The Mechanisms of Damping of Geomagnetic Pulsations

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The physical mechanisms that might determine the damping rates and resonance widths of Pc3-5 pulsations are considered. To estimate the role of ionospheric dissipation, a numerical model, based on a calculation of the complex eigenfrequencies of Alfvén field line oscillations within the approximation of a “thin” ionosphere and for a realistic ionosphere has been used. At middle latitudes the Joule dissipation in the ionosphere may account for the experimentally determined width of the resonance peak of the fundamental Alfvén mode. The meridional spatial structure of higher harmonics is determined by nondissipative mechanisms. The calculations predict a rapid decrease of the Q factor of the magnetospheric Alfvén resonator at $L < 1.5$. The effect is related to the violation of the thin ionosphere approximation at low latitudes. Experimental support for the calculations was found while analyzing the meridional distribution of the damping rates of P_{sc}3 pulsations observed along the 210° magnetic meridian network.

1. Introduction

At present, dissipation in the ionosphere is assumed to be the main mechanism for damping of geomagnetic pulsations. The basic sources of experimental information on dissipation mechanisms in the magnetosphere are the damping rate γ of transient ultra low frequency (ULF) signals excited by rapid disturbances of the magnetosphere and the spatial width δ of the resonance peak of quasi-stationary pulsations. Determination of such ULF parameters as γ or δ on the basis of ground data seems to give information on the height-integrated Pedersen conductivity of the ionosphere (Glassmeier *et al.*, 1984).

The damping decrements γ_0 produced by ionospheric dissipation were numerically calculated by Newton *et al.* (1978), Allan and Knox (1979), and Gough and Orr (1984). All these calculations were performed with the thin ionosphere approximation. The results of the calculations have revealed a sharp difference between the dissipative properties of the dayside and nightside magnetospheric resonators and have thereby explained the distinction between temporal forms of daytime and nighttime pulsations. The developed scheme of calculations was applied elsewhere for the interpretation of experimentally observed damping rates or the width of meridional amplitude maxima of Pc3-5 pulsations at the ground.

In the real magnetosphere, however, there are processes that may outweigh the ionospheric dissipation in the formation of observed γ or δ . When the magnetosphere provides conditions for stimulating the global cavity mode (Allan and Poulter, 1983), then as a result of intermodal coupling, the decrement of Alfvén oscillations may diminish due to energy feeding from the cavity mode. So far, only one event has been reported in which the observed Q factor of P_{sc}5 pulsations ($Q = 6.2$) was about four times higher than might be anticipated according to the simultaneous data on the ionospheric conductivity (Crowley *et al.*, 1987).

The inhomogeneity of Alfvén frequency in the magnetosphere produces phase mixing of the initial disturbance of geomagnetic field lines (Tataronis and Grossmann, 1973; Timofeev, 1976). This mixing provokes an increased decrement, observed on the ground, in damping of geomagnetic pulsations compared with γ_0 (Poulter and Allan, 1985; Barsukov *et al.*, 1986; Leonovich and Mazur, 1991). This nondissipative mechanism of damping could be most substantial in regions of sharp gradients of Alfvén

frequency (e.g. the plasmopause) (Gokhberg *et al.*, 1985).

In this paper all the above mechanisms will be analyzed and their damping rates will be estimated. However, at low latitudes a considerable part of the geomagnetic field line turns out to be submerged in the ionospheric plasma, which makes using the approximation of a thin ionosphere questionable. So a new model for calculating ionospheric dissipation will be used. This new model accounts for the real ionosphere. The results of this model differ drastically from those using the thin ionosphere approximation at $L < 1.5$. By way of experimental illustration of this effect, we have examined a train of $P_{sc}3$ pulsations recorded with the meridional network along the 210° magnetic meridian.

