Magnetic Field of Equatorial Electrojet —An Analysis by Use of Hermite Functions—

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Electrojet field in Peru is analysed by making use of Hermite function technique developed by RIKITAKE (1952, 1989) that is especially useful for analysing an isolated anomaly. External and internal parts of H and Z fields thus obtained are in good agreement with those obtained by FORBUSH and CASASPER (1961). It should be noted, however, that the present analysis is free from such an assumption that the distribution periodically repeats outside the space range in question when a Fourier series analysis is used.

The external field is distributed fairly regularly suggesting an overhead flow of electric currents in the east-west direction. On the contrary, the internal field is distributed somewhat anomalously probably reflecting a complicated distribution of electrical conductivity in the interior of the earth beneath the Peruvian west coast.

1. Introduction

Fourier series and/or infinite integrals derived from Fourier transforms are customarily used for analysing geomagnetic and gravity anomalies. When we use Fourier series, however, it is necessary to assume periodic repetition of anomaly outside the space range in question. In the case of infinite integral approach, great care should be taken of the limits of numerical integration for an actual problem. In order to avoid such difficulties, RIKITAKE (1952) proposed to make use of a series of orthogonal function system involving Hermite polynomials.

The Hermite function method is especially useful for analysing an isolated anomaly because each term composing the Hermite function series, that represent an anomaly distribution, tends to vanish at a great distance from the center of anomaly. However, full applications of the method were demonstrated only recently by RIKITAKE (1989, 1990) because of computational difficulties in evaluating infinite integrals involved. The present-day development of computer makes it possible to materialize the idea put forward some forty years ago.

In this paper, the writers will analyse the magnetic field associated with the electrojet in Peru. As the jet field is concentrated around the dip equator and it tends to diminish as observation points are located far away from the equator, it is most appropriate to apply the

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Hermite function analysis technique to the problem. A two-dimensional approach may provide a high-degree approximation because the jet current flows in an east-west direction. The jet field data are taken from FORBUSH and CASASVERDE (1961).

In Section 2 will be described the theory for achieving separation of jet field into external and internal origin parts by use of Hermite functions. The results of separation made for actual data will be presented in Section 3. Basing on the relation between the external and internal fields, possible anomaly of the electrical conductivity deep in the earth under the west coast of Peru will be discussed in Section 4.

2. Separation of a Geomagnetic Field into External and Internal Origin Parts

Let us first of all define a function $\phi_n(x)$ which will be extensively used in this paper. $\phi_n(x)$ is given by

$$\phi_n(x) = \exp(-x^2/2) \tilde{H}_n(x) \quad (1)$$

in which $\tilde{H}_n(x)$ is a normalized Hermite polynomial defined by

$$\tilde{H}_n(x) = 2^{-n/2} (n!)^{-1/2} \pi^{-1/4} H_n(x) \quad (2)$$

where $H_n(x)$ is an Hermite polynomial defined by

$$\left. \begin{aligned} H_{2m}(x) &= \sum_{s=0}^m (-1)^s \frac{(2m)!}{(2m-2s)!s!} (2x)^{2m-2s}, \\ H_{2m+1}(x) &= \sum_{s=0}^m (-1)^s \frac{(2m+1)!}{(2m-2s+1)!s!} (2x)^{2m-2s+1}, \end{aligned} \right\} \quad (3)$$

respectively for even and odd degrees. The first several polynomials become as follows;

$$\left. \begin{aligned} H_0 &= 1, \\ H_1 &= 2x, \\ H_2 &= 4x^2 - 2, \\ H_3 &= 8x^3 - 12x, \\ H_4 &= 16x^4 - 48x^2 + 12, \\ H_5 &= 32x^5 - 160x^3 + 120x, \\ H_6 &= 64x^6 - 480x^4 + 720x^2 - 120, \\ H_7 &= 128x^7 - 1344x^5 + 3360x^3 - 1680x, \\ H_8 &= 256x^8 - 3584x^6 + 13440x^4 - 13440x^2 + 1680, \\ H_9 &= 512x^9 - 9216x^7 + 48384x^5 - 80640x^3 + 30240x, \\ &\vdots \end{aligned} \right\} \quad (4)$$

As can be seen in (3) and (4), coefficients for an Hermite polynomial become extremely large as n increases. This is the reason why we use $\tilde{H}_n$ instead of H_n in actual calculation.

