An Average Equator for the Geomagnetic S_q -Field*

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Abstract

In the preceding paper [1], we obtained a coordinate system for the geomagnetic S_q -field. The equator of this spherical coordinate may be regarded as an average equator for the S_q -field.

This equator is in good agreement with a line of eastward maximum current density calculated by the dynamo-theory taking account of the discrepancy between the geomagnetic and geographic axes and of the high electric conductivity in the equatorial zone.

These results seem to suggest the fairly deep-rooted effect of the dynamo-action due to the discrepancy of above two axes upon the S_q -field on one hand, and the high electric conductivity in the equatorial zone on the other.

1. Introduction

Since an anomalous daily variation of horizontal intensity at Huancayo had been pointed out by H.E. Johnston and A.G. McNish [2], this interesting problem of diurnal variation of geomagnetism in equatorial regions has been discussed by many workers. M. Hasegawa has for a long time entertained the idea that these anomalous regions encircle the Earth, and he, with M. Ota, estimated such a line (say *MS-equator*) from an analysis of the data for the Polar Year, 1932-33 [3]. Recently A.T. Price and G.A. Wilkins [4] have also estimated a line of maximum ΔH . These two lines are in good agreement within the limits of error (Fig. 1). As a result of temporary observations in Africa, India and America, J. Egedal [5] has pointed out that these anomalous regions are situated along the magnetic equator. But from a result of temporary observations in south India and Ceylon by B.L. Gulatee [6], and by S.K. Pramanik and S.Y. Narayanan [7], it should seem that the line of maximum ΔH does not lie closely on the magnetic equator.

If S_q is due to an electric current flowing in the so-called S_q -layer, there must be expected to be an anomalous concentration of electric current (say *electrojet* by S. Chapman [8]) in these regions. On the origin of an equatorial electrojet, A.G. McNish [9] attributed it to an effect of the discrepancy between the magnetic and geographic equators upon the dynamo-action in the S_q -layer, and D.F. Martyn [10] to an increase of the electric conductivity near the magnetic equator. Recently M. Hirono [11], K. Maeda [12], T.G. Cowling [13], D.F. Martyn and W.G. Baker [14] have discussed,

* Contribution of Geophysical Papers dedicated to Prof. M. Hasegawa on his sixtieth birthday.

independently, the conductivity of the ionosphere from a same point of view (Hall polarization mechanism), and given an explanation for the enhanced conductivity near the magnetic equator.

We find here a line of eastward maximum current density at 11h 16m local time, at which H is nearly maximum, on the basis of the dynamo-theory, taking account of the discrepancy between the geomagnetic and geographic axes and of high conductivity in the equatorial zone, and compared with the equator of our coordinate system for the S_q -field.

2. An Average equator for the S_q -field

The equator of our coordinate system obtained in the preceding paper [1] may be

Fig. 1—Equator of our coordinate system as compared with several equators, where 1=magnetic equator, 2=line of maximum ΔH (After A.T. Price and G.A. Wilkins), 3=line of maximum a_1 (After M. Hasegawa and M. Ota), 4=equator of our coordinate system, 5=geomagnetic equator.

Fig. 2—Lines of eastward maximum current density at 11 h 16 m local time in the cases of $f=1$ ①, $f=5$ ②, and $f=100$ ③, as compared with the geomagnetic eqnator ④.

Fig. 3—Variation of current density along the line of its maximum in the case of $f=5$.

Fig. 5—Departure of the line of maximum current density ①, of the equator of our coordinate system ② and of the geomagnetic equator ③ from the geographic equator on the meridian of $\Lambda = 0$.

regarded as an average equator for S_q , and is illustrated in Fig. 1 as compared with other several equators. This average equator for S_q is in good agreement with that derived from the dynamo-theory as below.

The notation used is as follows:

θ, λ = geographic colatitude and east longitude,

τ = geomagnetic local time,

Θ, A = geomagnetic colatitude and east longitude,

T = geomagnetic universal time (*i.e.* -69° meridian time),

U, V = southward and eastward components of air velocity,

ϕ = velocity potential of air,

R = electric current function,

K = electric conductivity,

H_z = vertical component of the earth's magnetic field,

a = radius of the S_q -layer.

