aligned} \quad (32)$$

where $V[nS_l^{mz}]$'s are constants and correspond to the magnitudes of prescribed radial dependence $\xi_l^{mz}(r)$.

For the rigid CMB and the rigid ICB, we adopt the function as used by MATSU-SHIMA and HONKURA (1989a) or model 3 of WATANABE and YUKUTAKE (1975) defined as

$$\xi_l^{mz}(r) = \xi(r) = r^2 y^3 (1 - y^2)^2, \quad (33)$$

where $y = (r - r_{ic}) / (r_{oc} - r_{ic})$. For the free CMB and the rigid ICB, we adopt the function defined as

$$\xi_l^{mz}(r) = \xi(r) = r^4 (x+1)^2 (x-1) \{ (5-3\eta)x - 7 + 3\eta \}, \quad (34)$$

where $\eta = r_{ic}/r_{oc}$ is the ratio of the inner to outer core radii and x is defined in (12). Here the radial dependence of poloidal velocity field is simply assumed to be independent of degree l and order m of spherical harmonics. The radial dependence for these functions is shown in Fig. 1. In this paper, we do not consider the case of free CMB and free ICB, since the boundary condition for toroidal magnetic fields at the

Fig. 1. Radial dependence of scalar function for the poloidal velocity field. Left-hand-side figures represent the scalar function given in (33), and right-hand-side figures the scalar function given in (34); (a) $\xi(r)$, (b) $\xi(r)/r^2$ and (c) $d\xi(r)/rdr$.

ICB becomes very complicated and another problem arises; a nearly free body-rotation of the outer core is allowed in this case (see Subsection 3.3).

The problem of calculating poloidal velocity fields is thus reduced to that of determining the magnitude $U[S_\gamma](t)$ for a specified epoch. This unknown increases the number of unknowns and now the total number of equation is less than the total number of unknowns. We introduce the boundary condition (19) into the simultaneous equation to be solved. This indicates that the magnitude of poloidal velocity field is constrained by the observed magnetic field. For the Gauss coefficients appearing in the right-hand side of (19), we use a geomagnetic secular variation model (MATSUSHIMA and HONKURA, 1988) which will be described in the next subsection.

Now unknowns are $U[S_l^{mz}]$, $V[nT_l^{mz}]$, $B[nS_l^{mz}]$, and $B[nT_l^{mz}]$ for a certain epoch. Setting the time derivatives at zero for steady solutions of Eqs. (6), (7) and (8), we

solve the non-linear simultaneous algebraic equations by using a Newton method.

It is convenient to treat the equations in dimensionless form. In actual calculations, we scale the equations in terms of typical strength of magnetic field B_s , typical magnitude of velocity field V_s , and typical length L_s . In this paper, we adopt $B_s = 10^{-2}$ T, $V_s = 10^{-4}$ m s $^{-1}$, and $L_s = 3.485 \times 10^6$ m (outer core radius).

2.4 Magnetic field data

Secular variations in the geomagnetic field have been investigated in terms of time variations of Gauss coefficients. In particular, special emphasis has been put on the westward drift of non-dipole, to be exact, non-axisymmetric magnetic fields (YUKUTAKE and TACHINAKA, 1969; WANG and QI, 1983; YUKUTAKE, 1985; SHIMIZU and HONKURA, 1986; MATSUSHIMA and HONKURA, 1988). YUKUTAKE and TACHINAKA (1969) pointed out that the non-axisymmetric magnetic field can be separated into standing and drifting parts. Meanwhile, MATSUSHIMA and HONKURA (1988) introduced fluctuations in the amplitude of the standing and also the drifting parts, in view of a non-linear nature of geodynamo process, and approximated them by periodic time variations.

For the magnetic field data, we use the model C of MATSUSHIMA and HONKURA (1988), since $\langle \Delta Z^2 \rangle$ values, which are root-mean-square residuals of the vertical component of the magnetic field, are smaller in general than those for the Yukutake and Tachinaka model, the Wang and Qi model, and the models A and B of MATSUSHIMA and HONKURA (1988). The total number of variables is different from one model to another, but the AIC values (AKAIKE, 1974), a criterion for judging which model is more significant in the case of the uneven number of variables, are generally the smallest for the model C. For $(l, m) = (6, 4)$ and $(6, 5)$, however, $\langle \Delta Z^2 \rangle$ values and AIC values in the model C are larger than the corresponding ones in the model B. This indicates that fluctuations are negligible for these modes of the standing part.

In the model C, Gauss coefficients g_l^m and h_l^m are represented, respectively, as

$$\begin{aligned} g_l^m(t) &= \left\{ F_l^m + E_l^m \sin \frac{2\pi}{T_{El}^m} (t - \tau_{El}^m) \right\} \cos \psi_l^m \\ &+ \left\{ K_l^m + D_l^m \sin \frac{2\pi}{T_{Dl}^m} (t - \tau_{Dl}^m) \right\} \cos mv_l^m (t - \tau_l^m), \\ h_l^m(t) &= - \left\{ F_l^m + E_l^m \sin \frac{2\pi}{T_{El}^m} (t - \tau_{El}^m) \right\} \sin \psi_l^m \\ &- \left\{ K_l^m + D_l^m \sin \frac{2\pi}{T_{Dl}^m} (t - \tau_{Dl}^m) \right\} \sin mv_l^m (t - \tau_l^m), \end{aligned} \quad (35)$$

where F_l^m and ψ_l^m denote the amplitude and the phase angle, E_l^m the amplitude of periodic variation, T_{El}^m the period, and τ_{El}^m the phase in units of time, for the standing parts. Similarly, K_l^m and τ_l^m denote the amplitude and the phase in units of time, D_l^m the amplitude of periodic variation, T_{Dl}^m the period, and τ_{Dl}^m the phase in units of time, for the drifting parts. A positive value of velocity v_l^m corresponds to the westward drift. The time t is defined here as $t = (T - 1800)/100$ with T indicating the year (AD). Parameter values used in the present study are the same as those used by MATSUSHIMA and HONKURA (1989a).

2.5 Physical constants and properties of the Earth and the core

Some physical constants and properties of the Earth and the core must be known in order for a realistic calculation of fluid motion in the core to be possible. Some parameters for properties of the Earth and the core are rather well determined from direct observations such as seismic ones; for example, the core radius, the mean density of the core, and the rotation rate of the Earth.