2. Possible Damping Mechanisms of ULF Waves

Now we will discuss the possible input of various dissipation mechanisms of magnetohydrodynamic (MHD) disturbances to ground-based observations of the temporal and spatial structures of daytime ULF pulsations. Our consideration starts with an analysis of the Joule dissipation losses of ULF waves in the magnetospheric Alfvén resonator, which is formed along a geomagnetic field line between the conjugated ionospheres. The parameters of the resonator and the wave field distribution in it are determined by the system of MHD and Maxwell's equations:

$$\begin{aligned} \text{rot} \mathbf{E} &= ik_0 \mathbf{B}, \\ \text{rot}_\perp \mathbf{B} &= -ik_0 \varepsilon_\perp \mathbf{E}, \end{aligned} \tag{1}$$

where $k_0 = \omega/c$ is a vacuum wave number, and c is the velocity of light. The complex permittivity $\varepsilon_\perp$ comprises the dielectric permittivity of the magnetosphere $\varepsilon_A = (c/A)^2$ (A denotes an Alfvén velocity) and the Pedersen conductivity of the ionosphere $\sigma_\perp$:

$$\varepsilon_\perp = \varepsilon_A + i(4\pi\sigma_\perp\omega^{-1}). \tag{2}$$

In an inhomogeneous plasma the differential equations, stemming from the system (1), for Alfvén and fast modes with an arbitrary azimuthal wave number m are coupled and the variables are not separable. Nevertheless, with the use of the methods of the qualitative theory of differential equations the spectral properties of the system (1) can be studied. In the spectral theory of MHD resonators (Krylov *et al.*, 1979, 1980, 1981; Chen and Cowley, 1989) the exact equations were found for the determination of continuous spectra of the Alfvén resonator in an ideal cold plasma in a complicated magnetic geometry. According to Krylov *et al.* (1979, 1980) the Alfvén continuum of system (1) is comprehensively described by two sets of nonlocal dispersion equations. These two ordinary differential equations along a field line coincide with the well-known Dungey's ordinary differential equations, derived previously from (1) only for the extreme cases when azimuthal wave number $m \ll 1$ (toroidal mode) or $m \gg 1$ (poloidal mode). In a more general case, when plasma gyrotropy and the ionospheric conductivities are finite, the equations for an Alfvén continuum nevertheless keep their form (Lifshitz and Fedorov, 1986). The Pedersen conductivity determines the Joule dissipation of a field line oscillation. At the same time, the Hall conductivity has no influence neither on spectral properties of the system nor on the ionospheric dissipation (Krylov *et al.*, 1981). By this reason the terms with Hall conductivity have not been included in the relation (2). In a dipole curvilinear coordinate system the equations corresponding to the toroidal Alfvén mode have the form

$$\frac{db_\varphi}{dx} = ik_0(1+3x^2)\varepsilon_\perp e_v, \quad (3)$$

$$\frac{de_v}{dx} = ik_0(LR_E)^2 b_\varphi,$$

where $x = \cos\theta$ denotes a field-aligned coordinate (θ is a colatitude measured from the North Pole), R_E is the Earth's radius. The covariant components of the Alfvén wave magnetic and electric fields, b_φ and e_v , are related with physical components, B_φ and E_v , by the relationships (A1). Some additional mathematical relationships for a dipole geomagnetic field are given in the appendix. The system (3) is equivalent to the well-known second order differential equation (A2), widely used for the numerical calculations of field line oscillations (Cummings *et al.*, 1969). The complex spectrum of the resonant frequencies $\omega_n = \omega_{An} - i\gamma_n$ of a magnetic shell with a certain L value is obtained from the solution of the Sturm-Liouville boundary problem for Eq. (3).

2.1 Thin ionosphere approximation

As a starting point, we consider Joule dissipation in a thin ionosphere, i.e., the case when the ionosphere may be approximated by a thin film with anisotropic height-integrated conductivities Σ_p and Σ_H . This approximation can be made, when the thickness of the Pedersen layer Δh is much less than the effective skin-depth δ_p of MHD waves in the ionosphere, i.e.,

$$\Delta h \ll \delta_p. \quad (4)$$

Here $\delta_p = c/(2\pi\sigma_p\omega)^{1/2}$ is determined by the averaged Pedersen conductivity of the ionosphere, i.e., $\sigma_p = \Sigma_p/\Delta h$. In the dayside ionosphere, condition (4) is fulfilled for ULF waves with periods $T > 10$ s. Using (4) after the integration of (3) over the ionospheric layer, when $\Delta h \rightarrow 0$ and $\Sigma_p = \text{const}$, we obtain the following boundary condition:

$$E_v = B_\varphi / X_p \quad \text{or} \quad b_\varphi(x = x_i) = e_v(x = x_i)\eta. \quad (5)$$

