On the Average Position of the Near-Earth Neutral Sheet

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Magnetic field data from magnetometers on board the ISEE 1 spacecraft are used to examine the position of the magnetotail neutral sheet, which varies in response to variations in the tilt angle of the earth's dipole axis. In the whole data set of the magnetic field from January 1978 to December 1980, 129 neutral sheet crossings are identified in the tail region $X < -10 R_E$, $|Y| < 18 R_E$ and $R < 22.6 R_E$. While it is confirmed that the average neutral sheet does warp when the tilt angle is not equal to zero, in contrast to previous warped-neutral sheet models, the points at which the neutral sheet crosses the XY plane move farther away from the sun-earth line, as the absolute value of the tilt angle increases. A simple neutral sheet model including this new finding is proposed, and its usefulness is discussed.

1. Introduction

The neutral sheet is characterized by a very weak X component of the tail magnetic field, separating the earthward and tailward magnetic fields in the magnetotail. The shape and position of the neutral sheet varies annually and diurnally in response to variations in the earth's dipole tilt (e.g., Murayama, 1966). The distance from the "average" neutral sheet position is frequently needed to identify the relative location of spacecraft in the magnetotail (e.g., Nakai *et al.*, 1991). Thus, estimating accurately the position of the neutral sheet is an important requirement in magnetospheric physics.

Russell and Brody (1967) fitted the shape of the neutral sheet to an empirical expression using a formula of ellipse, of which the semimajor axis is on the solar-magnetospheric equatorial plane (the XY plane). Fairfield (1980) modified the expression by proposing the "displaced" ellipse model:

$$DZ = \begin{cases} \left[(H_0 + D)(1 - Y^2/Y_0^2)^{1/2} - D \right] \sin(T) & \text{for } |Y| \leq Y_0 \\ -D \sin(T) & \text{for } |Y| > Y_0, \end{cases} \quad (1)$$

where DZ is the deviation of the neutral sheet from the XY plane, and T is the tilt angle of the dipole axis (see Fig. 1). The semimajor and semiminor axes of the ellipse are given by Y_0 and $(H_0 + D)\sin(T)$, respectively. The best fit values for the neutral sheet in the range $-40 < X < -20 R_E$ are $H_0 = 10.5 R_E$, $Y_0 = 22.5 R_E$, and $D = 14.0 R_E$ (Fairfield, 1980). Further, using magnetic field data obtained in the range $-19 < X < -8 R_E$, Gosling *et al.* (1986) reached the best fit values as $H_0 = 9.0 R_E$, $Y_0 = 13.5 R_E$, and $D = 7 R_E$.

In the displaced ellipse model, the neutral sheet crosses the XY plane at

$$Y_c = \pm Y_0 \left(1 - \left(D / (H_0 + D) \right)^2 \right)^{1/2},$$

to balance the magnetic fluxes in the northern and southern hemispheres. Although this crossing point has

Fig. 1. A schematic cross section of the magnetotail showing the displaced ellipse model of the neutral sheet.

been assumed to be independent of the tilt angle, it has no observational ground to date. The purpose of this paper is to obtain the average neutral sheet for various values of the tilt angle, and to derive a new empirical model of the neutral sheet.

2. Data

Magnetic field data used in the present study have been obtained from the University of California, Los Angeles, flux gate magnetometers on board the ISEE 1 satellite for 1978–1980 (Russell, 1978). Data are organized in geocentric solar-magnetospheric coordinates. The tilt angle of the earth's dipole axis is defined as an angle between the Z axis and the northern axis of the earth's dipole field. As is in earlier papers, the tilt angle is defined to be positive when the northern hemisphere is in summer.

We have analyzed the ISEE 1 data in the region $X < -10 R_E$, and $|Y| < 18 R_E$. The farthest boundary of our data-set region is at $R = 22.6 R_E$, which is the perigee of ISEE 1. It is expected that in the midnight meridian the neutral sheet is parallel to the sun-earth line at $Y = 0$ in this X region (Fairfield *et al.*, 1987). Searching neutral sheet crossings in the magnetic field data set has been conducted to identify the reversals in the X component of the magnetic field. Crossings that were subsequently followed by a re-crossing within a few tens of minutes, have not been accepted in the present analysis, since they might result from temporal motions of the magnetotail. It has also been observed at times that the satellite crossed the neutral sheet repeatedly during a few hours. In such multiple-crossing events, only one crossing that occurred at about the center of the duration has been included in our data set. 129 crossings, satisfying these selecting conditions, have been retained for this study.

3. The Shape of the Neutral Sheet

The data set of neutral sheet crossings has been divided into five subsets according to the absolute values of the tilt angle. Each tilt angle bin has a 7-degree width. Assuming that the shape of the neutral sheet in winter is symmetric to that in summer with respect to the solar-magnetospheric equatorial plane, the data points with the negative tilt angle have been mapped onto the opposite hemisphere. Figure 2 shows the distribution of neutral sheet crossings plotted in solar-magnetospheric YZ coordinates. Assuming a symmetry of the neutral sheet configuration with respect to the Y axis, all the data points were folded into the plane $Y > 0$, and then, they were folded back to the plane $Y < 0$.