One unknown parameter is the electrical conductivity σ of the core, but it has been deduced in various ways. ELSASSER (1946) took the value as $\sigma = 10^6 \text{ S m}^{-1}$, inferring from the electrical conductivity of iron under ordinary laboratory conditions, while BULLARD (1949) took the value as $\sigma = 3 \times 10^5 \text{ S m}^{-1}$. Experiments made for core material at temperatures and pressures far lower than those in the core, indicate that $\sigma = 1 \times 10^5 \sim 6 \times 10^5 \text{ S m}^{-1}$ (GARDINER and STACEY, 1971) and $\sigma = 2.7 \times 10^5 \text{ S m}^{-1}$ (JOHNSTON and STRENS, 1973). JAIN and EVANS (1972) proposed $\sigma = 5 \times 10^5 \sim 1 \times 10^6 \text{ S m}^{-1}$ from a model for electrical transport properties of core material. An upper limit, 10^6 S m^{-1} , for the electrical conductivity of the core was imposed by considering the amount of heat transferred from the outer core to the mantle (STACEY, 1972). Recent high-pressure experiments indicate that the electrical conductivity of the outer core is close to $1 \times 10^6 \text{ S m}^{-1}$ (e.g. KNITTLE *et al.*, 1986).

HONKURA and MATSUSHIMA (1988a) estimated a lower limit for the electrical conductivity of the core, considering the magnetic Reynolds number R_m defined as $R_m = \mu_0 \sigma L_c V_c$, where L_c and V_c denote the characteristic length and velocity, respectively. For various values of σ , V_c was evaluated using the method of MATSUSHIMA and HONKURA (1989a). Then from the viewpoint of a steady dynamo, HONKURA and MATSUSHIMA (1988a) claimed that the smallest possible R_m for dynamo action is the order of ten and concluded that the electrical conductivity of the outer core σ must be larger than about $1 \times 10^5 \text{ S m}^{-1}$. Thus the electrical conductivity of the core is supposed to be in the range of $10^5 \sim 10^6 \text{ S m}^{-1}$, and hence in this paper we adopt $\sigma = 3 \times 10^5 \text{ S m}^{-1}$.

The kinematic viscosity ν of the fluid core seems to be the most uncertain parameter. For example, BACKUS (1968) inferred the value as $\nu = 5 \times 10^{-3} \text{ m}^2 \text{ s}^{-1}$. SUZUKI and SATO (1970) estimated the value as $\rho\nu = 3 \times 10^9 \sim 7 \times 10^9 \text{ N m}^{-2} \text{ s}$ from attenuation of seismic waves. GANS (1972) theoretically derived the value as $\rho\nu = 3.7 \times 10^{-3} \sim 18.5 \times 10^{-3} \text{ N m}^{-2} \text{ s}$. If we take the mean density of the core as $\rho = 1.1 \times 10^4 \text{ kg m}^{-3}$, the kinematic viscosity of the core is in the range of $10^{-7} \sim 10^6 \text{ m}^2 \text{ s}^{-1}$.

It is probable that smaller values correspond to molecular viscosity and larger ones to eddy viscosity. In usual description of dynamics of large-scale motions, the interaction between large-scale and smaller-scale motions is ignored, but it is important when the transfer of energy and momentum between the scale of interest and the smaller scale is to be discussed (PEDLOSKY, 1986). In meteorological phenomena, eddy viscosity is usually considered; for example, a cellular structure like Bénard convection and a vortex street similar to the Kármán vortex street seen in a photo of cloud pattern are explained in terms of eddy viscosity (KIMURA, 1988). In this sense, eddy viscosity would be relevant to this study, although it is beyond the present scope to consider the effect of smaller-scale motions like turbulence on magnetic field generation. Here

Table 1. Physical constants and properties of the Earth and the core.

Quantity	Symbol	Value
Magnetic permeability of vacuum	μ_0	$4\pi \times 10^{-7} \text{ H m}^{-1}$
Equivolume sphere radius of Earth	r_e	$6.371 \times 10^6 \text{ m}$
Core radius	r_{oc}	$3.485 \times 10^6 \text{ m}$
Inner core radius	r_{ic}	$1.217 \times 10^6 \text{ m}$
Mean density of core	ρ	$1.1 \times 10^4 \text{ kg m}^{-3}$
Rotation rate of Earth	Ω	$7.29 \times 10^{-5} \text{ s}^{-1}$
Electrical conductivity	σ	$3 \times 10^5 \text{ S m}^{-1}$
Kinematic viscosity	ν	$1 \text{ m}^2 \text{ s}^{-1}$

we assume that the kinematic viscosity of the fluid core amounts to $\nu = 1 \text{ m}^2 \text{ s}^{-1}$ [†].

Values of physical constants and properties of the Earth and the core used in this paper are summarized in Table 1.

3. Results

3.1 The previous results

MATSUSHIMA and HONKURA (1989a) derived fluid motion in the core, presuming that a strong toroidal magnetic field $\mathbf{B}[T_2^0]$ would be generated by the interaction between a differential rotation $\mathbf{V}[T_1^0]$ and the axial dipole magnetic field $\mathbf{B}[S_1^0]$ in the core. The factors taken into consideration in the calculation are $\nabla \times (\mathbf{V}[T_1^0] \times \mathbf{B}[S_1^0])$ (the ω -effect), $\nabla \times (\mathbf{V}[S_l^m] \times \mathbf{B}[T_2^0])$ as induction processes, the magnetic diffusion, and the balance, in spherical shells, between the electromagnetic torque due to the Lorentz force $(\mu_0)^{-1}(\nabla \times \mathbf{B}[T_2^0]) \times \mathbf{B}[S_1^0]$ and the effective torque due to the angular momentum transfer expected from virtual displacement of fluid particles. The torque balance yielded the differential rotation $\mathbf{V}[T_1^0]$. MATSUSHIMA and HONKURA (1989b) further extended the method and took into consideration the meridional dependence of differential rotation as well; that is, $\mathbf{V}[T_l^0]$ and $\mathbf{B}[T_l^0]$ ($l \leq 6$) were all considered.

We first show the results of MATSUSHIMA and HONKURA (1989b; unpublished result) in Fig. 2; (a) the distribution of angular velocity, (b) the distribution of strength of the zonal toroidal magnetic field, and (c) the poloidal velocity field on the equatorial and the two meridional planes at the epoch of 1980 AD.

It is clearly seen from Fig. 2(a) that the differential rotation cannot be represented by $\mathbf{V}[T_1^0]$, even in an approximate sense, contrary to MATSUSHIMA and HONKURA (1989a), and the meridional dependence is striking. The zonal toroidal motion is westward near the equator and eastward near the poles. The zonal toroidal magnetic field generated by the interaction between the axial dipole magnetic field and the differential rotation mainly consists of $\mathbf{B}[T_2^0]$ and $\mathbf{B}[T_4^0]$ with the magnitude of the order of $10^{-3} \sim 10^{-2} \text{ T}$, as seen in Fig. 2(b). The magnitude of poloidal velocity field shown in Fig. 2(c) is much smaller than that of MATSUSHIMA and HONKURA (1989a). This discrepancy is interpreted as follows.