Here $X_p = (4\pi\Sigma_p/c)\sin^{-1}I$ is the dimensionless ionospheric admittance, $\eta = X_p(4 - 3/L)^{1/2}/LR_E$ is the covariant admittance, I is the magnetic field inclination at the ionospheric level, and $x_i = \pm(1 - 1/L)^{1/2}$ is the intersection of a field line with Earth's surface. For the ionosphere with vertical magnetic field lines ($I = 90^\circ$), the boundary condition (5) reduces to the one used by Newton *et al.* (1978):

$$B_\varphi = (4\pi\Sigma_p/c)E_v. \quad (6)$$

The plasma density variation along the magnetospheric field lines can be described by the power law model (Cummings *et al.*, 1969):

$$N(x) = N_{eq}(LR_E/r)^q = N_{eq}(1-x^2)^{-q} \quad (7)$$

where N_{eq} denotes plasma density at the magnetospheric equator, and r is the radial distance from a particular point of a field line to the center of Earth.

The characteristic features of the mechanism of ionospheric dissipation can be seen for the simplest model of a homogeneous Alfvén resonator. In this case ($q = 6$) the solution of (3) can be expressed with

elementary trigonometric functions. For the symmetric ionospheres the set of eigenfrequencies ω_{An} and relative decrements γ_n/ω_{An} for the harmonics with numbers $n = 1, 2, \dots$ is as follows:

$$\omega_{An}(L) = (\pi n / 2x_i) \Omega(L), \quad (8)$$

$$\gamma_n / \omega_{An} = (2 / \pi n) \arctan \left\{ \left(1 + 3x_i^2 \right)^{-1/2} X_A / \eta \right\}.$$

Here $X_A = c/A$ stands for effective admittance of the homogeneous magnetosphere, $\Omega = A/(LR_E)$, and A is the Alfvén velocity in the homogeneous magnetosphere. For the dayside ionosphere when Σ_p is high enough, so that $X_A \ll \eta$, the Q factor of the n -th harmonic in the magnetospheric resonator is approximately given by

$$Q_n = (2\gamma_n / \omega_{An})^{-1} \approx (\pi n / 4)(4 - 3/L)^{1/2} (\Sigma_p / \Sigma_w \sin I) \quad (9)$$

where $\Sigma_w = c^2/4\pi A$ is the wave conductivity of the magnetosphere. Equation (9) shows that the Q factor of a magnetospheric Alfvén resonator is proportional to the ionospheric conductivity and to the harmonic number n . Equation (9) also predicts that Q factor should steadily increase with decreasing latitude due to the geometrical factor, $\sin I = 2[(L-1)/(4L-3)]^{1/2}$. This dependence is caused by the increase of an effective “field-aligned” thickness of the ionospheric conductive layer at small L values in the thin ionosphere approximation. From (8) and (9), the simple estimate for the damping rate follows: $\gamma \cong (10\Sigma_p L)^{-1}$, which was used by Glassmeier *et al.* (1984).

More accurate estimates of the damping rates based on numerical calculations of eigenoscillations in a dipole geomagnetic field with the boundary conditions (6) were given by Newton *et al.* (1978). The main focus of attention in their paper was poloidal oscillations, i.e., Alfvén waves with large azimuthal wave numbers ($m \gg 1$). In the calculations it was assumed that cold plasma is distributed in the magnetosphere according to the “gyrotropic” law:

$$N(L, x) = 10^3 (B(L, x) / B_E)^3 \quad (10)$$

and the results were presented in the form of plots. Most low-latitude dayside geomagnetic pulsations correspond to Alfvén toroidal mode. For this mode, similar to what Newton *et al.* (1978) did, we have calculated the eigenfrequencies and damping rates for a set of various L values with the plasma distribution (10) and the boundary conditions (6). We have summarized the results of the numerical calculations in the following approximate analytical expression for the Q factor of the magnetospheric resonator at various L and for arbitrary N_{eq} :

$$Q_n \cong C_n X_p (4 - 3/L)^{1/2} N_{eq}^{-1/2} L^{-3}. \quad (11)$$

The coefficient C_n in (11) has been calculated from the spline approximation of numerical results. For the first three harmonics the coefficients are approximated by the following expressions:

$$\begin{aligned} C_1 &= 1.95 + 0.28 \ln L, \\ C_2 &= 3.98 + 2.48 \ln L, \\ C_3 &= 5.47 + 6.24 \ln L. \end{aligned} \quad (12)$$

The approximate formulas (11) and (12) describe with good accuracy the behavior of Q_n for a wide range of L . The difference between the results of exact numerical calculations for a particular L value and the approximate analytical formula (11) with coefficients (12) does not exceed 3%. (For the case $q = 6$ the coefficients are given by $C_n = 1.77n$ and do not depend on L .)