It is well known that the current function R for the stationary state must satisfy the following equation in the case of $K = \text{const.}$:

$$\frac{\partial^2 R}{\sin \theta \partial A^2} + \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial R}{\partial \theta} \right) = aK \left\{ \frac{\partial (VH_z)}{\partial A} + \frac{\partial (UH_z \sin \theta)}{\partial \theta} \right\} \quad (1)$$

where $H_z = C \cos \theta$ and C is a constant. If U and V can be derived from the velocity potential ϕ , and ϕ is assumed to be expressed by surface harmonics.

$$\begin{aligned} \phi &= \sum_n \sum_m \phi_n^m \\ &= \sum_n \sum_m k_n^m P_n^m(\cos \theta) \sin(m\tau + a_n^m) \quad \dots \dots \text{in geographic coordinate,} \\ &= \sum_n \sum_m k_n^m \sum_\mu c_\mu^m P_n^\mu(\cos \Theta) \sin(\mu A + \beta_\mu^m) \quad \dots \dots \text{in geomagnetic coordinate,} \\ &\quad \text{(see our preceding paper [1], p. 48)} \end{aligned}$$

where

$$c_m^0 = A_{0m} \sin(mT + a_n^m),$$

$$c_m^\mu = \{ A_{\mu m}^2 \sin^2(mT + a_n^m) + B_{\mu m}^2 \cos^2(mT + a_n^m) \}^{\frac{1}{2}},$$

$$\beta_m^0 = \frac{\pi}{2},$$

$$\tan \beta_m^\mu = -\frac{A_{\mu m}}{B_{\mu m}} \tan(mT + a_n^m),$$

$A_{\mu m}$ and $B_{\mu m}$ = notations used in our preceding paper [1], relation (3).

then the solution of eq. (1) is

$$R = -KC \sum_n \sum_m k_n^m \sum_\mu [c_m^\mu S_n^\mu + C_{II}^\mu p^\mu + C_{IV}^\mu q^\mu] \sin(\mu\lambda + \beta_m^\mu), \quad (2)$$

where

$$S_n^\mu = \frac{n[(n+1)^2 - \mu^2]^{\frac{1}{2}}}{(n+1)(2n+1)} P_{n+1}^\mu + \frac{(n+1)[n^2 + \mu^2]^{\frac{1}{2}}}{n(2n+1)} P_{n-1}^\mu,$$

$$p^\mu = \frac{\sin^\mu \Theta}{(1 + \cos \Theta)^\mu}, \quad q^\mu = \frac{\sin^\mu \Theta}{(1 - \cos \Theta)^\mu},$$

C_{II}^μ, C_{IV}^μ = arbitrary constants.

Assuming that the electric conductivity of the layer concerned is fK_0 for region II and K_0 for regions I and III (Fig. 4), the solution for these three regions is

$$\left. \begin{aligned} R_I &= -K_0 C \sum_n \sum_m k_n^m \sum_\mu [c_m^\mu S_n^\mu + C_{II}^\mu p^\mu] \sin(\mu\lambda + \beta_m^\mu), \\ R_{II} &= -fK_0 C \sum_n \sum_m k_n^m \sum_\mu [c_m^\mu S_n^\mu + C_{II}^\mu p^\mu + C_{IV}^\mu q^\mu] \sin(\mu\lambda + \beta_m^\mu), \\ R_{III} &= -K_0 C \sum_n \sum_m k_n^m \sum_\mu [c_m^\mu S_n^\mu + C_{IV}^\mu q^\mu] \sin(\mu\lambda + \beta_m^\mu), \end{aligned} \right\} \quad (3)$$

