Intensity Problem in the Deflection of Cosmic Rays in the Solar Magnetic Field

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(Read May, 1951, Received Oct. 1, 1956)

Abstract

Assuming uniform parallel beam of cosmic rays at infinity is deflected in the general magnetic field of the sun, cosmic-ray intensities at each point of the earths' orbit were calculated. Problem was treated only in the equatorial plane for simplicity. As the intensities change with season, the solar diurnal variation averaged over all season never tends to zero due to the heliomagnetic deflection of cosmic rays.

We will assume that there exists a anisotropic distribution of cosmic rays in a space outside the solar system, due to any origin. This origin, for instance, may be the rotation of our galaxy, well known as Compton-Getting effect.¹⁾ Because of the deflection of cosmic-ray particles by the sun's general magnetic field, apparent solar diurnal variation of cosmic rays will show an annual variation not only in its phase, but also its amplitude. Thus, even if this apparent solar diurnal variation will be averaged over one year, it will not vanish and remain as mean solar diurnal variation. In order to study this possibility, following calculation was carried out. Deflection of cosmic rays in the solar magnetic field was already calculated in our previous paper,²⁾ which will be denoted as [1] hereafter, and this work is continuation of [1].

Now, problem was treated only in the equatorial plane, under the assumption that the sun's and the earth's magnetic dipole moment are parallel to each other. In Fig. 1, M_s is the sun's dipole moment, which is directing towards the back of paper perpendicular to the plane of paper, that is, towards the negative direction of Z -axis. Circle C is the earth's orbit and P is the position of earth, which is designated by radius r_s and angle φ_s counted positively clockwise from Y -axis. Now, we assume that there exists a uniform parallel beam of cosmic rays parallel to Y -axis at infinity. Furthermore,

Fig. 1 Circle C is the earth's orbit and P is the position of earth. Axes and symbols used in the paper are shown.

we take r -axis perpendicular to Y -axis at sufficiently far distance from the earth and x -axis perpendicular to the direction of incidence of cosmic rays at P , and we consider that particles existing in the region $r \sim r + \Delta r$ at infinity are incident into the region $0 \sim \Delta x$ at P . Then, if we denote cosmic-ray intensities per unit time per unit area perpendicular to the direction of incidence of cosmic rays at infinity and at P as $j(r)$ and $j(\varphi_s)$ respectively, then

$$j(\varphi_s) dt dx dZ = j(r) dt dr dZ.$$

If we take the direction of x -axis in a definite way and $j(r) \equiv 1$, then

$$j(\varphi_s) = \left| \frac{dr}{dx} \right| = \left| \frac{\partial r}{\partial \varphi_s} \frac{d\varphi_s}{dx} + \frac{\partial r}{\partial r_s} \frac{dr_s}{dx} \right| = \left| \frac{\cos \theta_s}{r_s} \frac{\partial r}{\partial \varphi_s} + \sin \theta_s \frac{\partial r}{\partial r_s} \right|, \quad (1)$$

where lengths are measured in Störmer's unit and r is two times r_1 , which appears in [1].

$\frac{\partial r}{\partial \varphi_s}$ and $\frac{\partial r}{\partial r_s}$ were computed by differentiating the curve $\varphi_s(r_1, r_s)$, which was obtained at the study of [1] and is shown in Fig. 4 a and 4 b of appendix 1. So, of course, the accuracy of computation is rather low. Intensities $j(\varphi_s)$ thus obtained is shown in Fig. 2, where full line and dotted line are two branches with different directions of incidence of cosmic rays as already mentioned in [1]. From this, it is

Fig. 2 Intensity $j(\varphi_s)$ of cosmic rays per unit time per unit area perpendicular to the direction of incidence are plotted against angle φ_s , shown in Fig. 1. Points, E, V, and W are those corresponding to incidence from eastern horizon, vertical incidence, and incidence from western horizon. The parts E-V-W of curves correspond to outer orbit.

recognized that intensity changes with season and there exists gloomy season covered with magnetic shadow and light season. Still more, twilight is incident after rotating many times around the sun, which is not shown in Fig. 2 wholly and extension of short line appearing at the right end of Fig. 2.

Here, we will see how the direction of incidence of cosmic rays changes on x -axis. In Fig. 3, angle of incidence θ_s is counted positively eastwards from PO' . (In the paper [1], θ_s was counted positively eastwards from PO' for outer orbit and from PO for inner orbit, respectively.) About the angle α from x -axis to the direction of incidence of cosmic rays, it is easily seen that $\Delta\alpha = \Delta\theta_s - \Delta\varphi_s$. Thus,

$$\frac{d\alpha}{dx} = \frac{d\theta_s}{dx} - \frac{d\varphi_s}{dx} = \frac{d\theta_s}{dx} - \frac{\cos\theta_s}{r_s}$$

$$\therefore \Delta\alpha \approx \Delta\theta_s - \cos\theta_s \cdot \frac{\Delta x}{r_s} \quad (2)$$

On the other hand, from $\sin\theta_s = \frac{r}{r_s} - \frac{1}{r_s^2}$,

$$\Delta(\sin\theta_s) \approx \frac{d(\sin\theta_s)}{dx} \Delta x = \left\{ \frac{dr}{dx} + \left(-\frac{r}{r_s} + \frac{2}{r_s^2} \right) \sin\theta_s \right\} \cdot \frac{\Delta x}{r_s} \quad (3)$$