Let us consider a function $f(x)$ which is finite around $x = 0$ and vanishes at $x \rightarrow \pm\infty$. In that case we can express $f(x)$ with sums of orthogonal functions $\phi_n(x)$'s multiplied by appropriate coefficients. We obtain

$$f(x) = \sum_{m=0}^{\infty} K_{2m} \phi_{2m}(x) + \sum_{m=0}^{\infty} K_{2m+1} \phi_{2m+1}(x) \quad (5)$$

in which

$$\left. \begin{aligned} K_{2m} &= \int_{-\infty}^{\infty} f(x) \phi_{2m}(x) dx, \\ K_{2m+1} &= \int_{-\infty}^{\infty} f(x) \phi_{2m+1}(x) dx, \end{aligned} \right\} \quad (6)$$

because of the orthogonality of $\phi_n(x)$.

It is here assumed that the H component of a geomagnetic field, which is uniform in the y direction, is given over a plane surface $z = 0$ by $H = f(x)$, this being the component in the x direction.

The magnetic field component H satisfies $\nabla^2 H = 0$ in free space, so that we obtain

$$\begin{aligned} H(x, z) &= \int_0^{\infty} A(u) e^{-uz} \cos ux du + \int_0^{\infty} B(u) e^{-uz} \sin ux du + \int_0^{\infty} C(u) e^{uz} \cos ux du \\ &\quad + \int_0^{\infty} D(u) e^{uz} \sin ux du \end{aligned} \quad (7)$$

the first and second terms representing the field arising from the origins located in the $z < 0$ region, while the third and fourth terms from those in the $z > 0$ region.

At $z = 0$, H is given by $f(x)$, so that we obtain

$$\left. \begin{aligned} \bar{A}(u) &= (2/\pi) \int_0^{\infty} \sum_m K_{2m} \phi_{2m}(\xi) \cos u \xi d\xi, \\ \bar{B}(u) &= (2/\pi) \int_0^{\infty} \sum_m K_{2m+1} \phi_{2m+1}(\xi) \sin u \xi d\xi \end{aligned} \right\} \quad (8)$$

from the Fourier integral theorem. $\bar{A}(u)$ and $\bar{B}(u)$ stand for $A(u) + C(u)$ and $B(u) + D(u)$, respectively.

As proved by RIKITAKE (1952) the following formulae hold good;

$$\left. \begin{aligned} \int_0^\infty H_{2m}(u) \exp(-u^2/2) \cos ux du &= (-1)^m \sqrt{\pi/2} H_{2m}(x) \exp(-x^2/2), \\ \int_0^\infty H_{2m+1}(u) \exp(-u^2/2) \sin ux du &= (-1)^m \sqrt{\pi/2} H_{2m+1}(x) \exp(-x^2/2), \end{aligned} \right\} \quad (9)$$

so that we have

$$\left. \begin{aligned} \int_0^\infty \phi_{2m}(u) \cos ux du &= (-1)^m \sqrt{\pi/2} \phi_{2m}(x), \\ \int_0^\infty \phi_{2m+1}(u) \sin ux du &= (-1)^m \sqrt{\pi/2} \phi_{2m+1}(x). \end{aligned} \right\} \quad (10)$$

Accordingly, (8) becomes

$$\left. \begin{aligned} \bar{A}(u) &= \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m K_{2m} \phi_{2m}(u), \\ \bar{B}(u) &= \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m K_{2m+1} \phi_{2m+1}(u). \end{aligned} \right\} \quad (11)$$

It is therefore apparent from (7) that the H component of geomagnetic field at $z = 0$ as denoted by H_0 can be written as

$$H_0 = \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m \left\{ K_{2m} \int_0^\infty \phi_{2m}(u) \cos ux du + K_{2m+1} \int_0^\infty \phi_{2m+1}(u) \sin ux du \right\}. \quad (12)$$

In a similar fashion, the vertical component at $z = 0$, Z_0 , is written as

$$Z_0 = \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m \left\{ L_{2m} \int_0^\infty \phi_{2m}(u) \cos ux du + L_{2m+1} \int_0^\infty \phi_{2m+1}(u) \sin ux du \right\}. \quad (13)$$

L_{2m} and L_{2m+1} are obtained from integrals similar to (6) with respect to Z component.