[†]We found that the derived fluid motions are not very different for ν 's ranging from $10^{-6} \text{ m}^2 \text{ s}^{-1}$ to $10^2 \text{ m}^2 \text{ s}^{-1}$. With larger ν (e.g. $10^4 \text{ m}^2 \text{ s}^{-1}$) the differential rotation is reduced considerably. In this paper we adopted $\nu = 1 \text{ m}^2 \text{ s}^{-1}$.

Fig. 2. (a) Distribution of angular velocity in the outer core. Solid contours indicate positive (eastward) values in units of $10^{-9} \text{ rad s}^{-1}$ and broken contours negative (westward) values. (b) Distribution of zonal toroidal magnetic fields. Solid contours indicate positive values in units of $2 \times 10^{-3} \text{ T}$ and broken contours negative values. (c) Poloidal velocity fields on the equatorial plane and on the meridional planes for the epoch of 1980 AD. The arrow for scale length corresponds to $5 \times 10^{-6} \text{ m s}^{-1}$.

We may define the magnetic Reynolds number characteristic of the ω -effect R_ω as

$$R_\omega = \omega_c L_c^2 / \nu_m, \quad (36)$$

where ω_c is a characteristic angular velocity and L_c a characteristic length scale. The radial motion which induces poloidal magnetic fields from toroidal magnetic fields tends to become weaker with increasing R_ω (ROBERTS and GUBBINS, 1987). In MATSUSHIMA and HONKURA (1989a), only $\mathbf{B}[T_2^0]$ was considered as the inducing magnetic field and strong toroidal magnetic field regions were located in the deeper parts of the core. In the case shown in Fig. 2, the inducing magnetic fields are $\mathbf{B}[T_l^0]$ ($l \leq 6$) and some of the strong toroidal magnetic field regions are located near the CMB. It is then expected that a poloidal magnetic field is easily induced by a weak poloidal velocity field in the presence of such a strong toroidal magnetic field. This represents one class of typical velocity field in $\alpha\omega$ -dynamos; that is, a strong ω -effect due to a strong zonal toroidal motion combined with a weak poloidal motion.

3.2 Reexamination of the previous results (Case 1)

We here reexamine the results of MATSUSHIMA and HONKURA (1989b; unpublished work) as Case 1. In the induction equation, we take into consideration $\nabla \times (\mathbf{V}[T_l^0] \times \mathbf{B}[S_1^0])$, $\nabla \times (\mathbf{V}[S_\alpha] \times \mathbf{B}[T_l^0])$, and the magnetic diffusion as in MATSUSHIMA and HONKURA (1989b), whereas in the Navier-Stokes equation for toroidal motion, the Lorentz force $(\mu_0)^{-1}(\nabla \times \mathbf{B}[T_l^0]) \times \mathbf{B}[S_1^0]$ and the advection terms are considered, since a differential rotation would arise from the non-linear momentum transport of convection (BUSSE, 1970).

No information is available on the radial dependence of the axial dipole magnetic field $\mathbf{B}[S_1^0]$ in the core. We simply adopt the radial dependence for the longest free-decay mode as in MATSUSHIMA and HONKURA (1989a). Then the scalar function is expressed as

$$B[S_1^0](r) = B_{S10} r^{1/2} J_{3/2}(\pi r / r_{oc}), \quad (37)$$

where $J_{3/2}(z)$ is a Bessel function. B_{S10} is a constant and can be determined by the boundary condition at $r = r_{oc}$.

We assume that both the CMB and the ICB are rigid boundaries as in MATSUSHIMA and HONKURA (1989a), and the radial dependence of poloidal velocity field given in (33) is used. We tentatively set the truncation levels L and N at 4 and 9, respectively. Unknowns are zonal toroidal velocity fields $V[_n T_l^0]$, non-axisymmetric poloidal velocity fields $U[_l S_l^{mz}]$, zonal toroidal magnetic fields $B[_n T_l^0]$, and non-axisymmetric poloidal magnetic fields $B[_n S_l^{mz}]$. The total number of unknowns is 300.

The result derived by solving both the induction and the Navier-Stokes equations is completely different from the previous one, as shown in Fig. 3; (a) the distribution of angular velocity and (b) the distribution of zonal toroidal magnetic field at the epoch of 1980 AD. The pattern of zonal toroidal velocity field may seem to be similar to that shown in Fig. 2(a) in that the flow is westward near the equator and eastward near the poles, but the magnitude of zonal toroidal motion is much smaller than that of zonal toroidal motion shown in Fig. 2(a). It is true that a zonal toroidal magnetic field is generated by the interaction between the dipole magnetic field and the differential rotation (shear motion), but a slow zonal toroidal motion results in insufficient shear, and hence the strength of zonal toroidal magnetic field, generated by the ω -effect,

shown in Fig. 3(b) is much weaker than that in Fig. 2(b). The magnitude of poloidal velocity field is much larger than that in Fig. 2(c), accordingly.

The overall velocity field is shown in Fig. 4. Velocity vectors are plotted, in Fig. 4(a), at mesh points regularly distributed in colatitude θ and longitude ϕ on a cylindrical equidistant projection map. The length of each vector is linearly proportional to the flow speed at each mesh point, and the arrow for scale length is shown at the lower right of the figure. The outlines of the continents are shown only for reference. In Fig. 4(b), are shown the velocity fields on the equatorial plane (center) and on the meridional planes passing through $\phi = 0^\circ$ – 180° (left) and $\phi = 90^\circ$ – 270° (right). The arrow for scale length is plotted as in Fig. 4(a). It is clearly seen from Fig. 4 that the poloidal velocity field is dominant and the flow speed is very fast. Judging from the result that the zonal toroidal magnetic fields are not so strong, the presumption that non-axisymmetric poloidal magnetic fields are generated only by the interaction between zonal toroidal magnetic fields and poloidal convective motions cannot be justified under the condition which we imposed for Case 1.

