Universal Time Variations in the $a_p$ and $D_{st}$ Indices and Their Possible Cause

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The universal time (UT) variation in the $a_p$ index for the years 1932–1956 and 1957–1986 is analyzed. Results are compared with the UT variation in the $D_{st}$ index and with that in the rate of energy input into the ring current estimated from the $D_{st}$ index. It is found that the UT variation of the $a_p$ index does not agree with that of the $D_{st}$ index, but is in agreement with the UT variation in the rate of energy input into the ring current. Results on the seasonal variation of the $a_p$ index indicate that the UT variation has mostly a minimum around 1030 UT for different seasons, though the average values have features similar to the well-known semiannual variation in geomagnetic activity. The averaged $a_p$ values and the numbers of events of the $a_p$ greater than 30, 50, or 100 reach a minimum around 1030 UT or during the UT time interval 0900–1200. These values are anti-correlated with the UT variation of the magnetic flux that would occupy the nightside auroral oval approximated by offset circles in corrected geomagnetic coordinate system. The modulation of $E \times B$ drift speed in the magnetosphere by the UT variation of the oval magnetic flux could be the source of the UT variation in the $a_p$ and the $D_{st}$ index.

1. Introduction

It has long been suggested that the solar wind plasma flow interacts or exchanges momentum and energy with the earth’s magnetospheric plasma in two basic ways. One is the viscous-like interaction between the solar wind and the magnetosphere, proposed by Axford and Hines (1961). It has been suggested that the Kelvin-Helmholtz instabilities generated at the magnetospheric boundary initiate the modulation of geomagnetic disturbance. The other is the reconnection of interplanetary magnetic field lines with the earth’s dipole magnetic field lines on the magnetospheric boundary (Dungey, 1961; Petschek, 1964). Several theories have been put forward which propose that the solar wind parameters and the interplanetary magnetic field influence geomagnetic activity (Dungey, 1961, 1963; Dessler and Fejer, 1963). These ideas have been confirmed by extensive analyses of the geomagnetic activity indices, such as $D_{st}$ and $AE$ indices (e.g., Kamide and Akasofu, 1974; Burton et al., 1975; Akasofu, 1981, 1983; Baumjohann, 1986).

The $K_p$ index, designed by J. Bartels, is intended to give a measure of the average world-wide activity and forms the basis for several other indices such as $a_p$. Eleven stations, ranging in geomagnetic latitude 63° to 46°, are used for the derivation of $K_p$. $K_p$ itself is a 3-hour index, and it has a linearized version, $a_p$, obtained using a standard conversion table (Mayaud, 1980). The $a_p$ index can be thought of as one-half of the typical 3-hour range of magnetic disturbance for a subauroral latitude station, and its unit is 2 nT. There are 28 steps in $a_p$, i.e., 0, 2, 3, 4, ..., 236, 300, 400, corresponding to the $K_p$ values 00, 0+, 1–, 1, ..., 8+, 9–, 9. One of the main criticisms in the derivation of the $a_p$ index concerns the geographical distribution of the station network, i.e., the circumstance that too great a weight is placed on the European stations and that stations are almost completely absent in the southern hemisphere. Therefore in the present analysis of the universal time variation in the $a_p$ index, we have attempted to compare the results with those from

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the $D_{st}$ index, by averaging hourly $D_{st}$ values over each of the same three-hourly intervals as for $a_p$.

The $D_{st}$ index (Sugiura, 1964; Sugiura and Kamei, 1991) is based on hourly averages of the $H$ component recorded at 4 low latitude stations. All four stations are, in geomagnetic coordinates, roughly 20° to 30° away from the dipole equator to be free from the equatorial electrojet effects and are nearly evenly distributed in local time. The main advantage in using the $D_{st}$ index is that it can measure the energy of the ring current because the $D_{st}$ index is directly proportional to the total kinetic energy of the ring current (see Stern, 1984). Shumilov (1987, private communication) has analyzed the Universal Time variation of the storm time $D_{st}$ and that of the number of occurrences of the $D_{st}$ minimum for the period from 1957–1980.

The purpose of the present analysis is to investigate the cause of the Universal Time variations of geomagnetic activity and their possible explanation, using geomagnetic indices $D_{st}$ and $a_p$. These indices are selected to focus our attention on the mechanism of the UT variation of the solar wind-magnetosphere coupling.