Fig. 4

and using the condition that the normal component of the current and the tangential component of the electric field are continuous at the two boundaries of the three regions, the arbitrary constants are determined as follows:

$$C_I^\mu = \frac{\Delta_I^\mu}{\Delta^\mu}, \quad C_{II}^\mu = \frac{\Delta_{II}^\mu}{\Delta^\mu}, \quad C_{III}^\mu = \frac{\Delta_{III}^\mu}{\Delta^\mu}, \quad C_{IV}^\mu = \frac{\Delta_{IV}^\mu}{\Delta^\mu},$$

where

$$\begin{aligned} \Delta^\mu &= (f-1)^2 - M^{2\mu}(f+1)^2, \\ \Delta_I^\mu &= (f-1)c_m^\mu W_n^\mu [(-1)^{\mu+m+1} 2fM^\mu - (f-1) - M^{2\mu}(f+1)], \\ \Delta_{II}^\mu &= (f-1)c_m^\mu W_m^\mu [(-1)^{\mu+m+1} M^\mu(f+1) - (f-1)], \\ \Delta_{III}^\mu &= (f-1)c_m^\mu W_n^\mu [M^\mu(f+1) - (-1)^{\mu+m+1}(f-1)], \\ \Delta_{IV}^\mu &= (f-1)c_m^\mu W_n^\mu [2fM^\mu - (-1)^{\mu+m+1} \{(f-1) + M^{2\mu}(f+1)\}], \end{aligned}$$

$$M^\mu = \left(\frac{1 + \cos \Theta_1}{1 - \cos \Theta_1} \right)^\mu, \quad W_n^\mu = \frac{S_n^\mu(\Theta_1)}{p^\mu(\Theta_1)}.$$

Table 1

$$V_e = r_0 \sum_n \sum_m E_n^m P_n^m \sin(mt + e_n^m)$$

m	n	E_n^m	e_n^m
1	2	7.22 γ	101 $^\circ$
2	3	3.70 γ	105 $^\circ$
3	4	1.10 γ	125 $^\circ$

(After M. Hasegawa and M. Ota)

From the assumption of constant conductivity of the layer, the values of $K_0 C k_n^m$ can be estimated from the observed magnetic potential [15]. Since the velocity potential ϕ_n^m contributes to the current function R_{n+1}^m , a solution in region II, using the result obtained by M. Hasegawa and M. Ota [3] (Table 1), becomes

$$\begin{aligned}
R_{II} = \sum_n f R_{n+1} \sum_{\mu} (-1)^{\mu} & \left[S_n^{\mu} + \frac{C_{II}^{\mu}}{c_m^{\mu}} p^{\mu} + \frac{C_{III}^{\mu}}{c_m^{\mu}} q^{\mu} \right] \\
& \times \left[\frac{1}{2} (A_{\mu_n} - B_{\mu_n}) \sin(nT + \mu A + e_{n+1}^n) \right. \\
& \left. + \frac{1}{2} (A_{\mu_n} + B_{\mu_n}) \sin(nT - \mu A + e_{n+1}^n) \right], \quad (4)
\end{aligned}$$

where

$$R_{n+1}^n = -\frac{1}{4\pi} \frac{2n+3}{n+2} \left(\frac{a}{r_0} \right)^{n+1} r_0 E_{n+1}^n.$$

Fig. 2 shows the lines of eastward maximum current density at 11 h 16 m local time in the cases of $f=1$, $f=5$, and $f=100$, and Fig. 3 the variation of current density along its line in the case of $f=5$. Fig. 5 shows a departure of the line of maximum current density from the geographic equator on the meridian of $A=0$. From these Figures we have found that;

- (i) a line of the maximum current density for $\Theta_1=80^\circ$ (as an example) and $f=5$ is in good agreement with the above-mentioned average equator for S_7 ,
- (ii) the value for current density along the line of its maximum is large at the places where the geomagnetic equator is situated far from the geographic equator, but the range of variation with longitude is merely 1.4 per cent,
- (iii) unless the highly conductive zone II is not very narrow, the position of maximum density may be shifted fairly well from the middle of the highly conductive zone.

3. Conclusion

Above considerations seem to suggest the fairly deep-rooted effect of the dynamo-action due to the discrepancy between the above two axes upon the S_7 -field on one hand, and the high conductivity in the equatorial zone (several times higher than in the middle latitude) on the other.

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The Meeting of the Society of Terrestrial Magnetism and Electricity :

The 13th General Meeting was held at the Geographic Survey Institute on May 15-17, 1953.

58 Reports were read, about 100 Members assembled,

The Tanakadate-Prize was awarded for the following excellent workers :

The 13th, Mr. S. Uyeda and Mr. S. Akimoto, at the 13th General Meeting :

Self-Reversal of Thermo-Remanent Magnetism of Igneous Rocks.