3.3 Introduction of meridional circulation (Case 2)

So far, meridional circulation has not been taken into consideration in deriving fluid motion in the core. Such a poloidal motion can create a zonal toroidal motion through the interaction term $(V[S_\alpha]S_1^\Omega S_\gamma)$ in (8). Therefore, we add a meridional poloidal

Fig. 3. (a) Distribution of angular velocity in the outer core. Solid contours indicate positive (eastward) values in units of $2 \times 10^{-12} \text{ rad s}^{-1}$ and broken contours negative (westward) values. (b) Distribution of zonal toroidal magnetic fields. Solid contours indicate positive values in units of $2 \times 10^{-5} \text{ T}$ and broken contours negative values. These were derived for Case 1.

Fig. 4. (a) Horizontal velocity field at $r = 0.95r_{oc}$ and (b) the overall velocity fields on the equatorial plane and on the meridional planes for the epoch of 1980 AD in Case 1. The arrow for scale length corresponds to $2 \times 10^{-2} \text{ m s}^{-1}$.

velocity field $\mathbf{V}[S_2^0]$, which gives rise to $\mathbf{V}[T_1^0]$ and $\mathbf{V}[T_3^0]$. This meridional motion possesses only one convection cell in the Northern and the Southern Hemispheres, respectively. The magnitude $U[S_2^0]$ is unknown, however, as long as the derivation of fluid motion in the core is based on the presumption that the inducing magnetic fields for the generation of poloidal magnetic fields are zonal toroidal magnetic fields alone, generated by the ω -effect. This is because the interaction of meridional motion with a zonal toroidal magnetic field $\nabla \times (\mathbf{V}[S_\alpha^0] \times \mathbf{B}[T_\beta^0])$ generates no poloidal magnetic field, as verified from one of the selection rules given in BULLARD and GELLMAN (1954).

We here take the magnitude $U[S_2^0]$ as a parameter to examine the effect of meridional circulation. This is Case 2.

In kinematic dynamo models considered by ROBERTS and STIX (1972), for example, the value of $U[S_2^0]$ is negative in the case that dynamo action occurs most easily. A negative value of $U[S_2^0]$ corresponds to a meridional flow in which the fluid rises near the equator and sinks near the poles. Here we consider only negative values of $U[S_2^0]$.

For $U[S_2^0] = -10^{-5}$ with the radial dependence given in (33), it turned out that the distributions of angular velocity and zonal toroidal magnetic fields are nearly the same as in Fig. 3; that is, too slow meridional circulation little affects zonal toroidal motion.

Once we take a larger value such as $U[S_2^0] = -10^{-3}$, however, the results are much different. Figure 5 shows the distributions of angular velocity and zonal toroidal magnetic field in the core for $U[S_2^0] = -10^{-3}$. Contour intervals are five times larger than those in Fig. 3. It is then expected that the magnitude of derived poloidal velocity field is reduced, since the inducing magnetic fields are more enhanced than in Case 1.

Figure 6 shows the overall velocity field, the pattern of which is different from the previous results, but the magnitude is not significantly different. This may be due to non-linearity of the problem or a slight difference in the zonal toroidal magnetic field distribution. Whichever is the case, a problem remains; that is, even if a meridional poloidal motion is introduced, the strength of zonal toroidal magnetic field is very

Fig. 5. (a) Distribution of angular velocity in the outer core. Solid contours indicate positive (eastward) values in units of $1 \times 10^{-11} \text{ rad s}^{-1}$ and broken contours negative (westward) values. (b) Distribution of zonal toroidal magnetic fields. Solid contours indicate positive values in units of $1 \times 10^{-4} \text{ T}$ and broken contours negative values. These were derived for Case 2.

Fig. 6. (a) Horizontal velocity field at $r = 0.95r_{oc}$ and (b) the overall velocity fields on the equatorial plane and on the meridional planes for the epoch of 1980 AD in Case 2. The arrow for scale length corresponds to $2 \times 10^{-2} \text{ m s}^{-1}$.

weak, of the order of 10^{-4} T . Hence we can not rely on the presumption that strong zonal toroidal magnetic fields, generated by the ω -effect, are the primary inducing magnetic fields for the generation of the non-axisymmetric poloidal magnetic fields, at least under the condition we imposed in Case 2.

In addition, we must consider another problem. We have so far employed rigid boundary conditions at the CMB and the ICB. These conditions may not be reasonable. The Ekman number E , defined as $E = \nu / \Omega L_c^2$, is of the order of 10^{-9} for the kinematic viscosity $\nu = 1 \text{ m}^2 \text{ s}^{-1}$, assumed in this paper. If ν is assumed to amount to 10^{-6}

$\text{m}^2 \text{s}^{-1}$, which would correspond to the molecular viscosity, E is further reduced to 10^{-15} . This suggests that an Ekman layer $10^{-1} \sim 10^2 \text{ m}$ thick would be set up, and in this layer, fluid flow would adjust itself to the physical boundary condition imposed for the contact with a solid body. It is impossible to resolve the boundary layer with the present truncation level for Chebyshev polynomials, $N = 9$ or so. It would be better, therefore, to impose stress-free boundary conditions.

If both the CMB and the ICB are stress-free, however, a problem arises for a zonal toroidal velocity field $\mathbf{V}[T_1^0]$. We have expressed the radial dependence, $V[T_1^0](r)$, in Chebyshev polynomials. Alternatively, it is possible to expand it into power series with respect to r . These should be equivalent to each other. For the term of r^2 in $V[T_1^0]$, the diffusion term vanishes; that is,

$$\left(\frac{d^2}{dr^2} - \frac{2}{r^2} \right) r^2 = 0.$$

Moreover, in the equation governing the boundary condition for free surfaces, the coefficient for the term of r^2 cannot be constrained, as is easily understood in (31). Then any free-body rotation of the outer core may be allowed at the CMB because the mantle is assumed to be an insulator. In reality, this is unlikely because strong shear at the ICB, due to the free-body rotation, would generate a strong magnetic field, which should work against the zonal motion. Hence, in the remainder of this paper, we regard the CMB as free and the ICB as effectively rigid.

3.4 The stress-free condition at the CMB

For the stress-free CMB and the rigid ICB, we take the radial dependence of poloidal velocity field given in (34). We take into consideration the same interaction terms as in Case 1, and derive fluid motion in the Earth's outer core. This is Case 3. Figure 7 shows the distributions of angular velocity and zonal toroidal magnetic field in the core. The distribution of angular velocity is clearly different from that shown in Fig. 3(a), whereas the distribution of zonal toroidal magnetic field is similar to that shown in Fig. 3(b). These resulted from the stress-free CMB condition.