2. Universal Time Variations in the $a_p$ and $D_{st}$ Indices

The $a_p$ data used in this analysis are divided into two periods to increase the confidence in the results. One is the period from 1932 to 1956, and the other from 1957 to 1986. Data in each period are classified into 3 classes. The criteria for the classification are; $a_p$ greater than 30, $a_p$ greater than 50, and $a_p$ greater than 100. Then these data are averaged for each 3-hour interval of Universal Time (UT). Also, the number of occurrences of $a_p$ greater than 30, 50, and 100 in each 3-hour interval of UT is determined for each of the three classes. The results are plotted against UT in Figs. 1(a) and 1(b). It is seen that the averaged $a_p$ and the number of events of $a_p$ greater than 30, 50, and 100 reach a minimum around 1030 UT or during the time interval 0900–1200 UT, and that this minimum is most pronounced for the class with $a_p$ values

![Fig. 1](image-url)
greater than 30. The same analysis as described above has been performed for different seasons for the period of 1957–1986 as shown in Fig. 2. The analogous figures (not shown) obtained for the period of 1932–1956 indicated similar results, that is, the minimum mostly occurs around 1030 UT for different seasons. It is also found that, on the average, the average value of $a_p$ and the number of events of $a_p$ greater than 30, 50, and 100 during the spring and autumn seasons are larger than those for the summer and winter seasons. These seasonal variations are well established with other geomagnetic indices (CHAPMAN and

Fig. 2. Seasonal dependence of the UT variation of the averaged $a_p$ index and the number of events of the $a_p$ greater than 30, 50, and 100 for the years 1957–1986.
Fig. 3. Histogram of the number of data points of: (a) $a_p$ values greater than 80 for each 3-hour UT interval; (b) $Dst$ values smaller than \(-80\) for each 3-hour UT interval; (c) ($dDst/dt$) values less than \(-8\) for each 3-hour universal time interval, for the years 1957–1984.

BARTELS, 1940; MCINTOSH, 1959; FRASER-SMITH, 1972; RUSSELL and MCPHERRON, 1973). Figure 3(a) shows a histogram of the number of data points of $a_p$ values greater than 80 for each 3-hour UT interval for the period of 1957–1984. It is seen that the number of events of $a_p$ greater than 80 becomes smallest during the time interval 0900–1200 UT which is consistent with the result shown in Figs. 1(a) and 1(b).

We compared the UT variation in $a_p$ with that in the $Dst$ index. The hourly $Dst$ values are based on hourly averages of the $H$ component recorded at 4 low latitude stations; Honolulu, San Juan, Hermanus and Kakioka (SUGIURA and KAMEI, 1991). In this analysis the hourly $Dst$ values are averaged over each of the same three-hourly intervals as for $a_p$ for the years from 1957 to 1984. Figure 3(b) shows a histogram of the number of data points of $Dst$ values smaller than \(-80\) for each 3-hour UT interval. It is seen that the number of data points of $Dst$ less than \(-80\) becomes smallest during the time interval 1200–1500 UT. This result does not agree with the universal time variation in the $a_p$ index but is consistent with the result obtained by Shumilov (1987, private communication).

The rate of the energy input to the ring current is proportional to \([dDst/dt + \alpha Dst]\), where $\alpha = 0.13$ is the ring current decay constant (BURTON et al., 1975). The histogram of the number of data points of this energy input thus calculated smaller than \(-8\) (nT/hour) is shown in Fig. 3(c). The number of data points of the energy input to the ring current less than \(-8\) becomes smallest during the time interval 0900–1200 UT. This result is the same as in the universal time variation of the $a_p$ index. This agreement strongly suggests that the UT variation is not the effect of the geomagnetic index itself (i.e., non-uniform distribution of the stations) but the actual UT variation in the efficiency of solar wind-magnetosphere coupling or that of the energy deposit to the inner magnetosphere.
3. Nightside Auroral Oval Magnetic Flux

One possible source of the UT variation is the longitudinally asymmetric geomagnetic field distribution. To investigate this possibility, we now calculate the nightside auroral oval magnetic flux using an assumption that the inner and outer boundaries of the oval are circles in a "corrected" geomagnetic coordinate system as was found by Holzworth and Meng (1975). It should be noted that the oval is "not" circle in geographic coordinate system nor in dipole coordinate system. The procedure adopted by these authors for mathematically representing the auroral oval is to represent Feldstein’s statistical ovals by a simple seven parameter Fourier series using corrected geomagnetic coordinates:

\[
\theta_m = A_1 + A_2 \cos(\lambda_m + A_3) + A_4 \cos(2\lambda_m + 2A_5) + A_6 \cos(3\lambda_m + 3A_7).
\]