Let us in the next place write the magnetic potential, that satisfies $\nabla^2 W = 0$, as

$$W = \int_0^\infty \left[\left\{ e_c(u) e^{-uz} + i_c(u) e^{uz} \right\} \cos ux + \left\{ e_s(u) e^{-uz} + i_s(u) e^{uz} \right\} \sin ux \right] du \quad (14)$$

from which we obtain

$$\left. \begin{aligned} H_0 &= \int_0^\infty \left[\{e_c(u) + i_c(u)\} u \sin ux - \{e_s(u) + i_s(u)\} u \cos ux \right] du, \\ Z_0 &= \int_0^\infty \left[\{e_c(u) - i_c(u)\} u \cos ux + \{e_s(u) - i_s(u)\} u \sin ux \right] du. \end{aligned} \right\} \quad (15)$$

Comparing (12) and (13) to (14), we have the following equations

$$\left. \begin{aligned} e_c(u) + i_c(u) &= \sqrt{2/\pi} \sum_m (-1)^m K_{2m+1} \frac{\phi_{2m+1}(u)}{u}, \\ e_c(u) - i_c(u) &= \sqrt{2/\pi} \sum_m (-1)^m L_{2m} \frac{\phi_{2m}(u)}{u}, \\ -e_s(u) - i_s(u) &= \sqrt{2/\pi} \sum_m (-1)^m K_{2m} \frac{\phi_{2m}(u)}{u}, \\ e_s(u) - i_s(u) &= \sqrt{2/\pi} \sum_m (-1)^m L_{2m+1} \frac{\phi_{2m+1}(u)}{u}, \end{aligned} \right\} \quad (16)$$

from which we can obtain e_c , i_c , e_s and i_s . With these coefficients we have the external and internal parts of H on $z=0$ as written below after some calculations, the suffixes e and i denoting the parts due to the origins in $z > 0$ and $z < 0$ respectively;

$$\left. \begin{aligned} H_{0e} &= \frac{1}{2} (H_0 + H'), \\ H_{0i} &= \frac{1}{2} (H_0 - H'), \end{aligned} \right\} \quad (17)$$

where

$$H' = \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m \left\{ L_{2m} \int_0^\infty \phi_{2m}(u) \sin ux du - L_{2m+1} \int_0^\infty \phi_{2m+1}(u) \cos ux du \right\}. \quad (18)$$

We also have

$$\left. \begin{aligned} Z_{0e} &= \frac{1}{2}(Z_0 - Z'), \\ Z_{0i} &= \frac{1}{2}(Z_0 + Z'), \end{aligned} \right\} \quad (19)$$

where

$$Z' = \sqrt{2/\pi} \sum_{m=0}^{\infty} (-1)^m \left\{ K_{2m} \int_0^{\infty} \phi_{2m}(u) \sin ux du - K_{2m+1} \int_0^{\infty} \phi_{2m+1}(u) \cos ux du \right\}. \quad (20)$$

It is thus brought out that the external and internal parts of H and Z components on $z = 0$ can be obtained separately provided the infinite integrals involved in (12), (13), (18) and (20) are evaluated.

3. External and Internal Parts of Jet Field

The theory developed in the last section is here applied to the Peruvian jet field. According to FORBUSH and CASAVERDE (1961), the average diurnal variation range of jet field at equinoxes is distributed as shown in Fig. 1. Magnetographs from permanent and survey stations are used. No detailed procedure for obtaining the graphs in Fig. 1 is given here because they are taken simply from FORBUSH and CASAVERDE (1961).

