Analysis of *F*-Region Incoherent Scatter Spectra during Periods of High Electric Fields

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In the presence of high electric fields in the auroral ionosphere, the ion distribution functions depart from their usual Maxwellian shape. The incoherent scatter radars have observed the signature of this phenomenon in the form of distorted spectra which have lost their double-humped characteristics. In consequence, the classical procedure of spectrum analysis, which assumes Maxwellian distributions to fit the plasma parameters, is no longer valid and has to be modified to account for a non-Maxwellian plasma. The observations of the European incoherent scatter radar EISCAT were at the origin of substantial developments, which are still in progress. We briefly review the main steps in the experimental and theoretical approaches of the non-Maxwellian behaviour of the ionospheric plasma, and we discuss particularly the most recently published results.

1. Introduction

The most spectacular effect of high electric fields on the auroral *F* region plasma is an increase of the ion temperature much above the neutral temperature. In fact, looking at the distribution of the different ionospheric parameters obtained by an incoherent scatter experiment, it is this clear increase in ion temperature that enables the experimenters to highlight the magnetically perturbed periods.

The increase in ion temperature is the very well known effect of frictional heating due to the relative velocities between ions and neutrals. Studying more closely the measured ion temperature, it has been shown that it was anisotropic with a larger increase in the direction perpendicular to the magnetic field. The first evidences of these anisotropies, using the incoherent scatter technique, have been published in 1984 for the O⁺ ions (Perraut *et al.*, 1984), and several years later for the molecular ions (Lathuillere *et al.*, 1991). An exhaustive review of the EISCAT observations of anisotropic ion temperature has been published recently by Lockwood *et al.* (1993).

In a parallel effort, it has been realised that the incoherent scatter spectrum made use of the assumption of Maxwellian ion velocity distribution-functions. This assumption did not hold during highly perturbed periods, resulting in incoherent scatter spectra whose shape could be distorted from the usual one. Such distorted spectra have been first observed with EISCAT by Lockwood *et al.* (1987), and by Winser *et al.* (1987) for ion velocities of about 2.5 km/s. The distortion of the spectra, which depends on the aspect angle of the measurement, i.e. the angle between the K vector of the observation and the magnetic field, is clearly seen on spectra recorded at aspect angles larger than 40°.

Using an analytical description of the non-Maxwellian ion-velocity distribution-function, Hubert and Lathuillere (1989) have been able to define a domain of validity of the standard EISCAT data analysis, as a function of the nature of the ions, the electric field strength and the angle of observation. Furthermore, Lathuillere and Hubert (1989) have shown that this domain was much smaller in the case of a mixture of ions than for a single species, due to the fact that the line-of-sight temperatures of the different ion species could be significantly different. During period of high electric fields, the molecular ions could become dominant at *F* region heights (see e.g. Sojka *et al.*, 1981; Lathuillere, 1987): an overestimation of the concentration of oxygen ions will lead to an underestimation of the real ion anisotropy.

Consequently, it became necessary to take into account the non-Maxwellian character of the ion-velocity distribution-function in the incoherent scatter spectra analysis, for obtaining a reliable description of the ionospheric plasma during disturbed periods.

In the first part of this paper we will review the different descriptions of non-Maxwellian ion-velocity distribution-function, which are available. This review is however limited to the description of the *F*-region where the ion-collision frequency is much smaller than the ion gyrofrequency. We discuss also how these descriptions are used in incoherent scatter analysis and with more details the method that we have developed recently (Gaimard *et al.*, 1994). The second part of the paper will present some tests of sensitivity of our method and its application on EISCAT observations.

2. The Analysis Methods of Incoherent Scatter Spectra

There are three main approaches for describing the non-Maxwellian state of the *F*-region plasma in the presence of large electric fields. The first one is the empirical description from a simplified model as proposed by Raman *et al.* (1981) following the work of St Maurice and Schunk (1974). The ion-velocity distribution function is written:

$$f_i = n_i \left(\frac{m_i}{2\pi k_B T^*} \right)^{3/2} I_0 \left\{ 2D^* \left[\frac{(v_x^2 + v_y^2)}{(2k_B T^* / m_i)} \right]^{1/2} \right\} \exp \left\{ -D^{*2} - \frac{(v_x^2 + v_y^2 + v_z^2)}{(2k_B T^* / m_i)} \right\} \quad (1)$$

where k_B is the Boltzmann constant, n_i is the density of the ions, m_i is their mass, v_x , v_y , v_z are the components of the ion-velocity in a coordinate system where z is the direction parallel to the magnetic field, and I_0 is the modified Bessel function. T^* , which has the dimension of a temperature and D^* , which is correlated to an electric field, are determined empirically.