In this equation, \(\theta_m\) is corrected geomagnetic co-latitude, \(\lambda_m\) is geomagnetic local time in angular measure (\(\lambda_m = 2\pi \times \text{local time}/24\ \text{hrs.}\)), and \(A_1, \ldots, A_7\) are best fit constants. Equation (1) represents the oval as a circle of radius (\(A_1\)) with periodic perturbations superimposed on it. The values of \(A_1, \ldots, A_7\) can be obtained by fitting to Feldstein’s ovals for the levels of activity from \(Q = 0\) (quiet) to \(Q = 6\) (active). A discussion between Eq. (1) and an offset circle approximating an oval is found in Holzworth and Meng (1975). In the present analysis we used the coefficients of the fit to the oval for the level activity \(Q = 3\) as an example which correspond to the moderate condition of geomagnetic activity.

In order to calculate the nightside auroral oval magnetic flux with an assumption as described above, we used the following equation:

\[
\Delta \Phi = \int_{-\pi/2}^{\pi/2} \int_{0}^{\theta^* = P_e} B_r(\theta', \lambda') R^2 F(\theta', \lambda'; \theta, \lambda) \sin \theta' \, d\theta' \, d\lambda' - \int_{-\pi/2}^{\pi/2} \int_{\theta^* = P_p}^{\theta^* = P_s} B_r(\theta', \lambda') R^2 F(\theta', \lambda'; \theta, \lambda) \sin \theta' \, d\theta' \, d\lambda' \tag{2}
\]

where \(B_r\) is the radial component of the magnetic field, \(R\) is the earth’s radius (\(\theta', \lambda'\)), represent oval coordinates, the origin of the coordinate is the center of the oval circle, \(F(\theta', \lambda'; \theta, \lambda)\) is the conversion factor of area element size between the oval coordinate system and the geographical coordinate system, \(P_e\) and \(P_p\) are radius of the circular fit of the equatorward edge and poleward edge of the auroral oval, respectively. The integral is taken over the nightside half oval circle area.

The first step to perform the integral in Eq. (2) is to calculate the position of center of the oval in geographic coordinate system. In this step, we assume (approximate) that the distance of the oval center from invariant (or corrected geomagnetic) pole does not depend on the UT. The \(\lambda_{ms}\) (longitude of the point antipodal to the subsolar point in geomagnetic coordinates) for given \(T\) (UT) and sun’s declination \(\delta\) (geographic latitude of the subsolar point) is calculated by using the matrix expression as below,

\[
\begin{pmatrix}
    x_{ms} \\
    y_{ms} \\
    z_{ms}
\end{pmatrix} =
\begin{pmatrix}
    x_s \\
    y_s \\
    z_s
\end{pmatrix}
\begin{pmatrix}
    G
\end{pmatrix}
\]

where
\[
G = \begin{pmatrix}
\cos \theta_0 \cos \lambda_0 & \cos \theta_0 \sin \lambda_0 & -\sin \theta_0 \\
-\sin \lambda_0 & \cos \lambda_0 & 0 \\
\sin \theta_0 \cos \lambda_0 & \sin \theta_0 \sin \lambda_0 & \cos \theta_0 
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
x_s \\
y_s \\
z_s
\end{pmatrix}
= \begin{pmatrix}
\sin \theta_s \cos \lambda_s \\
\sin \theta_s \sin \lambda_s \\
\cos \theta_s
\end{pmatrix}
= \begin{pmatrix}
\cos \delta \cos T \\
-\cos \delta \sin T \\
-\sin \delta
\end{pmatrix}
\]

VERTICAL COMPONENT: Z (nT)

YEAR=1985.0  MODEL=ICRF85  Contour Interval=2500

Fig. 4. The approximate locations of the nightside auroral oval for the northern hemisphere during the magnetic flux maximum and minimum that occur around 0730 and 2330 UT, respectively.
The θ₀ and λ₀ are colatitude and east longitude of the northern (invariant or corrected geomagnetic) pole in geographic coordinates, \( \theta_0 = \delta + \pi/2 \) and \( \lambda_0 = -T \).

Equation (3) can be written as

\[
\begin{pmatrix}
  x_{ms} \\
y_{ms} \\
z_{ms}
\end{pmatrix} =
\begin{pmatrix}
  \sin \theta_{ms} \cos \lambda_{ms} \\
  \sin \theta_{ms} \sin \lambda_{ms} \\
  \cos \theta_{ms}
\end{pmatrix}
\]  

Thus \( \theta_{ms} = \cos^{-1}z_{ms} \) and \( \lambda_{ms} = \tan^{-1}(y_{ms}/x_{ms}) \).