Figure 2 confirms the finding of Russell and Brody (1967) that the neutral sheet warps more with

Fig. 2. Distribution of neutral sheet crossings. The data are divided into five bins of the tilt angle with 7 degrees in width. The solid lines indicate the curves fitted by using the least squares method. The dashed lines show the model neutral sheet yielded from Eq. (3).

increasing tilt angle. It can be noticed, however, in the bottom panel that the data points distribute preferably in the northern hemisphere at $Y = 10\text{--}15 R_E$ in contrast to what Eq. (1) for $T = 31.5^\circ$ (the median value between 28 and 35°) indicates: that empirical formula yields $DZ = -3.7 R_E$ for $Y > 13.5 R_E$ (Gosling *et al.*, 1986). To show that this is not an artifact resulting from our somewhat biased data set, all the satellite trajectories for $|T| > 28^\circ$ are shown by thin lines in Fig. 3, where those for $T < -28^\circ$ are plotted in the opposite hemisphere. Neutral sheet crossings for $|T| > 28^\circ$ are also shown by using dots. From Fig. 3 it can be claimed that although, unfortunately, only twelve neutral sheet crossings are available in the tilt angle bin for $|T| > 28^\circ$, the neutral sheet displaced in the northern hemisphere is a real existence rather than virtual one generated by a biased distribution of the ISEE 1 orbits in our data set.

The data points in each panel of Fig. 2 are fitted by using a quadratic formula,

$$DZ = a^* - b^* Y^2, \quad (2)$$

Fig. 3. ISEE 1 trajectories with $|T| > 28^\circ$. The dots indicate the distribution of neutral sheet crossings.

Table 1. The coefficients used in Eq. (2) and the point in Y coordinate at which the fitted curve crosses the solar magnetospheric XY plane; $Y_c = (a^*/b^*)^{1/2}$.

Tilt angle ($^\circ$)	a^*	b^*	$Y_c (R_E)$
0–7	0.16	2.27×10^{-3}	8.3
7–14	1.48	7.83×10^{-3}	13.7
14–21	1.78	1.27×10^{-2}	11.8
21–28	1.58	4.39×10^{-3}	19.0
28–35	3.10	1.27×10^{-2}	15.6

which is shown by solid lines in Fig. 2. The coefficients a^* and b^* calculated by the least squares method are listed in Table 1. The point Y_c where the fitted curve crosses the XY plane can be determined from $(a^*/b^*)^{1/2}$, which is also listed in Table 1. It is noted that the crossing points seem to be further from the sun-earth line with a larger tilt angle.

Assuming that

$$a^* = aT,$$

and

$$b^* = bT^{1/2}$$

with T in degrees, we obtain

$$DZ = aT - bT^{1/2}Y^2. \quad (3)$$

The correlation coefficient between $T - (b/a)T^{1/2}Y^2$ and observed DZ has been calculated for the whole data set of neutral sheet crossings for various values of b/a . The highest correlation coefficient is 0.53 at $b/a = 2.15 \times 10^{-2}$. By using the least squares method, the coefficient a is calculated to be 0.089.

Taking into account the aberration in the tail axis due to the earth's rotation around the sun, the

Fig. 4.

Fig. 5.

Fig. 4. The average neutral sheet derived from Eq. (3) for various values of the tilt angle; $T = 3.5, 10.5, 17.5, 24.5, 31.5^\circ$.

Fig. 5. Observed neutral sheet crossings plotted as a function of DZ predicted by using Eq. (3). The solid line indicates $DZ(\text{observed}) = DZ(\text{predicted})$.

correlation coefficient between observed and predicted DZ has also been calculated. However, we have been unable to find a higher correlation between them. For this reason and also for simplicity we do not attempt to introduce the aberrated coordinate system in the present study.

From Eq. (3) the crossing point Y_c is represented by

$$Y_c = \pm 6.8 T^{1/4},$$

which agrees reasonably well with the crossing points derived by using Eq. (2) (see the forth column in Table 1). Equation (3) is shown in Fig. 2 by dashed lines. The tilt angle is assumed to be the median value of each tilt angle bin. These five curves are reproduced in Fig. 4.

The DZ values of the observed neutral sheet crossings are plotted in Fig. 5 as a function of DZ predicted by using Eq. (3). The average of the predicted DZ ($=0.65 R_E$), is nearly equal to that of the observed DZ ($=0.71 R_E$). The correlation coefficient is 0.53. The standard deviation of the difference between $DZ(\text{observed})$ and $DZ(\text{predicted})$ is $1.20 R_E$. This residual deviation might be attributed to (1) the temporal motion of the neutral sheet (e.g., Lui, 1984), (2) the deviation of the solar wind velocity from the sun-earth line, (3) the twist of the neutral sheet which is resulted from the penetration of the IMF B_y component into the magnetosphere (e.g., Fairfield, 1979), and/or (4) fluctuations in the tail radius coming from changes in the solar wind pressure.