The magnitude of angular velocity shown in Fig. 7(a) is larger than that in Fig. 3(a), but no marked difference can be seen between the strength of zonal toroidal magnetic fields shown in Figs. 7(b) and 3(b). This implies that the distribution of angular velocity in the core under the stress-free CMB condition cannot generate a strong zonal toroidal magnetic field. Hence the presumption that non-axisymmetric poloidal magnetic fields are generated only by the interaction between zonal toroidal magnetic fields and poloidal convective motions would also fail even if we adopt the stress-free CMB condition.

As Case 4, we add meridional circulation, as in Case 2, under the stress-free CMB condition. In Fig. 8, we show the distributions of angular velocity and zonal toroidal magnetic field for $U[S_2^0] = -10^{-4}$. We should remember that the radial dependence of poloidal velocity field is different from that in Case 2. We now consider the root-

Fig. 7. (a) Distribution of angular velocity in the outer core. Solid contours indicate positive (eastward) values in units of $2 \times 10^{-10} \text{ rad s}^{-1}$ and broken contours negative (westward) values. (b) Distribution of zonal toroidal magnetic fields. Solid contours indicate positive values in units of $4 \times 10^{-5} \text{ T}$ and broken contours negative values. These were derived for Case 3.

mean-square (rms) velocity defined as

$$V_{\text{rms}} = \left(\frac{\int V^2 dv}{\int dv} \right)^{1/2}, \quad (38)$$

where the volume integral is performed for the outer core, for $U[S_2^0] = -10^{-5}$. Then the radial dependence (33) leads to $V_{\text{rms}}[S_2^0] = 3.90 \times 10^{-10} \text{ m s}^{-1}$, whereas (34) leads to $V_{\text{rms}}[S_2^0] = 1.54 \times 10^{-8} \text{ m s}^{-1}$; the difference is remarkable.

Although the meridional circulation thus introduced in Case 4 also enhances the magnitude of zonal toroidal motion, and the zonal toroidal magnetic fields are strengthened, they are not sufficient enough to justify the basic presumption. The stress-free CMB would rather tend to align the lines of constant differential rotation on meridional planes in parallel to the magnetic lines of force for the axisymmetric poloidal magnetic field. No strong zonal toroidal magnetic fields would then be generated.

4. Time-Dependent Behavior of the Magnetic Field

As mentioned in the previous section, we should somehow check the validity of solutions. We examine the results obtained so far from a different point of view.

Fig. 8. (a) Distribution of angular velocity in the outer core. Solid contours indicate positive (eastward) values in units of $2 \times 10^{-9} \text{ rad s}^{-1}$ and broken contours negative (westward) values. (b) Distribution of zonal toroidal magnetic fields. Solid contours indicate positive values in units of $2 \times 10^{-4} \text{ T}$ and broken contours negative values. These were derived for Case 4.

Respective results have been obtained by solving the induction and the Navier-Stokes equations for the steady-state case on some assumptions. Whether the magnetic field is in fact steady under the conditions we imposed can be tested by examining time-dependent behavior of the magnetic field with the derived velocity field fixed. If the assumption made in deriving fluid motion in the core is inappropriate, the magnetic field would vary rapidly within a short time, because some induction terms, which are important in maintaining a steady magnetic field, must have been omitted.

4.1 The numerical method

We employ the same mathematical formulation as described in Section 2; the velocity and the magnetic fields are expressed in terms of toroidal and poloidal vector fields, and their scalar functions are expanded in spherical harmonics for the θ - and ϕ -dependence and in Chebyshev polynomials for the r -dependence as given in (5) and (10). Equations (6) and (7) obtained from the orthogonality of toroidal and poloidal fields and of spherical harmonics are satisfied at the mesh-points given in (15) except the boundaries, where boundary conditions given in (21) to (26) are imposed. In addition, the magnetic field must be finite at $r = 0$, and we obtain

$$B^{(i)}[T_\gamma](0, t) = 0, \quad (39)$$

$$B^{(i)}[S_\gamma](0, t) = 0. \quad (40)$$

There is no motion inside the solid inner core, and the induction equation is reduced to the diffusion equation. The scalar functions for magnetic fields inside the inner core $B^{(i)}[T_\gamma](r, t)$ and $B^{(i)}[S_\gamma](r, t)$ are also expanded in spherical harmonics and Chebyshev polynomials. The radial coordinate r in the inner core is transformed into coordinate x by

$$x = \frac{2r - r_{ic}}{r_{ic}}. \quad (41)$$

Since we have expanded all the scalar functions $B[T_\gamma](r, t)$, $B^{(i)}[T_\gamma](r, t)$, $B[S_\gamma](r, t)$, and $B^{(i)}[S_\gamma](r, t)$ in Chebyshev polynomials, the partial differential equations (6) and (7) are reduced to ordinary differential equations with respect to t for $B[nT_l^{mz}](t)$, $B^{(i)}[nT_l^{mz}](t)$, $B[nS_l^{mz}](t)$ and $B^{(i)}[nS_l^{mz}](t)$. We then have $2(N+1)$ variables and $2(N+1)$ equations for a set of (l, m, z) of toroidal or poloidal magnetic field; $2(N-1)$ equations correspond to the induction equations at the mesh-points x_k , given in (15), for $k = 1$ to $k = N-1$ in the inner and outer cores, and the other four are the boundary conditions. The $2(N+1)$ equations are combined into one matrix equation. The diffusion and induction terms on the right-hand-side of the matrix equation are calculated at each mesh-point. The matrix, $2(N+1)$ by $2(N+1)$, is independent of the order m of spherical harmonics and time t , since the matrix for the poloidal magnetic field depends only on the degree l of spherical harmonics through the boundary condition at the CMB. Then one matrix for each degree l of poloidal magnetic field and that for toroidal magnetic field are subjected to LU decomposition and stored onto the computer memory. We use the fourth-order Runge-Kutta scheme for time integration. In order to guarantee the stability of numerical calculation, the time step Δt should be less than

$$\left| \frac{\Delta r}{V_r} \right|_{\min}, \quad \left| \frac{r}{\{L(L+1)(V_\theta^2 + V_\phi^2)\}^{1/2}} \right|_{\min}, \quad (42)$$

where Δr is the distance between mesh-points, V_r , V_θ and V_ϕ denote the r -, θ - and ϕ -components of velocity field, respectively (e.g. GLATZMAIER, 1984); that is, Δt is the time during which a fluid particle at a mesh-point does not proceed beyond the next mesh-point. We should also be careful in selecting Δt to avoid strange behavior originating in the diffusion term.

