Experimental researches and numerical analyses of non-orthogonal rainfall infiltration on the surfaces of unsaturated sand slopes

Wang Chenghua i), Wan Zhengyi ii) and Zhang Chenglin iii)  

i) Professor, School of Civil Engineering, Tianjin University, Tianjin 300072, China.  
ii) Postgraduate, School of Civil Engineering, Tianjin University, Tianjin 300072, China.  
iii) Postgraduate, School of Civil Engineering, Tianjin University, Tianjin 300072, China.

ABSTRACT

To investigate the phenomenon and infiltration behavior of non-orthogonal rainfall infiltration into the surfaces of unsaturated sand slopes, a series of model tests of unsaturated sand slopes under different rainfall intensity, slope angle, and void ratio were carried out with a self-designed artificial rainfall device. For the purpose of contrast with the test results, a large amount of the numerical simulations was also conducted using classic methods for treating normal component of rainfall as infiltration boundary condition. Comparisons of the results of numerical simulation and of the tests indicate that there are differences which show that the orthogonal infiltration condition is unsuitable for analysis of rainfall induced seepage in unsaturated slopes.

Keywords: laboratory test, numerical analysis, non-orthogonal infiltration, rainfall, unsaturated sand

1 INTRODUCTION

As infiltration of rainfall is one of the most important factors that triggering landslides, it is often taken as a second class boundary condition in numerical simulations of seepage in soils, especially in analyses of stability of unsaturated soil slopes. But, nowadays and world widely, a normal component of rainfall has been taken as a flow boundary condition that is set normally to the surfaces of soil slopes by decomposing of the rainfall intensity only mathematically according to the slope direction, as in the works done by Fredlund and Rahardjo (1993), Ng and Shi (1998) and Cai and Ugai et. al. (1998) which does not reflect the real phenomena and conform to the mechanism of actual non-orthogonal infiltration of rainfall into unlevelled surfaces of soil slopes, and from which a severe error could induced in evaluation of the amount of water into the slope and hence suction in slope stability analysis, as firstly conceptually analyzed by Wang (1999).

To investigate the phenomenon and infiltration mechanism of non-orthogonal rainfall infiltration into the surfaces of unsaturated sand slopes, a series of model tests of unsaturated sand slopes under different rainfall intensity, slope angle, and void ratio were carried out with a self-designed artificial rainfall device.

In addition, for the purpose of contrast with the test results, a large amount of the numerical simulations was also conducted using classic methods for treating normal component of rainfall as infiltration boundary condition.

In both of the tests and numerical simulations, the variation of infiltration rate, seepage rate and sandy soil water storage increment with respect to infiltration time were obtained, and the influence of rainfall intensity, slope angle and void ratio on the process of seepage and the amount of seeped water were analyzed.

Finally, comparisons of the results of numerical simulation and test results indicate that there were differences in infiltration rate, seepage rate and sandy soil water storage increment and demonstrate that there are significant differences between the results of numerical simulation and test results under the large rainfall intensity which is larger than the saturated permeability coefficient of the sandy soil.

From this study, the phenomenon of non-orthogonal infiltration was observed and its mechanism verified which shows that the orthogonal infiltration condition is unsuitable for analysis of rainfall induced seepage in unsaturated slopes.

2 LABORATORY MODEL TESTS

2.1 Setup of test devices

In order to investigate the infiltration phenomenon of water rainfall on sloping surface of sandy soil, an artificial rainfall infiltration test system was designed, which is schematically shown in Fig. 1. The artificial rainfall of a given intensity \( i_0 \) was simulated by
supplying water which is of a given flux $q$ through a water-supply pipe with a sprinkler composing of a group of densely arranged syringe needles, and the rain water with given intensities was designed to infiltrate into the surface of a sand soil with given void ratio $e$ filled in a test chamber, whose dimensions are $230$ mm in length, $130$ mm in width and $100$ mm in height. The Sloping surface of a soil can be formed by rotating the chamber around an axis in the left end line at the bottom of the chamber, and rotation of angle $\alpha$, i.e., the angle of the sloping surface ranges from $0^\circ$ to $45^\circ$.

Measurements of total amounts and rates of runoff water from the soil surface by weighting the water flow from a pipe to container No. 3, and the seepage flows through the lateral and bottom boundaries of the chamber by weighting the water that flow from a pipe to container No. 2 and that flow from three pipes to container No. 1.