Taking the origin of x at 13° in the south latitude, H and Z are assumed as zero outside the range $-8 \leq x \leq 8$ or $23^\circ \geq \text{south latitude} \geq 0^\circ$. We shall hereafter omit suffix 0 for H and Z for the sake of brevity.

With the readings of H with 0.1 interval, the integrations involved in (6) can readily be performed over the range outside of which $\phi_n(x)$'s are assumed as zero with a high accuracy. $\phi_n(x)$'s for a large n can be evaluated by means of recurrence formulae. Since $\phi_n(x)$'s involve $\exp(-x^2/2)$, the integral limits $0-\infty$ can be replaced by $0-8$ with a great accuracy. As a result, we obtain K_n 's as given in Table 1. L_n 's for Z are obtained as given in Table 2 in a similar fashion.

Making use of K_n 's and L_n 's thus obtained, H' and Z' , respectively defined by (18) and (20), are readily calculated, the numerical integrations involved being performed taking the integration range $0 \leq u \leq 8$ as before.

H_e, H_i, Z_e and Z_i are then obtained from the relations given by (17) and (19) and illustrated in Fig. 2 in which we see that H_i and Z_i are not distributed quite regularly although the H_e and Z_e distributions are fairly smooth suggesting a flow of overhead electric current in the east-west direction.

It is interesting to compare the external and internal parts of jet field to those obtained by FORBUSH and CASAVERDE (1961) who used the so-called Kertz operator (KERTZ, 1954), which is essentially based on a Fourier series method. We see that the external and internal fields

Fig. 1. Diurnal range of electrojet field along the south-north meridian in Peru obtained by FORBUSH and CASAVERDE (1961) averaged for equinoxes from 1957–1959.

Fig. 2. External and internal parts of H and Z fields as denoted by H_e , H_i , Z_e and Z_i , respectively.

Table 1. K_n 's for H .

m	K_{2m}	K_{2m+1}
0	.1572E + 03	-.2648E + 01
1	.3486E + 02	-.2433E + 01
2	.1955E + 01	.3230E + 01
3	.6346E + 01	.5972E + 00
4	.5247E + 01	.5036E + 00
5	-.6918E + 00	.1623E + 01
6	.2161E + 01	.2304E + 00
7	.3789E + 00	.3624E + 00
8	-.8882E + 00	.3994E + 00
9	.1240E + 01	-.6953E + 00
10	-.1126E + 01	.6606E + 00
11	.5789E + 00	-.7514E + 00
12	-.1063E + 00	.6157E + 00
13	-.2475E + 00	-.1887E + 00
14	.5460E + 00	-.2288E + 00
15	-.7213E + 00	.4517E + 00
16	.7113E + 00	-.5109E + 00
17	-.5323E + 00	.4972E + 00
18	.2515E + 00	-.4498E + 00
19	.4790E - 01	.3562E + 00
20	-.2986E + 00	-.2044E + 00

Table 2. L_n 's for Z .

m	L_{2m}	L_{2m+1}
0	.3821E + 02	-.1147E + 03
1	.3240E + 02	-.5358E + 02
2	.1618E + 02	-.2619E + 02
3	.7897E + 01	-.2199E + 02
4	.6280E + 01	-.9973E + 01
5	.1622E + 01	-.7501E + 01
6	.8102E + 00	-.2796E + 01
7	-.3402E + 00	-.1482E + 01
8	.5296E - 02	.4540E - 01
9	-.1882E + 00	-.2174E - 01
10	-.3386E - 01	.5312E + 00
11	-.7378E - 01	-.1546E + 00
12	.1684E + 00	.8926E - 01
13	.7422E - 01	-.2142E + 00
14	-.2890E + 00	.1849E + 00
15	.2702E + 00	-.2469E + 00
16	-.1686E + 00	.3921E + 00
17	.1124E + 00	-.3535E + 00
18	-.1084E + 00	.1595E + 00
19	.1448E + 00	-.6505E - 01
20	-.1997E + 00	.1462E + 00

shown in Fig. 2 coincide fairly well with those obtained by FORBUSH and CASAVERDE (1961) even for minor structures. The fact that the present study agrees with the Forbush and Casaverdes' results may indicate that the Kertz operator technique used by them is after all workable.