In actual calculations, the induction equation is expressed in non-dimensional form. As mentioned in Section 2, we scale the equation by measuring the magnetic field $\mathbf{B}$ in units of typical strength of magnetic field B_s , the velocity field $\mathbf{V}$ in units of typical magnitude of velocity field V_s , length in units of typical length L_s . Time t is measured in units of the magnetic diffusion time L_s^2/ν_m . Then the induction equation, in vector form, becomes

$$\frac{\partial \mathbf{B}}{\partial t} = R_m \nabla \times (\mathbf{V} \times \mathbf{B}) + \nabla^2 \mathbf{B}, \quad (43)$$

where R_m is the magnetic Reynolds number defined as $R_m = V_s L_s / \nu_m \sim 130$. One magnetic diffusion time unit is about 1.45×10^5 years.

4.2 Preliminary result

As a preliminary calculation, we attempt to examine time-dependent behavior of the magnetic field in Case 1. Specifically, the initial values are given from the result for Case 1, and only the same induction terms as adopted in Case 1 are considered;

Fig. 9. Time-dependent behavior of magnetic fields for Case 1 up to $1000\Delta t$ (the time step $\Delta t = 6.92 \times 10^{-7}$). The thick lines indicate non-dimensional values of radial functions at $x = 1$ ($r = r_{oc}$) for poloidal fields and at $x = 0.8$ ($r = 0.93r_{oc}$) for toroidal fields. In this kinematic dynamo calculation, only the induction terms assumed in Case 1 are taken into consideration.

they are $(V[T_\alpha^0]B[S_\beta^0]T_\gamma)$ and $(V[S_\alpha]B[T_\beta^0]S_\gamma)$. The time step Δt is taken as $\Delta t = 6.92 \times 10^{-7}$. The results of time integration up to $1000\Delta t$ for $B[S_1^{1s}]$, $B[S_2^{1s}]$, $B[S_2^{2s}]$, $B[S_3^{1s}]$, $B[S_3^{2c}]$, $B[S_3^{3c}]$, $B[T_1^0]$, and $B[T_4^0]$, with the velocity field fixed, are shown in Fig. 9, where the values of scalar function at $x = 1.0$ ($r = r_{oc}$) are plotted for the dimensionless poloidal magnetic field, and those at $x = 0.8$ ($r = 0.93r_{oc}$) for the dimensionless toroidal magnetic field. It is clear that the steady state is actually realized as long as the same induction terms as adopted in Case 1 are taken into consideration.

4.3 Time-dependent behavior of the magnetic field for various cases

We apply the program code to the results obtained in the previous section and examine time-dependent behavior of the magnetic field; that is, we regard the derived magnetic field as the initial state, and examine a time-dependent kinematic dynamo

Fig. 10. Time-dependent behavior of magnetic fields for Case 1 up to $100\Delta t$ (the time step $\Delta t = 6.92 \times 10^{-7}$). The thick lines indicate non-dimensional values of radial functions at $x = 1$ ($r = r_{oc}$) for poloidal fields and at $x = 0.8$ ($r = 0.93r_{oc}$) for toroidal fields.

for the derived velocity field.

First, we examine time evolution of the magnetic field for the result obtained in Case 1. The velocity field to be considered is shown in Fig. 4. Time integration was performed for a time step $\Delta t = 6.92 \times 10^{-7}$. The result is shown in Fig. 10, which exhibits the values of the scalar functions for poloidal and toroidal magnetic fields at the same r as in Fig. 9. The total time $100\Delta t$ corresponds to 10 years, during which the magnetic fields vary considerably. It is indeed unreasonable that within such a short time interval the equatorial dipole magnetic field $B[S_1^{1c}]$ becomes stronger than the axial dipole magnetic field $B[S_1^{0c}]$.

This result indicates that the velocity field derived on the assumption in Case 1 is unrealistic. The fluid motion in the core was derived on the assumption that strong zonal toroidal magnetic fields are generated by the interaction between the axial dipole magnetic field and the zonal toroidal motion. However, the calculated zonal toroidal magnetic field turned out to be much weaker than we have expected, resulting in a

too large poloidal velocity field. In actual time evolution of the magnetic field, the induction terms which we have neglected are also significant. This is the reason why the magnetic fields vary considerably with time. It is concluded, therefore, that the interaction terms assumed in Case 1 are not sufficient for estimating fluid motion in the core.

Second, time evolution of the magnetic field is examined for the result derived on the assumption in Case 2. The velocity field used in the calculation is shown in Fig. 6; that is, the result for $U[S_2^0] = -10^{-3}$ is adopted here, since there is no significant difference between the velocity fields derived for $U[S_2^0] = 0$ and $U[S_2^0] = -10^{-5}$ as mentioned in Subsection 3.3. Time integration is performed up to $200\Delta t$ with the same time step $\Delta t = 6.92 \times 10^{-7}$ as in the previous calculation, and the result is shown in Fig. 11. During the time interval $200\Delta t$, corresponding to 20 years, the magnetic fields vary considerably. The axial dipole magnetic field $B[S_1^0]$ grows by three times the initial value within such a short time interval. This also indicates that the velocity

Fig. 11. Time-dependent behavior of magnetic fields for Case 2 up to $200\Delta t$ (the time step $\Delta t = 6.92 \times 10^{-7}$). The thick lines indicate non-dimensional values of radial functions at $x = 1$ ($r = r_{oc}$) for poloidal fields and at $x = 0.8$ ($r = 0.93r_{oc}$) for toroidal fields.

field derived on the assumption made in Case 2 is unrealistic even though we take into consideration the meridional circulation $V[S_2^0]$ which gives rise to zonal toroidal velocity fields $V[T_1^0]$ and $V[T_3^0]$. The reason would be the same as for Case 1.

For Cases 3 and 4, we also examined the time evolution of the magnetic field. We then obtained similar results, although we do not show them here. It is concluded that as long as large-scale fluid motion in the core is considered, no strong zonal toroidal magnetic fields are generated through the ω -effect; that is, the geodynamo presently operative in the core is unlikely to be of $\alpha\omega$ -type.

5. Discussion

Case 1 is aimed at reexamining MATSUSHIMA and HONKURA (1989b) in which it is presumed that strong zonal toroidal magnetic fields are generated by the interaction between the axial dipole magnetic field and zonal toroidal motions (the ω -effect), and also that the non-axisymmetric poloidal magnetic fields are generated only by the interaction between such strong zonal toroidal magnetic fields and poloidal convective motions.