The plan for the laboratory tests is briefly shown in Table 1, while for the detailed description of the test procedure readers are referred to the work by Zhang (2013). Totally nine normal tests were conducted, considering the three factors that influencing the states of infiltration, i.e., rainfall intensity $i_0$, soil density $e$, and angle of slope surface $\alpha$.

<table>
<thead>
<tr>
<th>Test group</th>
<th>Flow $q$ (ml·min$^{-1}$)</th>
<th>Void ratio $e$</th>
<th>Slope angle $\alpha$°</th>
<th>$i_0$ (m·s$^{-1}$)</th>
<th>$i$ (m·s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{0a}$</td>
<td>60</td>
<td>0.58</td>
<td>0</td>
<td>2.81×10$^{-5}$</td>
<td>2.81×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{0b}$</td>
<td>60</td>
<td>0.58</td>
<td>30</td>
<td>3.01×10$^{-5}$</td>
<td>2.91×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{0c}$</td>
<td>60</td>
<td>0.58</td>
<td>30</td>
<td>3.28×10$^{-5}$</td>
<td>2.84×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100a}$</td>
<td>20</td>
<td>0.58</td>
<td>30</td>
<td>1.37×10$^{-5}$</td>
<td>1.34×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100b}$</td>
<td>100</td>
<td>0.58</td>
<td>30</td>
<td>5.67×10$^{-5}$</td>
<td>4.90×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100c}$</td>
<td>60</td>
<td>0.64</td>
<td>0</td>
<td>2.79×10$^{-5}$</td>
<td>2.79×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100d}$</td>
<td>60</td>
<td>0.64</td>
<td>30</td>
<td>3.31×10$^{-5}$</td>
<td>2.87×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100e}$</td>
<td>60</td>
<td>0.73</td>
<td>0</td>
<td>3.01×10$^{-5}$</td>
<td>3.01×10$^{-5}$</td>
</tr>
<tr>
<td>$q_{100f}$</td>
<td>60</td>
<td>0.73</td>
<td>30</td>
<td>3.29×10$^{-5}$</td>
<td>2.85×10$^{-5}$</td>
</tr>
</tbody>
</table>

2.3 Analysis of test results

(1) Results under different rainfall intensities.

Fig. 2 shows seepage rate of undersurface increase with the increase of rainfall intensity in the test in which the void ratio $e$ is 0.58 and slope angle $\alpha$ is $30^\circ$. The test $q_{20}$ (the flow $q$ is equal to 20 ml·min$^{-1}$, similarly for as $q_{0a}$ and $q_{100}$) shows that the time for the bottom surface seepage appears is 30 minutes, as showed at the intersect point of the curve with horizontal ordinate axis in Fig. 2. With the increase of controlled flow $q$, the wetting fronts move faster and faster, while the starting time for the bottom surface seepage become earlier and earlier. The curves in Fig. 2 indicate that the wetting front in the soil moves slowly under small rain intensity, in other words, the initial seepage time of undersurface increase with the decrease of rainfall intensity.

In initial period of a test, the rate of seepage increase with the time till it becomes stable. It can be seen by comparing the curves in Fig. 2 that the time when seepage rate becomes stable decreases and the value of stable seepage rate increases with the increase of rainfall intensity.

The variations of soil water storage increment $\Delta Q$ with time are given in Fig. 3. Though seepage occurs at different times in test $q_{20}$, $q_{60}$ and $q_{100}$, the values of $\Delta Q$ at times when seepage occur in all the tests remain the same. This is the indication that there is no influence of rainfall intensity on $\Delta Q$ when the seepage just occurs. But, after the initial seepage point, $\Delta Q$ increase slowly and tend to be stable. As shown in Fig. 2, $\Delta Q$ for test $q_{20}$, represented by $\Delta Q_{q_{20}}$, is the least and $\Delta Q$ for test $q_{20}$, represented by $\Delta Q_{q_{20}}$, is the maximum. So, the $\Delta Q$ in stable seepage stage have a rising trend with the increase of the rainfall intensity.

An important finding is that there was no runoff appeared on the surface of sand slope, and all rainwater infiltrates into soil mass. In fact, the infiltration rate is constant value which is equal to the rainfall intensity which is also constant in the whole process of a test.

Fig. 1. The artificial rainfall infiltration test system

2.2 Test scheme design

The soil used in this is alluvial sand which specific gravity $d$ is 2.67. The of void ratio of the sand was set to three values, i.e., 0.58, 0.64 and 0.73, and the values of saturated seepage coefficients $k_s$ of the sand corresponding to the void ratios are $2.28 \times 10^{-5}$ m/s, $4.64 \times 10^{-5}$ m/s and $7.59 \times 10^{-5}$ m/s, respectively. The initial moisture content $w$ of the soil was controlled as 1.6%.

