Bearing capacity of strip footing in reinforced granular bed over soft non-homogeneous ground stabilized with granular trench

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ABSTRACT

This paper presents a method to estimate the bearing capacity of a strip footing embedded in a geosynthetic reinforced granular bed over soft non-homogeneous ground, i.e., increase of undrained shear strength with depth, stabilized with granular trench. Madhav and Vitkar’s solution for bearing capacity of granular trench-supported strip footing in soft ground together with Meyerhof’s punching failure mechanism for dense sand overlying soft clay, extended to include the effect of axial tension in the reinforcement, form the basis of the analysis. A parametric study is performed and charts are developed to quantify the contributions of various parameters on the degree of bearing capacity improvement. The results of the present study are compared with experimental results from literature.

Keywords: Non-homogeneity, bearing capacity ratio (BCR), granular trench, granular fill, geosynthetic reinforcement

1 INTRODUCTION

A versatile technique of improving ground is to use granular piles, also known as stone columns. Two-dimensional plane strain version of a granular pile is a granular trench. In the field, a layer of compacted granular material is often placed over the soft deposit to provide an elevated platform for the construction machinery to operate. The granular fill functions as a strong base, distributes the load over a wider area and facilitates increased load to be applied over it. The bearing capacity can be improved further by providing a geosynthetic layer near the interface of the soft ground and the granular fill, within the fill. Geosynthetic layers resist destabilising forces by mobilising tensile forces in the reinforcement.

Recent marine deposits often exhibit undrained shear strength increasing linearly with depth (Fig. 1a). If the deposit gets aged or gets weathered, the deposit may develop larger strength over the full depth (Fig. 1b) or possess relatively high but nearly constant undrained strength over the thickness of the crust while retaining the original linearly increasing variation at further depths (Fig. 1c). Most solutions for bearing capacity of footings in soft ground consider deposits to be homogeneous neglecting natural or stress induced non-homogeneity. This paper presents a method to estimate the bearing capacity of a strip footing embedded in a geosynthetic reinforced bed over soft non-homogeneous ground stabilized with a granular trench.

2 LITERATURE REVIEW

Madhav and Vitkar (1978) proposed a solution for the bearing capacity of a strip footing on granular trench-reinforced ground considering a general shear failure mechanism. Hamed et al. (1986) presented laboratory model test results for the ultimate bearing capacity of a surface strip foundation installed in soft ground and supported by a granular trench of the same width as the foundation. Rao et al. (1994) used the results of drained triaxial compression tests on sand specimens reinforced with circular discs and
micro-meshes of woven and non-woven geotextiles and geogrids, to assess the influence of such reinforced material on the ultimate bearing capacity of a strip footing in granular trench-improved weak clay. Bouassida and Hadhri (1995) investigated the improvement in bearing capacity of soft purely cohesive soils reinforced by granular piles and granular trench using the yield design theory. Unnikrishnan and Rajan (2012) studied the influence of providing a Granular Trench (GT) below strip footings on loose sand deposits. Abhishek et al. (2014) presented a method for the estimation of bearing capacity of a strip footing in a geosynthetic reinforced foundation bed over soft homogeneous ground stabilized with granular trench.

3 PROBLEM DEFINITION AND FORMULATION

A strip footing of width, \( B_t \), is embedded at depth, \( D_t \), below the ground surface in a reinforced granular fill of thickness, \( H \), over soft non-homogeneous ground stabilized with granular trench (Fig. 2). The undrained strength, \( s_u \), of non-homogeneous ground increases from a value, \( s_{u0} \), at the granular fill-soft ground interface, linearly with depth, \( z \), at a rate, \( \rho \). A single layer of geosynthetic reinforcement of length, \( L_r \), is placed just above the granular fill-soft ground interface, but within the granular fill itself. The width of the granular trench is \( B_t \). The cohesion, angle of shearing resistance and unit weight of the trench material are \( c_t \), \( \varphi_t \) and \( \gamma_t \), respectively. The unit weight of soft non-homogeneous ground is \( \gamma \). The granular fill is characterized by its angle of shearing resistance, \( \varphi_r \), and unit weight, \( \gamma \). The interface/bond resistance between the reinforcement and the fill is \( \varphi_r \) and the axial tension mobilized in the reinforcement is \( T_F \).

Davis and Booker (1973), using the method of characteristics, obtained plasticity solution for the bearing capacity of smooth and rough strip footings on non-homogeneous deposit (Fig.1), which is significantly different from those on a homogeneous deposit. The ultimate bearing capacity, \( q_{ult} \), of a strip footing on the surface of a deposit whose undrained shear strength increases linearly with depth is

\[
q_{ult} = F \left[ s_{u0} N_c + \frac{1}{4} \rho B \right]
\]

where \( N_c = 5.14 \) is a bearing capacity factor; \( s_{u0} \) - the undrained shear strength of the deposit at the base of the footing; \( \rho = \frac{ds_t}{dz} \) - the rate of increase of undrained shear strength, \( s_u \), of the deposit with depth, \( z \); \( B \) - the width of the footing and \( F = \left[ (\rho B / s_u) - \right] \) - a correction factor.