The next step is to express (\( \theta', \lambda' \)) in geographic coordinates (\( \theta, \lambda \)) to find the location of the oval center, by using the matrix expression as below:

\[
\begin{pmatrix}
  x_{ms} \\
y_{ms} \\
z_{ms}
\end{pmatrix} =
\begin{pmatrix}
  \sin \theta_{ms} \cos \lambda_{ms} \\
  \sin \theta_{ms} \sin \lambda_{ms} \\
  \cos \theta_{ms}
\end{pmatrix}
\]

Fig. 5. The approximate locations of the nightside auroral oval for the southern hemisphere during the magnetic flux maximum and minimum that occur around 1330 and 0030 UT, respectively.
\[
\begin{pmatrix}
    x \\
y \\
z
\end{pmatrix} = \begin{pmatrix}
    \sin\theta \cos\lambda \\
    \sin\theta \sin\lambda \\
    \cos\theta
\end{pmatrix} = \left( G^{-1} \right) \begin{pmatrix}
    \sin\theta' \cos\lambda' \\
    \sin\theta' \sin\lambda' \\
    \cos\theta'
\end{pmatrix}
\]  

(7)

where, \( G^{-1} \) is the inverse matrix of \( G \) and

\[
K = \begin{pmatrix}
    \cos\alpha \cos\lambda_{ms} & -\sin\lambda_{ms} & \sin\alpha \cos\lambda_{ms} \\
    \cos\alpha \sin\lambda_{ms} & \cos\lambda_{ms} & \sin\alpha \sin\lambda_{ms} \\
    -\sin\alpha & 0 & \cos\alpha
\end{pmatrix}
\]  

(8)

The parameter \( \alpha \) in Eq. (8) is the offset of the center of circle in the corrected geomagnetic (i.e., invariant latitude) coordinate system. (The \( \alpha \) correspond to the parameter \( A_2 \) in Eq. (1)). The conversion of each point from invariant latitude to geographic latitude to calculate the integral in Eq. (2) was done numerically by using a conversion table between them.

In the calculation of the nightside auroral oval magnetic flux we used the IGRF85 geomagnetic main field model and the following values:
- The earth’s radius: \( R = 6371.2 \) km.
- Position of the invariant pole:
  - For the northern hemisphere: \( \theta_0 = 9.2^\circ, \lambda_0 = 278.7^\circ \).
  - For the southern hemisphere: \( \theta_0 = 15.7^\circ, \lambda_0 = 305.6^\circ \).
- Coefficient of the fits to the oval:
  - Poleward edge: \( \alpha_p = 1.9^\circ, \lambda_p = 16.0^\circ \).
  - Equator edge: \( \alpha_e = 5.0^\circ, \lambda_e = 21.0^\circ \).

The \( \alpha_p \) and \( \alpha_e \) correspond to the \( \alpha \) in Eq. (8).

The results of the nightside auroral oval magnetic flux calculation for the northern and southern hemispheres in invariant coordinates which are nearly equal to corrected geomagnetic coordinates are shown in Fig. 6(a). It is found that for the northern and southern hemispheres the maximum value of magnetic flux occurs at 0730 and 1330 UT, and the minimum value of magnetic flux occurs at 2330 and 0030 UT, respectively. The approximate locations of the nightside oval during these times are shown in Figs. 4 and 5.

![Graphs showing the universal time variations of the nightside auroral oval magnetic flux and area](image-url)

**Fig. 6.** (a) The universal time variations of the nightside auroral oval magnetic flux \((N_i, S_i)\) for the northern and southern hemisphere, respectively. (b) The universal time variations of the nightside auroral oval area for the northern \((N_a)\) and southern \((S_a)\) hemisphere, respectively.
On the average, the maximum value of the nightside auroral oval magnetic flux occurs around 1030 UT and the minimum value around midnight. This variation is anti-correlated with the result on the universal time variation of the $a_p$ index; that is, the minimum value of the averaged value of $a_p$ index and the number of data points of the $a_p$ greater than 30, 50, and 100 occur almost at the same time with the time when the nightside auroral oval has maximum magnetic flux. This suggests that the minimum value of average $a_p$ and the number of occurrences of $a_p$ greater than 30, 50, and 100 are correlated with the maximum magnetic flux that would occur at the nightside auroral oval. The UT variation of the nightside auroral oval area, which come from the distortion of the auroral oval in real space (i.e., in geographic coordinate system), is also shown in Fig. 6(b). For the southern hemisphere, the UT variation of both magnetic flux and oval area are larger than that for the northern hemisphere.