It follows from (12) that the Q factor of the harmonics in an inhomogeneous magnetosphere grows nonequidistantly with the number n . The physical reason for this fact may be explained by the partial reflection of higher- n Alfvén harmonics from the inhomogeneity of Alfvén velocity as Alfvén waves propagate along the geomagnetic field line from the magnetospheric equator.

Experimentally, the Q factor can be determined from the width δ of the resonant peak in the amplitude distribution of eigen field line oscillations or through the damping rate γ of transient ULF signals. For the first method the estimate

$$Q = |2\delta / a|^{-1} \quad (13)$$

can be used, which stems from the well-known relationship between damping rate γ and width δ (Inhester, 1987). Here $a = (d(\ln \omega_A)/dx)^{-1}$ denotes the typical scale length of an Alfvén frequency inhomogeneity.

2.2 Nondissipative mechanisms influencing spatial and temporal structures of ULF waves at the ground

In a real magnetospheric situation, a number of nondissipative (or geometrical) effects might influence ground-based measurements of the dissipative properties of the magnetospheric resonator.

An additional increase in an apparent temporal damping of ground pulsations may be provided by the phase mixing of Alfvén oscillations. In a magnetospheric resonator with an inhomogeneous transverse distribution of an Alfvén frequency ω_A , an initial disturbance begins to split up in its evolution in time. In other words, the effective transverse wave number k_x increases with time as $k_x(t) = k_x(0) + \omega t/a$ (Tataronis and Grossmann, 1973; Timofeev, 1976). Since the ULF field observed on the Earth's surface is an integral effect of ionospheric currents induced by a set of the oscillating field lines with different frequencies, the damping of oscillations due to spatial integration (Poulter and Allan, 1985) will follow the law $H(t) \sim H_0 \exp(-\tilde{\gamma} t)$, where decrement $\tilde{\gamma}$ contains an additional term (Barsukov *et al.*, 1986; Leonovich and Mazur, 1991):

$$\tilde{\gamma} = \gamma_0 + \omega_A h / a. \quad (14)$$

Here h denotes an effective height of the ionospheric conductive layer. At times $t/T_A \sim (a/2\pi\Delta h) \gg 1$, when the scale of disturbance has been diminished to the extent that $k_x(t)\Delta h \sim 1$, the damping decrement should increase (Leonovich and Mazur, 1991). At middle latitudes the latter effect may be noticeable for low-damping ULF trains after several periods. The Q factor determined by the effects of phase mixing and spatial integration for ground oscillations is expressed by the formula

$$Q_* = |a / 2h|. \quad (15)$$

In fact, (14) and (15) summarize the results of the numerical calculations of Poulter and Allan (1985).

Estimates according to (15) show that for sharp inhomogeneities (e.g., the plasmopause, where $a \sim (0.1-1)R_E$), the nondissipative mechanism of damping is predominant for all harmonics of Alfvén oscillations, i.e., $Q_n \gg Q_*$. In other parts of the magnetosphere the nondissipative damping turns out to be comparable to the ionospheric dissipation of a fundamental harmonic, i.e. $Q_1 \approx Q_*$.

Spatial integration also influences the ground-based determination of the spatial scale of the meridional distribution of resonant ULF pulsations. According to Fedorov *et al.* (1990), the resonance width δ_g at the Earth's surface will be larger than the resonance width δ in the ionosphere, as follows:

$$\delta_g = \delta + h. \quad (16)$$

Equation (16) means that there is a critical value Q^* , determined by (15), that limits the ground-based observation of a very thin resonant structure with $Q \gg Q^*$. In the latter case the ground spatial structure will be totally determined by the geometrical spreading, i.e., $\delta_g \cong h$.

2.3 Dissipation in the real ionosphere

To study the effect of the real ionosphere on the properties of the Alfvén resonator, we have developed a computer model based on the numerical solution of the Sturm-Liouville problem for (3). The model enables us to determine complex eigenvalues and eigenfunctions and hence all the related parameters of the magnetospheric resonator. Below, we shall see that the dissipative properties of the Alfvén resonator between the realistic ionospheres change drastically at low latitudes compared to the properties of the thin ionosphere model.

