The Energy Collection and Dissipation in the Compressible Magnetotail

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(Received December 12, 1994; Revised May 18, 1995; Accepted November 14, 1997)

In this paper, we have derived the nonlinear wave equations of the shear flow at the magnetotail from compressible MHD equations by the two-dimensional reductive perturbation method, and then discussed the nonlinear processes of energy dissipation and transfer at different region in the magnetotail. At last, the effects of disturbed electric field was discussed.

1. Introduction

The earth magnetotail is an important region where the strong interaction of the solar-magnetosphere-ionosphere occurs. The non-linear waves in magnetotail may play important roles in the processes of energy and momentum transfer. J. F. Wang *et al.* (1992) and X. M. Wang *et al.* (1993) derived the nonlinear wave equations of the shear flow at the magnetotail from incompressible MHD equations by the two dimensional reductive perturbation method, and discussed the nonlinear processes of energy dissipation, transfer and the effects of disturbed electric field in the magnetotail. In this paper, following their methods, we have developed a compressible model to investigate the energy dissipation and transfer at the magnetotail.

2. Basic Equations of Compressible Magnetotail

At the magnetotail, the two-dimensional (x, z) compressible MHD equations can be reduced to

$$\frac{\partial \rho}{\partial t} = \rho \nabla \cdot \vec{v} - (\vec{v} \cdot \nabla) \rho, \quad (1)$$

$$\frac{\partial \vec{v}}{\partial t} = -(\vec{v} \cdot \nabla) \vec{v} - \left(\frac{s^2}{\rho} \right) [\mu_0 \nabla p + (\nabla \Psi) \nabla^2 \Psi] + \nu \Delta \vec{v}, \quad (2)$$

$$\frac{\partial \Psi}{\partial t} = (\vec{v} \cdot \nabla) \Psi + \mu_0 \eta \nabla^2 \Psi, \quad (3)$$

$$\frac{\partial p}{\partial t} = (\vec{v} \cdot \nabla) p - \gamma p \nabla \cdot \vec{v} \quad (4)$$

where ρ and p are the plasma density and pressure, u and w are the velocity components, $\vec{v} = u \hat{e}_x + w \hat{e}_z$, Ψ is the magnetic scalar potential, $\vec{B} = \nabla \Psi \times \hat{e}_y$. We assume that ε is a small dimensionless constant, $(\eta \mu_0)^{2/3} = \varepsilon$, and ρ, u, w, p , and Ψ can be written as sum of background and perturbation quantities

$$\rho = \rho_0(z) + \varepsilon \rho^1, \quad u = u_0(z) + \varepsilon u^1, \quad w = \varepsilon w^1, \quad p = p_0(z) + \varepsilon p^1, \quad \Psi = \Psi_0(z) + \varepsilon \Psi^1, \quad (5)$$

then assume that ρ , u , w , Ψ , and p have power series expansions in ε

$$\begin{aligned} \rho^1 &= \rho_1 + \varepsilon \rho_2 + \varepsilon^2 \rho_3 + \dots, & u^1 &= u_1 + \varepsilon u_2 + \varepsilon^2 u_3 + \dots, & p^1 &= p_1 + \varepsilon p_2 + \varepsilon^2 p_3 + \dots, \\ \Psi^1 &= \Psi_1 + \varepsilon \Psi_2 + \varepsilon^2 \Psi_3 + \dots, & w^1 &= w_1 + \varepsilon^{1/2} w_1 + \varepsilon^{3/2} w_2 + \dots. \end{aligned} \quad (6)$$

Furthermore, for carrying out a nonlinear expansion of Eqs. (1)–(4), we introduce the “stretched” variables

$$x \Rightarrow \varepsilon^{1/2}(x - c_1 t), \quad z \Rightarrow z, \quad t \Rightarrow \varepsilon^{3/2} t. \quad (7)$$

In 1st order approximate equations given in Appendix, we assume

$$u_1 = n(x, t) F'(z), \quad \Psi_1 = n(x, t) M(z),$$

then Eqs. (A1)–(A5) give

$$w_1 = -n_x(x, t) F(z), \quad \rho_1 = \rho_0' n(x, t) F(z) / (u_0 - c_1), \quad p_1 = -\Psi_0' M'(z) n(x, t) / \mu_0. \quad (8)$$