4. Electrical Conductivity in the Earth's Interior underneath the Western Peru

Going back to Fig. 2, it should be stressed that the internal origin part of H and Z component is largely different from what is expected for an earth having a uniform electrical conductivity. Taking H_e and Z_e as the inducing field components, the H component trace for the field arising from electric currents induced in a uniform earth should take a smooth form similar to H_e trace. The maximum of H_i should occur at 13°S agreeing with that of H_e although the peak value of H_i is smaller than that of H_e depending upon the depth and conductivity of the underground layers.

Since only the distribution of field components in one direction is given, no detailed study of underground conductor distribution can be made. In the circumstances the internal and external field relation is examined assuming a simple earth model which is perfectly conducting beneath a depth $z = D$ from the earth's surface. The conductor is assumed to be overlain by a non-conducting layer. The conductor extends infinitely in the x and y direction. Although such a simple model cannot lead us to an exact configuration of the conductor, this is the most we can do in this paper.

Theory of electromagnetic induction (e.g. RIKITAKE, 1966), in the above conductor by an external field of which the potential is given by

$$W_e = ee^{-kz} \cos kx \quad (21)$$

leads us to an internal field given by

$$W_i = ie^{kz} \cos kx \quad (22)$$

where

$$i = e^{-2kD} e \quad (23)$$

so that we obtain

$$H_i / H_e = e^{-2kD}. \quad (24)$$

We assume that the wavelength of the external inducing field amounts to the order of 1000 km from Figs. 1 and 2. From Fig. 2, H_i/H_e at 13°S is estimated as 0.042 or so. In that case D is obtained about 250 km from (24). It is thus surmised that the high conducting layer lies at a depth of 250 km or so beneath the Peruvian coast although the evaluation is very crude.

It has been reported by SCHMUCKER *et al.* (1964) that no coast effect for rapid geo-magnetic variations is observed at the west coast of Peru, where we have not only a sharp

conductivity contrast between land and sea but also a deep trench in the offing nearby. Rather, a reverse coast effect is detected at some temporary stations. Such an apparently anomalous phenomenon has been accounted for assuming that the conducting mantle lies at a fairly large depth beneath the sea and that it is upheaved to a shallower depth under the land area (e.g. RIKITAKE, 1968). SCHMUCKER (1969) actually showed that the high conductor is uprising to a depth of 60 km from the deep immediately below the Andes. The conductor, of which cross section is approximated with a semi-circle having a radius of 200 km, comes up from the conducting mantle at a depth of 260 km.

The depth of conducting mantle obtained here from the analysis of electrojet field is therefore in harmony with the conductivity structure in Peru although it is hard to say that the estimate is accurate.

DUHAU and OSELLA (1982) separated the electrojet field due to FORBUSH and CASAVERDE (1961) into the external and internal origin parts without making any assumption on the relation between the external and internal fields of normal Sq . The method, that relies on the Hilbert transform (SIEBERT and KERTZ, 1957) involving an infinite integral, is based essentially on the Kertz operator. They further studied the conductivity structure in the upper mantle basing on the external and internal relation of the electrojet field thus obtained assuming an elliptical uprise of mantle conductor (OSELLA and DUHAU, 1985). The analysis in this paper is less sophisticated and more straightforward than these studies.

5. Concluding Remarks

Separation of electrojet field into the external and internal origin parts is successfully made by making use of Hermite functions. In contrast to the smooth distribution of external jet field, that suggests an east-west flow of overhead electric currents, the internal field is distributed in a somewhat irregular way probably reflecting an anomalous conductivity distribution in the earth's interior.

In order to account for the external and internal relation thus brought out, it is supposed that the depth of highly conducting mantle below the Peruvian Pacific coast is as deep as 250 km or so in contrast to a small value of its depth at Andes, 60 km say.

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