The rms velocity, defined by (38), for the poloidal velocity field $V_{\text{rms}}[S] = 3.95 \times 10^{-3} \text{ m s}^{-1}$ is much larger than that for the toroidal velocity field $V_{\text{rms}}[T] = 3.01 \times 10^{-5} \text{ m s}^{-1}$. Thus the poloidal velocity field turns out to be dominant even on the assumption made in Case 1. Since the magnitude of the zonal toroidal motion is small, resulting zonal toroidal magnetic fields, which are supposed to be the primary inducing magnetic fields for the generation of non-axisymmetric poloidal magnetic fields, are also weak, amounting to 10^{-4} T at most. This means that the presumption is not justified. The magnitude of derived poloidal velocity field is too large, as reflected in time-dependent behavior of the magnetic field with the derived velocity field fixed; that is, the magnetic fields vary rapidly. This indicates that the induction term $(V[S_\alpha]B[T_\beta^0]S_\gamma)$, which is assumed to be predominant in the generation of steady poloidal magnetic fields, is not actually dominant; that is, we must take into consideration other interaction terms as well. In view of the fact that weak zonal toroidal magnetic fields are due to a weak zonal toroidal motion, we attempted to increase the interaction terms.

In Case 2, we introduced meridional circulation which affects zonal toroidal motions through the interaction $(V[S_\alpha]S_1^\Omega S_\gamma)$ in (8), and considered the same induction processes as in Case 1. The adopted axisymmetric poloidal velocity field is $\mathbf{V}[S_2^0]$, which can create $\mathbf{V}[T_1^0]$ and $\mathbf{V}[T_3^0]$. Zonal toroidal magnetic fields then become stronger than in Case 1.

The reason why we introduced the axisymmetric poloidal velocity field is as follows. Let us consider steady-state fluid motion, for example, and neglect the advection, the Lorentz force and the viscous terms. Then we obtain

$$2(\boldsymbol{\Omega} \cdot \nabla)\mathbf{V} = -\mathbf{g} \times \nabla C, \quad \mathbf{g} = -g(r)\hat{\mathbf{r}}, \quad (44)$$

taking the curl of (2). The horizontal components of (44) are written as

$$2\Omega \left(\frac{\partial V_\theta}{\partial z} - \frac{V_r \sin \theta}{r} \right) = \frac{g(r)}{r \sin \theta} \frac{\partial C}{\partial \phi}, \quad (45)$$

$$2\Omega \frac{\partial V_\phi}{\partial z} = -\frac{g(r)}{r} \frac{\partial C}{\partial \theta}, \quad (46)$$

which are known as the thermal-wind equations (e.g. PEDLOSKY, 1986). According to BLOXHAM and GUBBINS (1987), even small lateral temperature differences, of the order of 10^{-6} K m^{-1} , can excite thermal winds, and hence zonal toroidal motions would be excited; for instance, if the horizontal dependence of $C(r, \theta, \phi)$ is expressed in terms of $P_2(\cos \theta)$, zonal toroidal motions $\mathbf{V}[T_1^0]$ and $\mathbf{V}[T_3^0]$ are excited. It is further expected from the Navier-Stokes equation (2) that $\mathbf{V}[S_2^0]$ motion would be excited by the $P_2(\cos \theta)$ term of $C(r, \theta, \phi)$. Thus, the meridional circulation term should not be neglected when the generation of toroidal magnetic fields is considered. The importance of meridional circulation is also understood from a different point of view. If Taylor's condition (TAYLOR, 1963), which is known as a dynamical constraint for a steady, slow, and inviscid fluid motion, is satisfied in an $\alpha\omega$ -dynamo without thermal winds, it is known that no $\alpha\omega$ -dynamo will be operative (ROBERTS and GUBBINS, 1987). In view of these, the introduction of meridional circulation would be significant.

In the present study, the magnitude $U[S_2^0]$ has been taken as a parameter. We must examine to what extent the assumed value is appropriate. In approaches to fluid motion near the CMB on the basis of the frozen-flux approximation, another constraint such as toroidal flow or geostrophic flow, for example, can remove the ambiguity of the solution. In the toroidal flow constraint, which would be expected if light material is concentrated near the CMB (FEARN and LOPER, 1981) or the sub-adiabatic temperature gradient is realized at the top of the core, as in some models of the Earth's thermal history (GUBBINS *et al.*, 1982), no poloidal motions are supposed to exist. In the geostrophic flow constraint, which is equivalent to the condition

$$\nabla_H \cdot (\mathbf{V} \cos \theta) = 0, \quad (47)$$

where ∇_H is the horizontal divergence operator given as

$$\nabla_H = \left(0, \frac{1}{r} \frac{\partial}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} \right), \quad (48)$$

axisymmetric poloidal motions should vanish (LE MOUËL *et al.*, 1985; BACKUS and LE MOUËL, 1986). Thus it is expected that even if meridional circulation exists, its magnitude would be very small. It should be noted, however, that constraints such as tangential geostrophy may not hold throughout the core and meridional circulation could be significant within the core.

Meanwhile, YOKOYAMA (1989) claimed that the magnitude of $\mathbf{V}[S_2^0]$ should amount to 5.0×10^{-7} m s^{-1} if a sixty year variation of the geomagnetic field is ascribed to a magnetohydrodynamic (MHD) oscillation in a thin layer beneath the CMB. In fact, MHD oscillations for $\mathbf{B}[S_1^0]$, $\mathbf{B}[T_2^0]$, $\mathbf{V}[S_2^0]$, and $\mathbf{V}[T_1^0]$ are possible in the thin layer through the Lorentz force, the Coriolis force, and induction processes. For the sixty year variation of the axial dipole magnetic field, the magnitude of $\mathbf{V}[S_2^0]$ amounts to 5.0×10^{-7} m s^{-1} and that of $\mathbf{V}[T_1^0]$ to 3.5×10^{-4} m s^{-1} as long as the fluctuation of $\mathbf{B}[S_1^0]$ is caused by the interaction between $\mathbf{V}[S_2^0]$ and $\mathbf{B}[S_1^0]$.

The magnitude of $\mathbf{V}[T_1^0]$ amounting to 3.5×10^{-4} m s^{-1} is nearly equal to the value derived on the frozen-flux approximation (e.g. MADDEN and LE MOUËL, 1982) and corresponds to the westward drift rate of the non-dipole magnetic field (about 0.2° year^{-1}). We suspect such an oscillation of large amplitude. We may claim, however, that 5.0×10^{-7} m s^{-1} would be a maximum estimate for $\mathbf{V}[S_2^0]$, as long as we

consider the balance between the Lorentz force due to zonal toroidal magnetic fields and the Coriolis force due to $\mathbf{V}[S_2^0]$. The value of 10^{-7} m s^{-1} for $\mathbf{V}[S_2^0]$ corresponds to 10^{-3} for $U[S_2^0]$, if the radial dependence of poloidal velocity is given by (33).