Raman *et al.* (1981) use this formulation in the calculation of incoherent scatter spectra and point out that such a description was limited to aspect angles larger than 30° . It has then been used by Lockwood and Winser (1988), Moorcroft and Schlegel (1988), Winser *et al.* (1989) and Suvanto *et al.* (1989) to analyse EISCAT data during magnetically disturbed periods. One of their results is it does not allow to reproduce accurately the measured incoherent scatter spectra with a pure O^+ plasma. Suvanto *et al.* (1989) have also shown that it was impossible to determine the ion composition, when molecular ions are NO^+ or (and) O_2^+ .

The second approach consists to use Monte-Carlo numerical simulations to evaluate the ion-velocity distribution-function as done by Barakat *et al.* (1983) and more recently by Winkler *et al.* (1992). Such Monte-Carlo simulations have been used by Kikuchi *et al.* (1989) to simulate incoherent scatter spectra, but up to now they have not been applied to the analysis of measured spectra.

The third approach was proposed by D. Hubert (1983, 1984). He found an analytical expression of the non-Maxwellian ion-velocity distribution function for the solution of the Boltzmann's equation which is the fundamental equation governing the auroral ion population. This solution is written as a polynomial expansion of the ion distribution function which generalises the bi-Maxwellian development as first proposed by St Maurice and Schunk (1977):

$$f_i = n_i \left(\frac{m_i}{2\pi k_B} \right)^{3/2} \left(\frac{1}{T_{\parallel}} \right)^{1/2} \frac{1}{T} \cdot \exp \left(-\frac{m_i T_n}{m_n T_{\parallel}} c_{\parallel}^{\prime 2} - \frac{m_i T_n}{m_n T} c_{\perp}^{\prime 2} - \alpha^2 D^{\prime 2} \right) I_0 \left\{ 2\alpha D' \left(\frac{m_i T_n}{m_n T} \right)^{1/2} c_{\perp} \right\} \left\{ 1 + a_{40} H_4(c_{\parallel}') M_0(c_{\perp}') + a_{21} H_2(c_{\parallel}') M_1(c_{\perp}') \right\} \quad (2)$$

where $T_{\parallel}$ is the ion parallel temperature, T_n is the neutral temperature, $c_{\parallel}'$ and $c_{\perp}'$ are the components parallel and perpendicular to the magnetic field of the random velocity normalised with respect to the neutral thermal velocity, $\alpha D'$ is an effective electric field normalised with respect to the neutral thermal velocity, T is a parameter which has the dimension of a temperature, and a_{40} and a_{21} are the coefficients of the expansion according to the Hermite polynomials H_2 , H_4 and to the generalised polynomials M_0 and M_1 . T and α are determined such that the approximation of zero order f_i^0 yields the exact values of the second and fourth order moments of the perpendicular velocity of Boltzmann's equation (Hubert, 1983), which in turn depend on the ion-neutral collisional cross-sections. The O^+ -O collision cross section chosen in this work is the model B of St Maurice and Schunk (1977) which agrees with the Monte-Carlo results of Winkler *et al.* (1992).

The approximation of the ion distribution as given in (2) has been discussed with respect to the Monte-Carlo simulation by Hubert and Barakat (1990) and Barakat and Hubert (1990).

This formulation has been used by Gaimard *et al.* (1994) who proposed a new method of analysis of incoherent scatter spectra during disturbed periods. This method assumes that the scattering processes between ions and neutrals are correctly described. It also neglects the ion-ion collisions as well as the energy exchange with the electrons. The ion temperature is thus given by:

$$T_i = T_n \left(1 + \frac{2}{3} \frac{M_{nO} + M_{nN_2} \frac{v_{iN_2}}{v_{iO}}}{M_{iO} + M_{iN_2} \frac{v_{iN_2}}{v_{iO}}} D_i''^2 \right) \quad (3)$$

where v_{iN_2} and v_{iO} are the ion-neutral collision frequencies, D_i'' the ratio between the ion-neutral differential velocity and the ion thermal velocity calculated with the neutral temperature instead of the ion temperature, $D_i'' = |\langle v_i \rangle - \langle v_n \rangle| \sqrt{m_i / (2k_B T_n)}$ and in which $M_{nO} = m_O / (m_i + m_O)$, $M_{iO} = m_i / (m_i + m_O)$, and equivalent expressions for M_{nN_2} and M_{iN_2} .