4. Comparison with Earlier Models

Figure 6 compares the neutral sheet models proposed by Fairfield (1980) and Gosling *et al.* (1986), and with what our data points show for $21 < |T| < 28^\circ$. These curves are calculated for the tilt angle of 24.5° . The apparent discrepancies between Fairfield's model and the present data points are seen in Fig. 6. In particular, the neutral sheet model of Fairfield crosses the XY plane at the point $Y = \pm 18.5 R_E$, which is further from the tail axis than those in the Gosling *et al.* model ($Y = \pm 12.1 R_E$) and our model ($Y = \pm 15.0 R_E$ with $T = 24.5^\circ$). Fairfield used magnetic field data obtained in the range $X = -40$ to $-20 R_E$, where the radius of the magnetosphere is generally larger than that in the range concerned in Gosling *et al.* and in the present study. This might be the primary cause for this discrepancy, although Fairfield (1980)

Fig. 6.

Fig. 7.

Fig. 6. Neutral sheet crossings for the tilt angle $|T| = 21\text{--}28^\circ$. The short- and long-dashed lines show the neutral sheet models proposed by Fairfield (1980) and Gosling *et al.* (1986), respectively.

Fig. 7. Neutral sheet crossings observed in the midnight region $|Y| < 6 R_E$. The solid line indicates the DZ value predicted by using Eq. (3). The short- and long-dashed lines indicate the predictions by Fairfield (1980) and Gosling *et al.* (1986), respectively.

suggested that neutral sheet width shows no significant variation with distance down the tail.

It is also noted in Fig. 6 that Fairfield's prediction lies at considerably higher Z locations than our data points in the midnight sector. A more detailed comparison for the data in the midnight sector ($|Y| < 6 R_E$) is performed in Fig. 7 in terms of the tilt angle dependence of DZ . It is evident that the values of DZ predicted by the previous two models are two times as high as the prediction from the present study. For example, when the tilt angle is 30° , the models of Fairfield and Gosling *et al.* estimate DZ at $Y = 0$ to be 5.3 and 4.5 R_E , respectively, while our model estimates it to be only 2.7 R_E . It is unlikely that neutral sheet crossings in our present study were found preferably at lower latitudes because of some bias in the distribution of the ISEE 1 orbits. The reason for this discrepancy is not clear.

In Fig. 6 systematic discrepancies are seen between Gosling *et al.*'s model and our data points. That is, the neutral sheet position predicted by Gosling *et al.*'s is higher in Z than what our data points indicate at the center of the magnetotail, and is lower at the magnetotail flanks. Gosling *et al.* (1986) utilized the ISEE 2 data obtained in 1978. To make clear these discrepancies, Figs. 8 and 9 plot the difference between the observed neutral sheet position and the position predicted by Gosling *et al.* (1986) for the 1978 data and the 1979–1980 data, respectively. To construct these figures, data points with the negative tilt angle have been mapped onto the opposite hemisphere. Gosling *et al.*'s formula does seem to fit to the observation reasonably well in Fig. 8, but it is clear in Fig. 9 that the prediction is low in the region $|Y| > 10 R_E$ and is high in the region $|Y| < 10 R_E$ for the data obtained in 1979 and 1980.

Figure 10 shows the neutral sheet crossings plotted on the Y - T (tilt angle) plane for 1978 (crosses) and for 1979–1980 (dots). It appears that there is only one crossing in the region $|Y| > 10 R_E$ and $|T| > 20^\circ$ for 1978, resulting from the unique ISEE 1 orbit configuration in 1978. This lack in the coverage of data points might significantly influence the determination of the parameters in Gosling *et al.* (1986). Using the whole data set, we have found the best parameter values: $H_0 = 5.0 R_E$, $D = 4.0 R_E$, and $Y_0 = 18.0 R_E$. The correlation coefficient between the observation and the model prediction is 0.49 for these parameters.

Although Eq. (3) correlates with the observation somewhat better than the displaced ellipse model with the above parameter values, the neutral sheets predicted by these two models would be nearly identical, at least, in the midnight sector. As described in the first section, the displaced ellipse model, i.e., Eq. (1), assumes that the crossing point Y_c is constant. According to our observations, however, Y_c seems

Fig. 8.

Fig. 9.

Fig. 8. The difference between the observed neutral sheet crossing and the prediction by Gosling *et al.* (1986) for 1978.
 Fig. 9. Same as Fig. 8 except for the year: 1979 and 1980.

Fig. 10. Scatter plot of the neutral sheet crossings on the Y - T (tilt angle) plane. The crosses indicate the data points obtained in 1978. The dots are for the data in 1979 and 1980.

to depend in reality on the tilt angle (see Table 1 and Fig. 2). Therefore, one must be careful in using the displaced ellipse model, for example, to sort out data obtained in the tail flank.

In conclusion, determining the average position of the neutral sheet is crucially needed in the study of the magnetic field and plasma configurations. We contend that a coordinate system such as Eq. (3) is quite useful to analyze magnetic and particle data measured in the near-earth magnetotail.

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