In 2nd order approximate equations given in Appendix, eliminating u_2 , w_2 , p_2 , and Ψ_2 , making use of Eq. (8), we find that $n(x, t)$ will evolve according to

$$n_t + n n_x - a n_{xx} + b n_{xxx} = 0 \quad (9)$$

which is just the Burgers-Korteweg-de Vries equation with

Fig. 1. Parameter a_1 versus the distance from the centre plan of current sheet.

$$b = \int_{Z_0}^{Z_1} a_3(z) dz / \int_{Z_0}^{Z_1} a_1(z) dz, \quad a = \int_{Z_0}^{Z_1} a_2(z) dz / \int_{Z_0}^{Z_1} a_1(z) dz \quad (10)$$

where

$$\begin{aligned} a_1 &= \rho_0' F / (u_0 - c_1) - \rho_0' M / \Psi_0' + \rho_0' (u_0 - c_1) (\rho_0' F' + \rho_0 F'') / (s^2 \Psi_0' \Psi_0'''), \\ a_2 &= \mu_0 \eta_0 \rho_0' M / \Psi_0' + 2\nu_0 \rho_0 (u_0 - c_1) F'' / (s^2 \Psi_0' \Psi_0'''), \\ a_3 &= \rho_0 (u_0 - c_1) [\rho_0 (u_0 - c_1) F - s^2 \Psi_0' M] / (s^2 \Psi_0' \Psi_0''') \end{aligned} \quad (11)$$

which are different from incompressible case (Wang *et al.*, 1991).

3. Energy Transfer and Dissipation in the Magnetotail

Assuming that the background distributions are

$$B_0(z) = B_{00} \tanh(z/\delta) \hat{e}_x, \quad u_0(z) = u_{00} \tanh(z/\delta) \hat{e}_x, \quad \rho_0(z) = \rho_{00} \operatorname{sech}(z/\delta),$$

where δ is the half-thickness of the current sheet, B_{00} and u_{00} are the values of B_0 and u_0 at the boundary of current sheet, and ρ_{00} is the maximum value of ρ_0 in the sheet, and taking $c_1 = 0.9$ and $s = \tau_R/\tau_A = (\delta^2 \mu_0 / \bar{\eta}) / (l_0 \sqrt{\mu \rho} / B_0) = 0.1$, we get values of parameters a_1 , a_2 , and a_3 , which were shown in Figs. 1–3. From Figs. 1–3, we have $a = 0.12 > 0$, $b = -0.01 < 0$ in the region of $|z/\delta| \leq 0.5$ (central region), $a = -0.02 < 0$, $b = 0.006 > 0$ in the region $0.5 < |z/\delta| \leq 1.0$ (peripheral region). Properties of the solution of Eq. (9) are determined by the interaction of dissipation and dispersion. In the central region of the sheet, $a > 0$ (positive dissipation) and $b < 0$ (negative dispersion), the travelling wave amplitude will increase step by

Fig. 2. Parameter a_2 versus the distance from the centre plan of current sheet.

Fig. 3. Parameter a_3 versus the distance from the centre plan of current sheet.

Table 1. The values of coefficients a (dissipation) and b (dispersion) in Eq. (9).

Model	Region	
	$ z/\delta \leq 0.5$	$0.5 < z/\delta \leq 1.0$
Compressible model	$a = 0.12 > 0, \quad b = -0.01 < 0$	$a = -0.02 < 0, \quad b = 0.006 > 0$
Incompressible model	$a = 0.321 > 0, \quad b = 0.014 > 0$	$a = 3.24 > 0, \quad b = 0.627 > 0$

step in the process of propagation, and some small vortexes will develop into a large one, so that the turbulence energy will gradually transfer from small vortexes to a large one. Then the energy will be collected. In the peripheral region, $a < 0$ (negative dissipation), it means that the waves will get energy from the background. If the input of outer energy keeps the constant background, the system will get energy from the outside. Because b is positive, a large-scale vortexes will transfer their energy into small ones. Then the plasma is heated. The results shown that the island can form in central region of the current sheet, but it can not form in the peripheral region. In the peripheral region of current sheet the turbulent energy from outside became the thermal energy. These results are qualitatively consistent with magnetotail observations.