In this case, the incoherent scatter spectrum is completely determined by 5 scalar parameters in addition to the ion velocity: the electron temperature and density (T_e and N_e), the ion composition (Comp = $n(O^+)/N_e$) as in the usual case, and instead of the ion temperature, one finds 2 new parameters: the neutral temperature (T_n) and the ion-neutral differential velocity ($V_{diff} = |\langle v_i \rangle - \langle v_n \rangle|$). In addition, it has been considered that the neutral atmosphere was composed of O, O₂ and N₂. The ratio of the molecular components to the atomic one ($R_{an} = (n(O_2) + n(N_2))/n(O)$), that is used in the calculation of the collision frequency ratio in (3), is calculated from the MSIS90 model (Hedin *et al.*, 1991).

Our method is quite different from the method based on Eq. (1), which leads to describe the deformation of the incoherent scatter spectrum in terms of a T^* and a D^* for each ion species, but do not assume a priori any collision model.

3. Sensitivity Tests of the New Method

Simulations using noise perturbed theoretical autocorrelation functions have been performed to test the new method. If theoretically, it is possible to fit simultaneously on the 5 parameters cited above, the tests have shown that, due to the presence of noise, only a 3-parameter fit gave good results. We will emphasise here with two examples the sensitivity of this new analysis method to the collision model and to the neutral atmosphere composition, as given by the parameter R_{an} .

Let us first imagine that, during a magnetically disturbed EISCAT experiment, one has a correlated measurement of the neutral wind and temperature. Such correlated measurements have been done by the WINDII interferometer on board the UARS satellite (Shepherd *et al.*, 1993) but are not yet fully validated (Gault *et al.*, 1994; Lathuillere, 1994). This will allow to fix the two parameters T_n and V_{diff} in the fit, V_i

Fig. 1. Simulation results at different aspect angles for a fit on N_e , T_e and the composition. In panel a, the ACFs have been generated with the collision model A of St Maurice and Shunk (1977) and analysed with the model B. In panel b and c, the ACFs have been generated with the neutral atmosphere parameter $R_{an} = 0.3$ and analysed with $R_{an} = 0.1$ (panel b) and $R_{an} = 0.5$ (panel c). The values of N_e , T_e and the composition used in the ACF generations are indicated by the dashed line.

being obtained by a classical analysis of the spectra recorded in the three EISCAT sites. Our method will allow to retrieve from a single incoherent scatter spectrum the electron temperature and density and the ion composition. It will also give the anisotropy of each ion species, which is defined as the ratio of the perpendicular temperature to the parallel one.

We show on Fig. 1, how the fitted parameters depend on the collision model and on the neutral atmosphere composition. Autocorrelation functions have been generated for 4 different aspect angles with $T_n = 1000$ K and $V_{diff} = 1500$ m/s for the two fixed parameters, $N_e = 2 \times 10^{11}$, $T_e = 3000$ K and $O^+/N_e = .4$ as indicated by the dashed line for the fitted parameters. Thirty different realisations of noisy autocorrelation functions have been analysed: the values presented in Fig. 1 correspond to the average values of the fitted parameters and their error bars to the average values of individual error bars.

In panel (a), the autocorrelation functions have been generated using the model A of St Maurice and Schunk (1977) and analysed with their model B. In panel (b) and (c) the ACFs have been generated with $R_{an} = 0.3$ and analysed with $R_{an} = 0.1$ (panel b) or 0.5 (panel c). The results are affected more strongly for small aspect angles. For large aspect angles, the results are not very sensitive to the neutral atmosphere

Fig. 2. Same as Fig. 1, but for a fit on N_e , V_{diff} and the composition.

composition and to the collision model. In particular the bias on the ion composition is of the order of magnitude of its error.

The second example that we have chosen supposes that independent measurements of the neutral and electron temperatures are available. The neutral temperature can be inferred from EISCAT measurements during quiet conditions and Hubert *et al.* (1993) proposed to deduce the electron temperature from plasma line measurements. The 3-parameter fit will give in this case the electron density, the ion composition, the ion-neutral differential velocity, and in addition the anisotropy of the two ion species. The sensitivity of these results to the collision model and the neutral composition is shown in Fig. 2. Panel a, b and c have the same meaning as that for Fig. 1. In this case, the bias introduced by a wrong assumption is almost entirely supported by the V_{diff} parameter. The value of the ion composition is good for all aspect angles.

Other examples of sensitivity studies can be found in Gaimard *et al.* (1994), who conclude that for large aspect angles the results are never sensitive to the neutral atmosphere composition and to the collision model.