The ionospheric plasma distribution has been simulated by the International Reference Ionosphere model (IRI-86). The plasmaspheric field-aligned plasma distribution is smoothly connected to the ionospheric topside profile by a cubic spline interpolation. The field-aligned tracing of plasma distribution into the magnetosphere is made by using Angerami-Thomas (AT) functions for the diffusive equilibrium distribution (Angerami and Thomas, 1964) for both conjugate ionospheres. The equatorial density N_{eq} and the temperature T_e , reconstructed from the northern and southern ionospheres, differ somewhat (a fact mentioned earlier by Rycroft and Jones (1987)). So, the parameters of AT functions have been corrected during the calculations to give mean values of N_{eq} and T_e .

To reveal the differences between models of the magnetospheric resonator with the thin ionosphere and with the realistic ionosphere, we chose the plasmaspheric density distribution in the equatorial plane, given by a power law

$$N_{eq}(L) = N_{eq}(L_0) (L / L_0)^{-p}. \quad (17)$$

The parameters of the AT functions could be adjusted to the given plasmaspheric density distribution in the equatorial plane. For the comparison, the typical parameters of the plasmaspheric plasma distribution at high solar activity were chosen: $N_{eq} = 3.2 \times 10^3 \text{ cm}^{-3}$ at $L_0 = 3$ and $p = 3.2$.

Fig. 1. Meridional distributions of the Pedersen height-integrated conductivities of the northern (solid curve) and southern (dashed curve) ionospheres, as derived from the IRI model ($R = 90$, $LT = 13$, month = 3).

Fig. 2. Meridional distributions of parameters of the fundamental Alfvén harmonic ($n = 1$) in the dayside magnetospheric resonator in the approximation of the thin ionosphere (dashed curve) and in the IRI model of ionosphere (solid curve). (a) Eigenperiods $T_A(L)$; (b) damping rates $\gamma_1(L)$; (c) Q factors.

The IRI model shows that the height-integrated Pedersen conductivity of the ionosphere decreases slowly with L (Fig. 1). For the comparison of results given by the two ionospheric models, the mean value of Σ_p was chosen, i.e., $\Sigma_p = 9 \times 10^7 \text{ km/s}$.

The obtained dependencies of the eigenperiod $T_A(L)$ of the fundamental harmonic on the parameter L are shown in Fig. 2(a). At middle latitudes ($L > 2$) the results of the calculations for the real ionosphere and the approximation for the thin ionosphere practically coincide, but at low latitudes ($L < 2$) the curves begin to diverge. The slope of $T_A(L)$ becomes less steep and even starts to increase while approaching the equator. Thus at $L \approx 1.3$, a minimum of the period of Alfvén oscillations is formed at which the meridional gradient of the Alfvén frequency changes its sign. A change in the profile of $T_A(L)$ at low latitudes is connected with a "mass loading" of oscillating field lines with heavy ionospheric ions (O^+ , NO^+ , O_2^+). This effect was modeled by Hattingh and Sutcliffe (1987) and Poulter *et al.* (1988).

However, the effect of “abnormal” $T_A(L)$ dependence will be hard to observe experimentally, because at low latitudes the damping of ULF waves should also greatly increase. This fact is clearly evident in Fig. 2(b). At midlatitudes ($L > 2$) the damping rates $\gamma(L)$ of the fundamental mode in the resonators with the thin ionosphere and the realistic ionosphere vary with L nearly identically. The small difference between them is caused by the difference in the field-aligned magnetospheric plasma distributions between the AT model and the power law model. At low latitudes ($L < 2$), however, the decrements of damping for the realistic ionosphere are considerably enlarged, despite some increase of Σ_p . This effect is more evident in the behavior of the quality factor $Q(L)$ of the magnetospheric resonator (Fig. 2(c)). While the theory of the thin ionosphere predicts a steady growth for the Q factor, in reality this parameter quickly drops below $L < 2$. Therefore the estimates of the effectiveness of Joule dissipation within the framework of the approximation of a thin ionosphere produce unrealistically high values of Q for all modes at low latitudes. The model of a thin ionosphere leads to a severe overestimation of Q for small L . The violation of the thin ionosphere approximation at low latitudes results in a sharp deterioration (by more than 1 order of magnitude) in the quality of the magnetospheric resonator. The region of a sharp increase in ionospheric dissipation may be a low-latitude border for the magnetospheric resonance effects.