Thus, except the case that $\frac{dr}{dx}$ is considerably large, $\Delta\alpha$ is the order of $\frac{\Delta x}{r_s}$. If we take fifty times earth's radius for Δx , which is considered to be region for cosmic rays being effectively influenced by the earth's magnetic field, $\frac{\Delta x}{r_s}$ is 2.1×10^{-4} and so $\Delta\alpha$ is negligible small. Thus, cosmic rays are incident into effective region of earth's magnetic field in a parallel beam approximately. Furthermore, if we take usual dimension of cosmic-ray meter for Δx in the case of deflection of cosmic rays by the earth's magnetic field, $\Delta\alpha$ is negligible small in this case also and cosmic rays are incident upon the earth in a parallel beam approximately. Thus, in the case that deflections of cosmic rays by the sun's and the earth's magnetic field are taken into account together, intensity distribution of cosmic rays on the earth in a certain season is such as the part corresponding to outer orbit in Fig. 2, but its relative magnitude changes with season in such a manner as shown in Fig. 2 also.

Next, at the limit of increasing intensity in Fig. 2, $j(\varphi_s)$ tends to ∞ due to increasing of $\frac{\partial r}{\partial \varphi_s}$ towards ∞ . Although this divergence is merely mathematical and integral intensity $\int_0^{\Delta x} j(\varphi_s) dx = \int_0^{\Delta x} \left| \frac{dr}{dx} \right| dx = \int_0^{\Delta r} dr$ is finite, the intensity is very strong at this point. But, at the same time, the angle θ_s of incidence of cosmic rays changes very largely near this point, as known from equation 3. Intensity problem in such case cannot be treated simply and was omitted out of the present consideration. However, it is only small region of φ_s to be excluded for this reason.

Now, the dependence of angle $\theta_s(\varphi_s)$ of incidence of cosmic rays upon φ_s is given by Fig. A2 of appendix 1. From Fig. 2 and Fig. A2, we can express the intensity of cosmic rays at each point of the earth's orbit with its direction of incidence expressed in solar time on harmonic dial. They are shown in Fig. 4 a, 4 b and 4 c. There, two full lines correspond to two branches with different directions of incidence of cosmic rays as seen in Fig. 2 and dotted line is vector sum of these two branches. The

Fig. 3 Arc C is the earth's orbit and P is the position of earth. Axes and symbols used in the paper are shown.

position on the earth's orbit is shown by angle φ_s written on the curves. When we consider cosmic rays of vertical incidence on the earth, OY'-axis and OY''-axis ought to be taken instead of OY-axis in the cases of $M_s=10^{34}$ and $M_s=2.5 \times 10^{33}$ gauss cm³, respectively.

Fig. 4 a For $r_s=4$ Störmer of cosmic-ray energy, intensity of cosmic rays at each point of earth's orbit with its direction of incidence are reproduced on harmonic dial. In the case that cosmic rays incident vertically on the earth are considered, axis OY' and OY'' are to be taken for o-hour axis instead of OY in the case of $M_s=10^{34}$ and 2.5×10^{33} gauss cm³, respectively. Two full lines correspond to two branches with different directions of incidence of cosmic rays as seen in Fig. 2 and dotted line is vector sum of these two branches. Vector A, B, and C are resultant vectors of two branches mentioned above and their vector sum.

Fig 4 b It is the same figure for $r_s=2$ Störmer as Fig. 4 a.

Fig. 4 c It is the same figure for $r_s=1$ Störmer as Fig. 4 a.

Furthermore, vector A, B and C are resultant vectors of above two branches and their vector sum. In the case of $M_s=10^{34}$ gauss cm³, $r_s=4$ and 2 Störmer of cosmic-ray energy correspond to $r_e=2$ and 1 Störmer, respectively. Then, mean intensities of $r_e=2$ and 1 Störmer of cosmic-ray energy given by resultant vectors C are about 1/5 and 1/25 of intensity at infinity, respectively. That is, mean intensity averaged over all season never tends to zero due to the general magnetic field of the sun, as mentioned at the beginning of this paper.

The author wishes to express his cordial thanks to professor Y. Sekido for his suggestion of the problem and his guidance throughout this work.

Fig. A1a

APPENDIX

1. In Fig. A1a and A1b, angle φ_s is plotted against r_1 , taking r_s for parameter. The parts A-B-C of curves correspond to outer orbit. In Fig. A2, angle θ_s is plotted against φ_s , taking r_s for parameter.

2. At the point where $\frac{\partial r}{\partial \varphi_s} \rightarrow \infty$, that is, $\frac{\partial \varphi_s}{\partial r} = 0$, $\frac{\partial \varphi_s}{\partial \theta_s} = 0$.

Furthermore, $\frac{\partial x_s}{\partial \theta_s} = \pm 1$ (+: inner orbit,

—: outer orbit) and thus $\frac{\partial t_n}{\partial t_e} = 0$, as

seen from equation (1), (2), and equations in appendix of the paper [1]. It should be noticed that the

point in Fig. 2 where $j(\varphi_s) \rightarrow \infty$ corresponds to the point in Fig. 4

of [1] where $\frac{\partial t_n}{\partial t_e} = 0$.

Fig. A1b

References

- [1] A.H. Compton and I. A. Getting, phys. Rev. 47, 817 (1935).
- [2] Y. Sekido and T. Yagi, Journ. Geomag. Geoelect. II, No. 3, 78 (1950), which is denoted as [1] in this paper.

Erratum in [1]: Equation (2) ought to be corrected as $r_s/r_e = D/a_e \times (M_e/M_s)^{1/2}$, instead of $r_s = D/a_e \times (M_e M_s)^{1/2}$ in that paper.

Fig. A2