The period of rainfall is set, based on the observations in prior tentative tests, to one hour for all the normal tests. The intensity of rainfall on horizontal surface of the bottom of the sprinkler $i_0$ is controlled through a gauge installed on the supply pipe by given amount of water flow per minute $q$, while the rainfall intensity on the sloping surface which e accepting rain water is $i = i_0 \cos \alpha$.

By measuring the variation of the total water amounts with time $t$, i.e., the total amount of the seepages from bottom of the chamber $Q_{1}(t)$, the total amount of seepages from front side $Q_{2}(t)$ and the total amount of the water of surface runoff $Q_{3}(t)$, The variations of water stored in soil, i.e., the increment water amount in soil $\Delta Q(t)$ can be determined and used in analyzing the infiltration behavior in each tests.
This means that the use of normal component of the total rainfall intensity by decomposing simply according to the slope angle as the flux boundary condition of unsaturated seepage analysis is not valid.

(2) Results under different void ratios.

In order to exam the influence of the void ratio of sand on the infiltration. Three tests \( q_0 \) of the same sloping angle \( (\alpha=30^\circ) \) with different values of void ratios \( e \), i.e., 0.58, 0.64 and 0.73 under same rainfall intensity 60 ml/min were conducted. The initial time of seepage the bottom surface are 13 min, 14 min and 16 min, respectively, as shown in Fig. 4 and Fig. 5. The results show that the differences of the initial seepage time are not obvious with the increase of void ratio. The initial seepage time from front side face is longer with the bigger void ratio.

Keeping the slope angle and rainfall intensity constant, different void ratios have great influences on rainfall infiltration. Fig. 4 and Fig. 5 show together that the seepage rate and cumulative flow of side face decrease with the increase of void ratio.

The points A and B in Fig. 6, show the water storage increments \( \Delta Q \) of test soils are of little difference, i.e., the variations of sandy soil water storage increments do not change much. In addition, the values of the total amount of cumulative seepages are very close. But, the seepage rates of front side face and bottom surface have obvious difference, which indicate the void ratio has great influence in the main seepage direction and flow path of soil and has less effect on the water storage capacity of sand soil.

(3) Results under different slope angles.

Fig. 7 shows the seepage rates of bottom surface. The slope angles are \( 0^\circ \), \( 15^\circ \) and \( 30^\circ \), for test cases with \( e=0.58 \), \( q=60 \text{ ml/min} \) condition. With the raise of slope angle, the initial seepage time \( t \) (at the intersect points of the curves with horizontal ordinate axis in the figure.) become longer, meanwhile the time when the seepage achieves stable state is shorter. With the increase in slope angle, the differences between seepages of bottom surface become less. The seepage rates in the tests with \( \alpha=0^\circ \) and \( \alpha=15^\circ \) are very close. The bottom seepage rate curve for the test with \( \alpha=0^\circ \) shows a peak before it achieves a stable stage, but the curves for tests with \( \alpha=15^\circ \) and \( \alpha=30^\circ \) have no peaks.
Fig. 8 shows that the curve of water storage increment $\Delta Q_{\alpha=0^\circ}$ with time for the test with $\alpha=0^\circ$ has a peak value at the initial seepage stage. But in the tests with $\alpha=15^\circ$ and $\alpha=30^\circ$ there are no peak values of $\Delta Q$. At the end of the tests, the test $\alpha=0^\circ$ is in steady state and the soil water storage increment is lower than that at initial seepage time (A, B in Fig. 8). At steady stage, $\Delta Q_{\alpha=15^\circ} > \Delta Q_{\alpha=30^\circ} > \Delta Q_{\alpha=0^\circ}$, the test results indicate that the sand water storage increments increase at first and then decrease with the angle increasing. Besides, the test $\alpha=0^\circ$ has no lateral seepage and the test $\alpha=15^\circ$ has less lateral seepage during the whole test. The test results show the lateral seepage ratios and cumulative seepage flows increase at first and then decrease with slope angle rising.