3. Comparison with Observational Data

Now we compare the results of the previous consideration to the existing experimental data. Note that the available experimental information for δ and γ is rather limited, because the input of resonant field line oscillations in the observed spectra of ULF waves may be masked by other effects. Therefore just the width of a spectral peak of ULF pulsations cannot be directly used to estimate the Q factor of the magnetospheric resonator. In the same way, an amplitude distribution of ULF pulsations measured at a meridional network of magnetometers does not indicate directly the value of δ , because of a relatively large separation between stations Δx in most cases (i.e., $\Delta x \geq \delta$).

At high latitudes the detailed measurements of the meridional amplitude and phase structure of Pc5 pulsations in the ionosphere were made with the STARE radar (Walker *et al.*, 1979). According to these data the resonance width δ in the ionosphere is about 50 km. The meridional profile of the damping rates of high-latitude Pc5 was determined by Glassmeier *et al.* (1984). The observed values of Q (≈ 2.4 – 2.9) correspond to reasonable values of the ionospheric conductivity at these latitudes. Both experimental observations prove that Joule dissipation is the dominant damping mechanism for ULF waves in the Pc5 range.

For midlatitudes a detailed study of the resonance spatial structure of Pc4 pulsations was done by Baransky *et al.* (1989). They used the gradient method to determine the set of local eigenperiods of the magnetospheric resonator at certain L . The resonance width δ was estimated to be $\approx 140 \pm 10$ km. The experimentally determined values of eigenperiods are indicated by asterisks in Fig. 3(a). Let us compare these experimental data with the mechanism of ionospheric dissipation. As was shown in the preceding sections at midlatitudes a sufficient accuracy can be obtained with the thin ionosphere approximation. According to the observed $T_A(L)$ distribution, the scale of the Alfvén frequency inhomogeneity a in the ionosphere should be ≈ 570 – 610 km. For a rough estimate of the plasma radial gradient in the equatorial plane of the magnetosphere, let us assume that $f_A \approx A_0/LR_E$. If $f_A \approx L^{-s}$, then the parameter s can be estimated as $s \approx 4 - p/2$, where p is the parameter of the magnetospheric plasma distribution in (17). Because a and s are related by the relationship $a \approx (R_E/s)(4L - 3)^{-1/2} \sin^{-1} I$, we get $s \approx 3.1$ – 3.2 from the predetermined values of a . These values of s correspond to parameter p as $p \approx 1.6$ – 1.8 . Choosing $p = 1.7$ and $N_0 = 250$ cm⁻³ at $L_0 = 4.0$ in (17) and $q = 2$ in (7), a good fit to the experimental values of T_A and its harmonic has been achieved (Fig. 3(a)).

The vertical profile of ionospheric conductivity has been constructed by using the IRI model with parameters corresponding to the period of observations. From that profile the effective height h and the height-integrated Pedersen conductivity Σ_p of the ionosphere were estimated as ≈ 100 km and $\approx 4 \times 10^7$ km/s, respectively. These values were then used as parameters in the numerical calculations of the

Fig. 3. Meridional distributions of parameters of the dayside magnetospheric resonator at midlatitudes. (a) Eigenperiods of fundamental (solid curve) and second (dashed curve) harmonics. Asterisks denote experimentally determined values. (b) Q factors for these harmonics. Vertical bars denote values determined from the ground-based gradient measurements.

eigenperiods and Q factors of the dayside magnetospheric resonator at middle latitudes. The results of calculations for the fundamental and second harmonics are shown in Figs. 3(a) and 3(b). For comparison with numerical calculations, the experimental Q factors were derived from the observed values by using the relationship $Q = |2\delta/a|^{-1}$, where $\delta = \delta_g - h$. The experimentally determined values of Q for two pairs of stations ($Q \approx 5.7-9.5$ and $Q \approx 6.1-10.2$) are in a reasonable agreement with the numerically calculated values (Fig. 3(b)). At the same time, the Q factor of the second harmonic (≈ 30), determined by Joule dissipation, is much higher than the critical value $Q_* \approx 3$. This difference shows that spatial structure and temporal damping of the second harmonic are determined totally by nondissipative mechanisms. Our consideration shows that the analyzed Pc4 pulsations are actually the fundamental mode, and not a second harmonic, as was assumed by Baransky *et al.* (1989) in their treatment.