4. Analysis of EISCAT Data

Gaimard *et al.* (1994) have presented results of the analysis of the event of 27 August 1986, that shows

Table 1. Results of the analysis of the CP3 EISCAT scan on 27th August 1986 between 13.00 and 13.30 UT. T_i represents the mean ion temperature and A the anisotropy.

Φ ($^\circ$)	62.19	54.60	44.27	30.42	12.01	3.59
V_{diff} (m/s)	1700	1700	1550	1800	1450	1450
O^+/N_e	.36	.39	.32	.04	.45	.43
T_i (K): O^+	3129	3129	2777	3381	2559	2559
T_i (K): NO^+	3429	3429	3019	3723	2767	2767
$A:O^+$	1.64	1.64	1.58	1.68	1.54	1.54
$A:NO^+$	1.24	1.24	1.22	1.24	1.21	1.21

distorted incoherent scatter spectra at 275 km altitude (Winser *et al.*, 1987). This event has been studied by Lockwood and Winser (1988) and Winser *et al.* (1989), using the analysis based on the Raman's description (Suvanto *et al.*, 1989), which gives the opportunity to test the quality of the new method.

Details of the experimental conditions and of the analysis can be found in the above cited papers. We will here only outline the common results and the differences obtained for the mean ion temperature (Eq. (3)), the anisotropy and the ion composition.

Assuming a single ion species O^+ , Gaimard *et al.* (1994) showed that the ion temperatures were very similar to the ones obtained by Winser *et al.* (1989), with values varying between 2000 and 2500 K in the positions of the EISCAT scan that correspond to aspect angles larger than 30° . The anisotropy that we found was ranging between 1.4 and 1.5. Winser *et al.* (1989) have also performed an analysis with a mixed ion composition. They have obtained values of D^* for the molecular ions larger than D^* for oxygen ions. This is not possible if the molecular ions are NO^+ . Thus they have explained their results by the presence of a majority of N_2^+ ions for which it is expected larger D^* than for O^+ .

The result of our analysis for a mixed ion composition is given in Table 1 for the 6 positions of the scan corresponding to a large ion velocity. The obtained ion composition shows that the NO^+ ions are in all positions (all aspect angles) more abundant than the O^+ ions with the largest value (minimum O^+/N_e) when V_{diff} is the largest.

The NO^+ temperature is larger than the O^+ temperature of about 200 to 350 K. This difference arises from our choice to consider a realistic neutral atmosphere of 2 constituents. The collision frequency ratio in Eq. (3) is larger for NO^+ ions than for O^+ ions, leading to larger NO^+ temperature than O^+ temperature in presence of electric fields. Such a difference has been predicted by Hubert and Kinzelin (1992) using the same formulation of the ion-velocity distribution-function. It has also been predicted by a numerical model of the ionosphere (Diloy *et al.*, 1995), although the value of the difference was quite larger. The anisotropy of the molecular ions is much smaller as expected than the anisotropy of the oxygen ions, with a difference ranging between .33 and .44. The largest temperatures and anisotropies are found again in the scan position where the differential velocity is the largest.

As far as our procedure is concerned, the observed incoherent scatter spectra can be interpreted in term of a mixture of NO^+ and O^+ ions, where the molecular ions are more abundant than the O^+ ions but less anisotropic. The suggestion of Winser *et al.* (1989) that N_2^+ ions were present during these observations is therefore not correct. Furthermore, simulations using a mixture of NO^+ and O^+ distribution functions, in an oxygen atmosphere, have shown that a fit on the 4 parameters T^* and D^* for each ion type, do not give reliable results (Hubert, private communication, 1995).

5. Conclusion

Up to now, there are two main methods to analyse incoherent scatter spectra during highly disturbed periods, each of them having its own limits. The method based on the description of the ion-velocity distribution-function as a function of a D^* and a T^* has the advantage to analyse the distortion of the incoherent scatter spectra without any strong assumptions on the collision processes between the ions and

the neutrals. It gives good results for the ion temperature providing that only one ion species is present. It is however not valid for small aspect angles and is not able to retrieve the ion composition in most of the cases.

The second method does not have these two last restrictions but however is strongly dependent, at least at small aspect angles, on the assumed collision processes and on the neutral atmosphere composition. In fact it is as good as the representation of the ion-velocity distribution-function by Eq. (2). New experimental results are needed to fully validate this method, i.e. the model of ion-velocity distribution-function. They include results of correlated experiments giving access to some of the free parameters such as the temperature, wind and composition of the neutrals. They also include results of experiments where the same ionospheric volume is sampled under very different aspect angles. Such a geometry is not possible with the actual EISCAT system but the addition of the new antenna in Svalbard will open the possibility of new geometrical configurations.

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