![Fig. 7. Variations of seepage rate of undersurface vs. time (Test $\varepsilon=0.58$, $q=60$ ml/min)](image)

In summary, the influences of rainfall intensity $i$, slope angle $\alpha$, and void ratio $\varepsilon$ on the test results are analyzed. These results suggest that the regularity of non-orthogonal rainfall infiltration on the surfaces of unsaturated sand slopes is different from the traditional orthogonal rainfall infiltration.

3 NUMERICAL ANALYSES

3.1 The objective of numerical analyses.

In order to justify the validity of traditional orthogonal infiltration that has been worldwide taken as the second class boundary condition in seepages analyses, a series of finite element numerical analyses of seepage in unsaturated sand slopes under rainfall infiltration were made, under the exact same conditions as that for the laboratory tests.

The total intensity of rainfall was decomposed according to the slope angle and only the intensity component that was orthogonal to the slope surface could be applied as the second class boundary condition for the analysis. It is in this way the differences between results from the experiments and the numerical simulations can be utilized in justifying the validity of orthogonal infiltration as the boundary condition in analysis of rainfall induced seepage of unsaturated slopes.

3.2 The model for numerical analyses.

The numerical analyses were made with international commercial software SEEP/W in GEOSTU10 Version 2007. The basic theories and detailed numerical techniques for the finite element analyses are same as described in the software manual by Geo-slope International Ltd. (2007). A typical finite element mesh and its boundary condition is shown in Fig. 9 for a case with a slope angle $\alpha= 30^\circ$. The dimensions and the soil properties for the numerical analysis model are consistent with the laboratory tests. In the model, the front lateral face 1 and bottom face 2 are permeable boundaries. The boundary 3 is defined as a second class boundary, and the flux was set as equal to $i = i_0 \cos \alpha$. The other boundaries are impervious boundaries.

![Fig. 9. Analysis model of 30° slope](image)

3.3 Soil-water characteristic curve.

The SWC-150 Fredlund was used to measure the Soil-water characteristic curves (SWCC) for the sand used in laboratory tests for the soil samples. The measured data fitted with Fredlund-Xing’s Equation (Fredlund and Xing, 1994) as Eq. (1):

$$\theta_w = \frac{1 - \ln \left[ \frac{1 + \frac{\psi}{\psi_f}}{1 + 10^\frac{\psi}{\psi_f}} \right]}{\ln \left[ e + \left( \frac{\psi}{\alpha} \right)^\gamma \right]} \cdot \theta_s$$

In where, $\theta_w$ is the volumetric water content of the soil,
$\psi$, the suction value of the residual water rate (kPa), $\psi$ the suction value (kPa), $b$, the saturated volumetric water content the soil, while $a$ is the coefficient related to air entry value (kPa), $h$ and $c$ the coefficients related to the residual water rate, $e$ is the natural logarithm.

The fitted results are shown in Table 2, where $R$ is coefficient of the fitting correlation.

Table 2. The fitting parameters of SWCC for the sand

<table>
<thead>
<tr>
<th>Void ratio $e$</th>
<th>The fitting parameters $a$ /kPa</th>
<th>$c$ /kPa</th>
<th>$\psi$ /kPa</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.58</td>
<td>3.69742</td>
<td>5.15481</td>
<td>0.66189</td>
<td>0.98535</td>
</tr>
<tr>
<td>0.64</td>
<td>3.17903</td>
<td>3.25464</td>
<td>0.91765</td>
<td>0.97947</td>
</tr>
<tr>
<td>0.73</td>
<td>3.1551</td>
<td>2.63899</td>
<td>1.17992</td>
<td>0.97603</td>
</tr>
</tbody>
</table>

3.4 The comparison of test results and numerical simulation results.

Based on the test results and numerical simulation results, three aspects including the infiltration rate, the sand water storage increment $\Delta Q$ and a lateral seepage flow $v_1$ are discussed to examine the differences between test results and numerical simulation results.

1 The infiltration rate

The Numerical simulations for rainfall cases $q_{60}$ and $q_{100}$ have two stages, i.e., rainfall controlled stage and infiltration ability controlled stage. When the rainfall intensity is larger than the saturated infiltration coefficients $k_s$, the infiltration turns into infiltration ability controlled stage from rainfall controlled stage. In this stage, as shown in Fig. 10, the rainwater, in numerical simulation, even from the part of orthogonal component, will be translated into runoff when the orthogonal component rainfall intensity is more than sand soil surface infiltration capacity. In rainfall controlled stage, say in $q_{20}$ cases, the simulation results show small difference with the experimental results. But, in infiltration ability controlled stage, the simulation results are different from the experimental results significantly. In the whole process of rainfall in all the tests, there is no runoff observed.