For experimental determination of the Q factor of the magnetospheric resonator at low latitudes, we have used data from the meridional network of magnetic stations along the 210° magnetic meridian (210 MM). The 210 MM network was organized by the Solar-Terrestrial Environment Laboratory of Nagoya University. Each station is equipped with a three-component ring-core flux-gate magnetometer with a noise level of about 0, 1 nT, flat frequency response in the range 0–2.5 Hz, and a multicomponent digital acquisition system with a sampling rate of 1 Hz. A more detailed description of the whole experiment is given by Yumoto *et al.* (1992).

For an experimental study of the resonance response of the magnetospheric resonator at different latitudes, it would be better to start with a probing signal with a wide spectrum, e.g., with a stepwise temporal wave form. A natural signal, whose temporal wave form is very similar to a step function, is storm sudden commencement (SSC) impulse. Figure 4 shows the H -component magnetograms of the very in-

H-component (March 24, 1991)

Fig. 4. Variations of the H component of the geomagnetic field along the 210° geomagnetic meridian during SSC on March 24, 1991, at 03:42 UT. Magnetograms are from the stations of the northern (right panels) and southern (left panels) hemispheres with the latitude decreasing from the top panel downward.

tense and clear SSC recorded on March 24, 1991, at stations in the 210° MM network in the southern and northern hemispheres (Yumoto *et al.*, 1992). The amplitude of the SSC main impulse is practically the same at all stations and is larger than 200 nT. The SSC excites a damped train of P_{sc3} pulsations. These oscillations, with a period $T \approx 21$ –22 s, are clearly evident at conjugated stations MSR and BSV ($L \approx 1.6$). The Q factor of the magnetospheric resonator determined by examining the visual train form is $Q \approx 9$ at this latitude. However, at stations at lower latitudes, i.e., KAG ($L = 1.22$), WEP ($L = 1.18$), and CBI ($L = 1.14$), P_{sc3} pulsations disappear. The absence of oscillations is probably not related to peculiarities of an excitation mechanism in the magnetosphere, because the amplitude and the growth rate of the SSC impulse are practically the same at all stations. From our point of view, this fact may illustrate the rapid increase of dissipative properties of the magnetospheric resonator at near-equatorial latitudes.

The data of conjugated stations (MSR and BSV) make it possible to determine the field-aligned structure of P_{sc3} by using the phase relationships between horizontal magnetic components. From the ideal MHD equations and the rule of Alfvén wave polarization rotation upon transmission through the ionosphere, the following criteria can be obtained (Lanzerotti and Fukunishi, 1974): for the odd (according to the magnetic components) mode (or symmetric mode according to the field line displacement), the H components in conjugated points should be in phase, while the D components are in antiphase. For the even mode, the relationships are reversed. In our case the H components of the P_{sc3} train are in phase (Fig. 5), so the recorded P_{sc3} pulsation is an odd mode, which has an antinode in the equatorial plane of the magnetosphere. According to the observed pulsation period and the numerical calculations, the excited

Fig. 5. Radial distribution of parameters of the magnetospheric resonator for the fundamental mode calculated by using the IRI-AT model with parameters $R = 90$, $LT = 13$, month = 3, and $\Lambda = 210^\circ$. (a) Eigenperiod, (b) Q factor. Asterisks denote experimentally observed periods of P_{sc3} pulsations. Dashed curve indicates the numerically estimated values of the Q factor governed by nondissipative processes. Positions of the stations of 210° geomagnetic meridian are indicated by arrows.

ULF wave is the fundamental harmonic of the magnetospheric resonator at these latitudes.

For a more detailed comparison of these observations with the model described above, the parameters of the Alfvén resonator have been calculated. The input parameters of the IRI model have been chosen to correspond to the geophysical situation during the SSC event of March 24, 1991. However, the plasmaspheric density greatly depends not only on a current set of geophysical parameters but also on the history of plasmasphere dynamics. The plasmaspheric plasma density was reconstructed from the IRI topside vertical profiles, as described in the previous section. A good correspondence between the periods of P_{sc3} observed at stations near $L = 1.6$ and the numerical results was obtained for the effective solar number $R = 100$. Taking into account the fact that the observations were made during a period of high solar activity, this value seems quite reasonable. The results of calculations for the fundamental harmonic are shown in Fig. 5(a). The period of P_{sc3} , observed at $L \approx 1.6$ (marked by asterisk), lies well on the curve of $T_A(L)$ (Fig. 5(a)).