4. Discussion

The $D_{st}$ index is a measure of the equatorial ring current in the magnetosphere, but to a lesser degree it is influenced by the current on the magnetopause. Following the work of BURTON et al. (1975), during the recovery phase of a storm, for intervals when there is no ring current injection, the ring current decay constant can be evaluated from $(dD_{st}/dt) = -aD_{st}$. The ring current decay constant was computed from thirty-two 1-hour intervals during storm recoveries and they found that the ring current decay constant is 0.13, which corresponds to a decay time of 7.7 hours. Thus, in the present analysis we have used their results to determine the values of $(dD_{st}/dt)$.

It is found that the universal time variation of the $a_p$ index does not agree with the universal time variation of the $D_{st}$ index found by Shumilov (1987, private communication), but that the former agrees with the universal time variation in the rate of energy input to the ring current estimated from the $D_t$ index. This means that during the time interval 0900–1200 UT, the rate of the energy input to the ring current is also a minimum. It is shown that during the time interval 0900–1200 UT the magnetic flux that occupies the nightside auroral oval is a maximum though the peak is not so clear. One of the possible explanation would be that during the magnetic flux maximum the plasma drift velocity in the plasma sheet becomes smallest because of the stronger magnetic field and therefore that the plasma flow from the geomagnetic tail is reduced if the size of the plasma sheet which correspond to the nightside oval is constant. This can be understood qualitatively by noting that the plasma drift velocity can be written as: \( V_{E\times B} \sim E/B \) where \( E \) denotes the perpendicular electric field and \( B \) the magnetic field magnitude of the magnetosphere, respectively. Thus if the magnetic field in the plasma sheet increases, the plasma drift velocity will decrease, assuming that the electric field remains constant. So that the plasma flow from the geomagnetic tail to the ring current region also decreases. If we use the geomagnetic field model such as IGRF, the longitudinal variation (asymmetry) of the field strength on geomagnetic equator near geostational distance (6.6 Re) is about 5%, though it may be not enough to explain the observed UT variation of $a_p$ or $D_{st}$. It is necessary in future to confirm the UT variation of the magnitude of the magnetic field in the plasma sheet and the ring current region by direct measurements. We assumed the results by HOLZWORTH and MENG (1975) which showed that the auroral oval is approximated by offset circles. However, it is also necessary in future to investigate the UT variation of the oval shape in a corrected geomagnetic (or invariant) coordinate system.

The results on the seasonal variation indicate the well-known variation in geomagnetic data. A theoretical explanation proposed by BOLLER and STOLOV (1970) is that the Kelvin-Helmholtz instability along the flanks of the magnetosphere exhibits a semi-annual variation, with instability maxima occurring at equinoxes and instability minima at solstices. RUSSELL and MCPHERRON (1973) proposed a different explanation for the seasonal variation in geomagnetic activity by invoking the influence of the polarity of the interplanetary magnetic field. This mechanism gives rise to an annual variation with a maximum in spring or in fall, corresponding to the interplanetary magnetic field $B_y$ being negative or positive. However, it is difficult to explain the UT variation by these mechanisms. The appearance of the minimum around 1030 UT in four different seasons suggests that the dipole tilt angle is not the cause of
the UT variation, because the inclination of Earth's rotation axis on the ecliptic plane would cause semi-
annual and annual change of the UT variation of the dipole tilt angle.

5. Conclusions

The main results and conclusions obtained by the present analysis are summarized below.

1) The universal time variation of the $a_p$ index does not agree with the universal time variation of the $D_{st}$ index, but agrees with the universal time variation in the rate of the energy input into the ring current estimated from the $D_{st}$ index.

2) The results on the seasonal variation in the $a_p$ index indicate that this variation has features similar to the well-known semiannual variation in geomagnetic activity obtained by using other geomagnetic indices. The UT variation has a minimum around 1030 UT for different seasons, suggesting that it is not caused by the variation of dipole tilt angle.

3) The averaged $a_p$ values and the numbers of events of the $a_p$ greater than 30, 50, and 100 reach a minimum around 1030 UT or during the UT time interval 0900–1200. These values are correlated with the maximum value of the magnetic flux that occupies the nightside auroral oval calculated by assuming the oval shape obtained by Holzworth and Meng (1975).

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