Fig. 10. Variations of infiltration rate of sandy soil slope vs. time ($e=0.58$, $\alpha=30^\circ$)

When the void ratio and slope angle are constants, with the increase of rainfall intensity the amounts of cumulative infiltration water from simulation results are more obvious different from those from tests. The cumulative infiltrations of $q_{20}$, $q_{60}$ and $q_{100}$ are shown in Fig. 11. Obviously, in both the test $q_{20}$ and numerical $q_{20}$ (from the part of orthogonal component) there is no runoff. The whole rainfall process is in rainfall controlled stage. So, the cumulative infiltrations are close. But, in the $q_{60}$ and $q_{100}$ cases, the difference in cumulative infiltration water between test data and numerical analysis data are obvious.

Fig. 11. Variations of cumulative infiltration vs. time ($e=0.58$, $\alpha=30^\circ$)

2) The side seepage results

When the slope angle and rain intensity are the same, the rules of the simulation results are contrary to the test results. As shown in Fig. 12, the test results show that the seepage rates decrease with the void ratio increasing, but the simulation results have contrary rules. When the seepage rates reach stable, the test results are obvious different from the simulation ones. The stable moments in the tests are as shown by points $a$, $b$ and $c$, while the stable moments in the simulations are as represented by points $a_1$, $b_1$ and $c_1$. The results indicate that the seepage in the tests is stable earlier with the void ratio increasing, which is quite contrary to the simulation results.

Fig. 12. Variations of seepage rate of side face vs. time ($q=60$ ml/min, $\alpha=30^\circ$)

4 DISCUSSIONS

As mentioned above, the differences of infiltration rate, seepage and soil water storage increment exist in between the results from laboratory tests and numerical analyses. These differences are mainly resulted from infiltration mechanism rather than from merely
The most obvious difference is the direction issue in infiltration phenomenon. The decomposing rainfall intensity merely based on geometric mechanism without considering the material properties of soils does not reflect the fact from field inspection and laboratory observations. The test results demonstrate surely that the orthogonal infiltration theory does not conform to the actual non-orthogonal rainfall infiltration.

The second important difference concerns with the infiltration ability. Many seepage analyses just set the second class boundary condition by simply “feed” a flux to a surface without considering whether the surface of the material could be able to “eat” it or not. In the infiltration ability controlled stage of rainfall, the ability is of great significance in simulating rainfall infiltration in unsaturated soils.

The third reason for the difference may exist in the ways in simulation of rainfall infiltration. Theoretical calculation treats the rainfall intensity as a continuous water flow both in spatial and temporal domains lead to the differences of the test results and the simulation results. Fig. 13 shows the pore water pressure of slope midpoint of the simulations in cases $q_{20}$, $q_{60}$ and $q_{100}$. The points A and B show that the pore water pressure $\psi_0$ is equal to 0 kPa and the surface soil is saturated. But, the pore water pressure at point C is less than 0 kPa. At the end of the tests, the measured mass moisture contents in the tests $q_{20}$, $q_{60}$ and $q_{100}$ are 11.83%, 14.87% and 17.40%, respectively. The saturated mass moisture content is 17.85%. So, the sand in tests $q_{20}$ and $q_{60}$ are in unsaturated state and $q_{100}$ in near saturated state. The moisture content of the simulation calculated with the orthogonal infiltration boundary is obvious different from the actual results.

5 CONCLUSIONS

1. The differences between the results of the laboratory tests and finite element numerical analyses of rainfall infiltration on unsaturated sand slopes indicate that the orthogonal infiltration theory considering runoff in decomposing rainfall intensity merely based on geometric mechanism without considering the material properties of soils does not conform to the actual non-orthogonal rainfall infiltration.

2. When rainfall intensity is larger than the saturated infiltration coefficients $k_s$, the results if numerical analyses present two stages, i.e., rainfall controlled stage and infiltration ability controlled stage. But the test results in this research have only shown one rainfall controlled stage.

3. Compared with soil void ratio, rainfall intensity and slope angle have obvious more effects on the soil water storage increment.

4. Significant differences exist both in lateral seepage and water storage increment between the test results and the numerical simulation results.

5. The traditional orthogonal rainfall infiltration boundary has obvious defects. So, it is necessary to study the non-orthogonal infiltration boundary equation, which could provide reasonable boundary conditions for seepage analyses due to rainfall infiltration on unsaturated soils.

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