From the B-KdV equations of incompressible MHD model (Wang *et al.*, 1991) and this compressible MHD model at the magnetotail we obtained the values of coefficients as Table 1.

It shows from Table 1 that for incompressible case the coefficient b is positive in the central region of the sheet, and the coefficient a is positive in the peripheral region. The sign of these two coefficients is contrary to the compressible case. The results indicate that the two regions have the same properties in the incompressible model. From above analysis, we can say that, for analyzing the energy reversion of the turbulence, MHD compressible model will be consistent with observations, and the incompressible model will not be satisfactory. However, the MHD turbulence are very complex, the mechanism of energy reversion of MHD is expected to be investigated further.

4. Effects of Disturbed Electric Field

The magnetotail is not a closed system. Disturbances of the solar wind and the embedded magnetic field may have greater influence on the magneto-fluid in the magnetotail. We assume that due to solar wind disturbances the disturbed electrical field propagates along the magnetotail in the form of sinusoidal wave and add the disturbance term on the right side of Eq. (9). Then we can derive the disturbed B-KdV equation as following

$$n_t + nn_x + bn_{xx} = \varepsilon [a_1 n_{xx} + \alpha \cos(\Omega \xi)] \quad (12)$$

where Ω is the disturbance frequency. α is proportional to the disturbance amplitude and associated with the background shear field. Following the methods using in incompressible model, we can derive Melnikov function of equation

$$\begin{aligned} M(\xi_0, a_1, \alpha, b, \Omega, u_1) &= \int_{-\infty}^{\infty} n'^* [a_1 n'^* + (\alpha / \Omega) \sin \Omega(\xi + \xi_0)] d\xi \\ &= -4.8a_1 u_1^{5/2} b^{-3/2} + 12\pi\alpha\Omega^2 \cos(\Omega\xi_0) \left[sh(\Omega\pi u_1^{1/2} b^{1/2}) \right]^{-1}. \end{aligned} \quad (13)$$

We expand shx in the power series, the criterion of chaos is obtained

$$\left(0.4u_1^{5/2} a_1 b^{-3/2} / \pi\Omega^2 \right) \left[\Omega u_1^{-1/2} b^{1/2} \Omega + (\Omega\pi)^3 u_1^{-3/2} b^{3/2} / 3! + \dots \right] \leq |\alpha|. \quad (14)$$

Assuming $u_1 \approx 50$ km/s and the amplitude of disturbed electrical field $\alpha \approx 5$ mV/m–10 mV/m and making order of magnitude estimate, we have approximately from Eq. (14) that

$$2.4 \times 10^{-5} \text{ Hz} \leq \Omega < 0.8 \times 10^1 \text{ Hz} \quad (15)$$

in contrast with

$$3.7 \times 10^{-4} \text{ Hz} \leq \Omega < 2.3 \times 10^1 \text{ Hz} \quad (16)$$

for incompressible model. Since the medium- and small-scale turbulent energy of the solar wind is distributed in the range of 10^{-5} – 10^1 Hz, ranges of values given for Ω in two models are not contradictory to observations. But the ranges of Ω for the incompressible model are less, and do not cover all ranges of the observations. The ranges of Ω for the compressible model do cover all ranges of the observations. It is clear that the compressible model of the magnetotail is more suitable to the study of influence on the magnetotail by the disturbed solar wind.

We thank Prof. H. Oya for assistance to WJF to attend the 8th ISSTP. This work is supported by the Natural Science Foundation of China Education Committee.