The velocity field derived for $U[S_2^0] = -10^{-5}$ was found to be nearly the same as that in Case 1, whereas the flow pattern for $U[S_2^0] = -10^{-3}$, shown in Fig. 6, becomes entirely different from the previous ones. This difference is quantitatively represented by the rms velocity; for $U[S_2^0] = -10^{-5}$, $V_{\text{rms}}[S] = 3.43 \times 10^{-3} \text{ m s}^{-1}$ and $V_{\text{rms}}[T] = 3.13 \times 10^{-6} \text{ m s}^{-1}$, whereas for $U[S_2^0] = -10^{-3}$, $V_{\text{rms}}[S] = 2.41 \times 10^{-3} \text{ m s}^{-1}$ and $V_{\text{rms}}[T] = 2.03 \times 10^{-5} \text{ m s}^{-1}$. The larger the magnitude of zonal toroidal motion, the stronger is the magnitude of zonal toroidal magnetic field. In fact, contour intervals in Fig. 7 are five times larger than those for $U[S_2^0] = -10^{-5}$.

As shown in Fig. 5(b), however, the zonal toroidal magnetic fields are not strong enough, and cannot be considered as the primary inducing magnetic fields for the generation of non-axisymmetric poloidal magnetic fields. This indicates that the assumption made in Case 2 is unrealistic. As in Case 1, the magnitude of the velocity field derived for Case 2 is also too large. For such an unrealistic velocity field, the magnetic field grows rapidly in the time-dependent kinematic dynamo calculation, as shown in Fig. 11.

The stress-free condition at the CMB yield the results which are quite different from those for the rigid condition. In the stress-free case, the lines of constant differential rotation on meridional planes tend to align in parallel to the magnetic lines of force for the axial dipole magnetic field. Strong zonal toroidal magnetic fields are then not generated. This is explained simply as follows. The induction equation (1) can be rewritten as

$$\frac{\partial \mathbf{B}}{\partial t} + (\mathbf{V} \cdot \nabla) \mathbf{B} = (\mathbf{B} \cdot \nabla) \mathbf{V} + \nu_m \nabla^2 \mathbf{B}. \quad (49)$$

If the two types of lines mentioned above are parallel to each other, the first term of the right-hand-side of (49) vanishes; that is, such a magnetic field is not induced. In the case of the stress-free CMB, the region in which the lines are parallel to each other is enlarged. Therefore, even if the magnitude of differential rotation is larger than that for the rigid CMB, zonal toroidal magnetic fields cannot be strengthened accordingly. In any case, the ω -effect is not sufficient enough to generate strong zonal toroidal magnetic fields.

In this paper, whole-core convection with the radial dependence given by (33) or (34) has been considered. If smaller-scale convective cells prevail in the core, like turbulence, the angular momentum transfer through the Reynolds stress would be more effective and differential rotation would be stronger. Such a problem involving a small-scale fluid motion may be important, but it is beyond the scope of the present study, and we have only considered a large-scale fluid motion in the core.

We have set the truncation level for the spherical harmonics at $L = 4$. When fluid motion at the core surface is estimated on the frozen-flux approximation, $L = 8$ or more is usually adopted. However, smaller-scale fluid motion would vary with shorter time-scale, and hence we must also take time variations into consideration. We have assumed a quasi-steady state in this paper, but for more detailed studies, non-steady cases would have to be treated.

Nevertheless, it would be necessary to examine the effect of truncation levels in order to confirm the result that no strong zonal toroidal magnetic field has been derived.

We attempted to derive fluid motion in the core for Case 3, setting the truncation levels L and N at 4 and 13, respectively. We then found that the fluid motion is not significantly different compared with that for Case 3 ($L = 4$ and $N = 9$) and that no strong zonal toroidal magnetic field could be derived. Even when the truncation levels L and N were set at 5 and 13, respectively, we could not obtain a strong zonal toroidal magnetic field. Hence, we may conclude that the low truncation levels, $L = 4$ and $N = 9$, would little affect the results obtained in the present study.

Finally, we point out that the estimation of differential rotation based on the concept of virtual displacement of fluid particles is inappropriate. As long as a large-scale fluid motion is considered, the ω -effect which has been supposed to generate strong zonal toroidal magnetic fields is not so strong in the core as we have presumed. Therefore, in estimating fluid motion within the core, we should not rely on any approximations for dominant induction terms, but take all the interaction terms into consideration. In other words, the geodynamo seems to be of α^2 -type rather than of $\alpha\omega$ -type.

6. Conclusions

We attempted to derive fluid motion in the Earth's outer core from geomagnetic field data by solving both the induction and the Navier-Stokes equations, in order to reexamine the results of MATSUSHIMA and HONKURA (1989a, b). In solving them, we prescribed the radial dependence of poloidal velocity field and derived fluid motion in the core. We have so far presumed that a strong ω -effect is operative in the core and strong zonal toroidal magnetic fields are generated by the interaction between the dipole magnetic field and differential rotation. We have further presumed that the non-axisymmetric poloidal magnetic fields are generated by the interaction between poloidal convective motions and the zonal toroidal magnetic fields. We assumed that the velocity and magnetic fields are at a quasi-steady state for a specified epoch. In order to check the validity of the derived solutions, we examined time-dependent behavior of the magnetic field with the velocity field fixed.

We considered four cases: in Cases 1 and 2, the rigid CMB and the rigid ICB were assumed, while in Cases 3 and 4, the stress-free CMB and the rigid ICB were assumed. In addition, an axisymmetric poloidal velocity field $\mathbf{V}[S_2^0]$, which gives rise to $\mathbf{V}[T_1^0]$ and $\mathbf{V}[T_3^0]$, was introduced in Cases 2 and 4. The conclusions are summarized as follows.

(1) The derived zonal toroidal magnetic fields are much weaker than we have presumed, so that they cannot be considered as the primary inducing magnetic fields for the generation of the non-axisymmetric poloidal magnetic fields.

(2) The introduction of axisymmetric poloidal velocity field enhances the zonal toroidal velocity fields, but the zonal toroidal magnetic fields are not strengthened accordingly.

(3) The assumption of steady state also fails, since the weak zonal toroidal magnetic field results in rather rapid time variations of the magnetic fields, as shown in kinematic dynamo calculations.

(4) As long as a large-scale fluid motion and a large-scale magnetic field are considered, we should not rely on any approximations for dominant induction terms, such as the ω -effect.

In this paper, we set the truncation level for the degree of spherical harmonics at $L = 4$. If the truncation level is to be increased, time variations of the geomagnetic field should also be examined, since small-scale geomagnetic fields would be subject to rather rapid secular variations.

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