A rapid deterioration of the resonant properties of the Alfvén resonator begins between $L = 1.6$ and $L = 1.2$ according to the observations. This range corresponds well to the numerically calculated variations of the Q factor (Fig. 5(b)). The same figure indicates the values of Q_* as estimated with (15) according to the numerically calculated gradients of the Alfvén frequency. The experimentally observed damping rates should be determined by the minimal of those two, Q and Q_* . At midlatitudes ($L > 1.5$), where $Q_* < Q_1$, the nondissipative damping of SSC-excited ULF waves dominates ionospheric dissipation. In contrast, at $L < 1.5$, where the gradients of the Alfvén frequency are smooth, Joule dissipation prevails.

4. Conclusion

Theoretical estimates and an analysis of experimental data show that at middle latitudes ionospheric dissipation and nondissipative mechanisms have a comparable input to the spatial structure and temporal damping of a fundamental Alfvén mode. Hence, in principle, ground-based observations of the fundamental field line oscillations can be used to deduce ionospheric conductivity at the observation point. For second and higher harmonics, however, Joule dissipation plays a minor role compared to that of nondissipative mechanisms. Thus the ground-based observational data on these harmonics cannot give any information on ionospheric conductivity.

Data for the meridional profile 210° MM show that the dissipation rate of the ULF oscillations stimulated by SSC increases sharply at near-equatorial latitudes. Analysis of possible dissipation mechanisms and the numerical calculations show that the observed effect could be explained by a breakdown of the approximation of a thin ionosphere at low latitudes.

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Appendix: Numerical Calculation of Alfvén Resonator Parameters

Dipole coordinates (here θ is a co-latitude measured from the North Pole).

$$\begin{aligned} \nu &= \sin^2 \theta / r = 1 / LR_E & (\mathbf{e}_\nu \text{ is normal to field line}), \\ \varphi &= \text{azimuthal angle} & (\mathbf{e}_\varphi \text{ is azimuthal direction}), \\ \mu &= \cos \theta / r^2 & (\mathbf{e}_\mu \text{ is field-aligned direction}). \end{aligned}$$

Lame's coefficients

$$\begin{aligned} h_\nu &= r^2 / \sin \theta (1 + 3 \cos^2 \theta)^{1/2}, \\ h_\varphi &= r \sin \theta, \\ h_\mu &= r^3 (1 + 3 \cos^2 \theta)^{-1/2}. \end{aligned}$$

Dipole geomagnetic field

$$B_0 = B_E / L^3, \quad \frac{B(\theta)}{B_0} = \frac{(1 + 3 \cos^2 \theta)^{1/2}}{\sin^6 \theta} = \frac{(1 + 3x^2)^{1/2}}{(1 - x^2)^3}$$

where $B_E = 0.312\text{G}$ – magnetic field at the Earth's equator. Field line equation

$$r = LR_E \sin^2 \theta = LR(1 - x^2).$$

Relationships between physical (B_ϕ , E_v) and covariant (b_ϕ , e_v) components

$$B_\phi = \frac{b_\phi}{h_\phi} = \frac{b_\phi}{r \sin \theta} = \frac{b_\phi}{LR_E (1 - x^2)^{3/2}}, \quad (A1)$$

$$E_v = \frac{e_v}{h_v} = \frac{\sin \theta (1 + 3 \cos^2 \theta) e_v}{r^2} = \frac{(1 + 3x^2)^{1/2}}{(LR_E)^2 (1 - x^2)^{3/2}} e_v.$$

The system (3) corresponds to the well-known Dungey's equation for toroidal mode (Gaussian units):

$$e_v'' + (\omega / \Omega)^2 (1 - x^2)^{6-q} e_v = 0 \quad (A2)$$

where $(\omega / \Omega)^2 = KC$ in the notations of Cummings *et al.* (1969). In the power law model (Cummings *et al.*, 1969),

$$\varepsilon_\perp = \left(\frac{C}{A} \right)^2 \frac{(1 - x^2)^{6-q}}{(1 + 3x^2)}$$

and for Alfvén velocity distribution along the field line,

$$A^2(x) = \frac{B^2}{4\pi\rho} = A_0^2 \frac{(1 + 3x^2)}{(1 - x^2)^{-q}}.$$

In (3), the transition to the (10) model can be done if the next substitution has been done:

$$N_{eq} = 10^3 / L^3, \quad q = 3, \quad \varepsilon_\perp \rightarrow \varepsilon_\perp (1 + 3x^2)^{1/2}.$$

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