Appendix. Approximate Equations

Making use of the “stretched” variables (7) and carrying out small parameter expansion in Eqs. (1)–(4), we have the 1st order approximative equations:

$$(u_0 - c_1) \frac{\partial \rho_1}{\partial x} + \rho_0 \left(\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} \right) + \rho_0' w_1 = 0, \quad (\text{A1})$$

$$\rho_0 (u_0 - c_1) \frac{\partial u_1}{\partial x} + \rho_0 u_0' w_1 + S^2 \left(\mu_0 \frac{\partial p_1}{\partial x} + \Psi_0'' \frac{\partial \Psi_1}{\partial x} \right) + v_0 \frac{\partial^2 u_1}{\partial z^2} = 0, \quad (\text{A2})$$

$$\mu_0 \frac{\partial p_1}{\partial z} + \Psi_0' \frac{\partial^2 \Psi_1}{\partial z^2} + \Psi_0'' \frac{\partial \Psi_1}{\partial z} = 0, \quad (\text{A3})$$

$$(u_0 - c_1) \frac{\partial \Psi_1}{\partial x} + \Psi_0' w_1 - \mu_0 \eta_0 \frac{\partial^2 \Psi_1}{\partial z^2} = 0, \quad (\text{A4})$$

$$\gamma_0 p_0 \left(\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} \right) = 0 \quad (\text{A5})$$

and the 2nd order approximative equations:

$$\frac{\partial \rho_1}{\partial t} + (u_0 - c_1) \frac{\partial \rho_2}{\partial x} + u_1 \frac{\partial \rho_1}{\partial x} + w_1 \frac{\partial \rho_1}{\partial z} + \rho_0 \left(\frac{\partial u_2}{\partial x} + \frac{\partial w_2}{\partial z} \right) + \rho_1 \left(\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} \right) + \rho_0' w_2 = 0, \quad (\text{A6})$$

$$\begin{aligned} \rho_0 \frac{\partial u_1}{\partial t} + \rho_1 (u_0 - c_1) \frac{\partial u_1}{\partial x} + \rho_1 u_0' w_1 + \rho_0 (u_0 - c_1) \frac{\partial u_2}{\partial x} + \rho_0 u_0' w_2 + \rho_0 \left(u_1 \frac{\partial u_1}{\partial x} + w_1 \frac{\partial u_1}{\partial z} \right) \\ + v_0 \frac{\partial^2 u_2}{\partial x^2} + v_0 \frac{\partial^2 u_2}{\partial z^2} + S^2 \left(\mu_0 \frac{\partial p_2}{\partial x} + \Psi_0'' \frac{\partial \Psi_2}{\partial x} + \frac{\partial \Psi_1}{\partial x} \frac{\partial^2 \Psi_1}{\partial z^2} \right) = 0, \end{aligned} \quad (\text{A7})$$

$$\rho_0 (u_0 - c_1) \frac{\partial w_1}{\partial x} + S^2 \left(\mu_0 \frac{\partial p_2}{\partial z} + \Psi_0' \frac{\partial^2 \Psi_2}{\partial x^2} + \frac{\partial^2 \Psi_2}{\partial z^2} + \Psi_0'' \frac{\partial \Psi_2}{\partial z} + \frac{\partial \Psi_1}{\partial z} \frac{\partial^2 \Psi_1}{\partial z^2} \right) + v_0 \frac{\partial^2 w_1}{\partial z^2} = 0, \quad (\text{A8})$$

$$\frac{\partial \Psi_1}{\partial t} + (u_0 - c_1) \frac{\partial \Psi_2}{\partial x} + u_1 \frac{\partial \Psi_1}{\partial x} + w_2 \Psi_0' + w_1 \frac{\partial \Psi_1}{\partial z} - \mu_0 \eta_0 \left(\frac{\partial^2 \Psi_2}{\partial x^2} + \frac{\partial^2 \Psi_2}{\partial z^2} \right) = 0, \quad (\text{A9})$$

$$(u_0 - c_1) \frac{\partial p_2}{\partial x} + p_0' w_2 + \gamma_0 p_0 \left(\frac{\partial u_2}{\partial x} + \frac{\partial w_2}{\partial z} \right) + \gamma_0 p_1 \left(\frac{\partial u_1}{\partial x} + \frac{\partial w_1}{\partial z} \right) = 0. \quad (\text{A10})$$

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