Madhav and Vitkar (1978) proposed a solution for the ultimate bearing capacity of a strip footing in soft ground stabilized with a granular trench considering general shear failure mechanism along with Coulomb’s criterion for yielding of soils (Fig. 3). The ultimate bearing capacity, \( q_{ult} \), of the strip footing in soft ground stabilized with granular trench is

\[
q_{ult} = c_2 N_c + \left( \frac{\gamma B}{2} \right) N_B + D_f \gamma_2 N_q
\]

where

\[
N_c = \frac{c_1}{c_2} N_{c1} + N_{c2}
\]

\[
N_B = \frac{\gamma_1}{\gamma_2} N_{B1} + N_{B2}
\]

and \( N_c, N_B, N_q \) and \( N_r \) are dimensionless factors that depend on the geotechnical properties of the trench and soft soil materials and the ratio \( B/B \). Values of the bearing capacity factors \( N_c, N_B, N_q \) have been given by Madhav and Vitkar (1978) for varying values of \( B/B \) and \( \varphi_t \).

Fig. 2. Definition sketch of strip footing on reinforced granular bed over soft non-homogeneous ground with granular trench.

Fig. 3. Failure mechanisms for strip footing in soft ground stabilized with granular trench (a) \( B/B \leq 1 \) and (b) \( B/B \geq 1 \) (after Madhav and Vitkar 1978)
Meyerhof (1974) proposed a punching mode of failure for a strip footing of width, B, and depth D, resting on a relatively thin, dense sand stratum of thickness, H with angle of shearing resistance, $\phi$ and unit weight, $\gamma$, overlying thick soft clay with undrained cohesion, c, (Fig. 4). A total passive force, $P_p$, inclined at an angle, $\delta$, acts on a vertical plane through the footing edge. The possible failure modes of the footing, namely, punching shear through a relatively thin sand layer (Fig. 4a) and general shear failure within a thick sand layer alone (Fig. 4b) are shown. As the footing punches through the granular layer into soft clay, shear stresses are developed on either sides of the sand column. The ultimate bearing capacity, $q_u$, of a strip footing in dense sand overlying soft clay is

$$q_u = cN_c + \frac{\gamma H^2}{B} (1 + \frac{2D}{H}) K_2 \tan \phi + \gamma D$$

limited by the ultimate bearing capacity of a thick deposit of sand as

$$q_t = \gamma D N_q + 0.5 \gamma B N_r$$

where $K_2$ is the coefficient of punching shearing resistance; $N_c$ (equal to 5.14 for soft clay with $\phi = 0$), $N_q$ and $N_r$ are Meyerhof’s bearing capacity factors.

3.1 Bearing capacity of strip footing on granular bed over soft non-homogeneous ground with granular trench

The ultimate bearing capacity, $q_{nct}$, of a strip footing in soft non-homogeneous ground stabilized with granular trench is obtained by incorporating Davis and Booker’s theory in Madhav and Vitkar’s solution, as

$$q_{nct} = \frac{F [s_{ud} N_c + \frac{1}{4} \rho B]}{1 + \frac{2D}{H}} + 0.5 \gamma_2 BN_r + \gamma D_N_q$$

where $N_c$, $N_q$ and $N_r$ are Madhav and Vitkar’s bearing capacity factors. Normalizing Eq. (7) with the undrained shear strength of soft ground, $s_{ud}$, the normalized ultimate bearing capacity, $N_{nct}$, of a strip footing installed at depth, $D$, in soft non-homogeneous ground stabilized with granular trench is

$$N_{nct} = \frac{F [s_{ud} N_c + \frac{1}{4} \rho B]}{s_{ud}} + 0.5 \frac{\gamma_2 B}{s_{ud}} N_r + \gamma \frac{B}{s_{ud}} \frac{D}{H} N_q$$

(8)

The ultimate bearing capacity, $q_{nct}$, of a strip footing in a two-layered system of granular fill over soft non-homogeneous ground stabilized with granular trench is obtained by coupling equations (1), (2) and (5), as

$$q_{nct} = \frac{F [s_{ud} N_c + \frac{1}{4} \rho B]}{s_{ud}} + 0.5 \gamma_2 BN_r + \gamma H^2 (1 + \frac{2D}{H}) K_2$$

(9)

where $K_2$ is the coefficient of punching shearing resistance -a function of the angle of shearing resistance of the granular fill, $\phi$, and the ratio $q_2/q_1$ where $q_1$ and $q_2$ are the ultimate bearing capacities of a strip footing on the surface of a thick granular bed and granular trench-reinforced soft non-homogeneous ground respectively. The ratio $q_2/q_1$ is given by

$$q_2 = \frac{F [s_{ud} N_c + \frac{1}{4} \rho B]}{s_{ud}} + 0.5 \gamma_2 BN_r + \gamma H^2 (1 + \frac{2D}{H}) K_2 \tan \phi$$

(10)

where $N_c$ and $N_r$ in the numerator correspond to those of Madhav and Vitkar (1978) while $N_r$ in the denominator is Meyerhof’s bearing capacity factor. Considering the total thickness of the granular fill as $H$ (Fig. 1), Eq. 9 gets modified as

$$q_{nct} = \frac{F [s_{ud} N_c + \frac{1}{4} \rho B]}{s_{ud}} + 0.5 \gamma_2 BN_r + \frac{\gamma H^2 (1 + \frac{2D}{H}) K_2 \tan \phi}{\gamma D_N_q}$$

(11)

Normalizing Eq. (11) with the undrained shear strength of soft ground, $s_{ud}$, the normalized ultimate bearing capacity, $N_{nct}$, of a strip footing in a two-layered system of granular fill over soft non-homogeneous ground stabilized with granular trench, is

$$N_{nct} = F [N_c + \frac{1}{4} \frac{\rho B}{s_{ud}}] + 0.5 \frac{\gamma_2 B}{s_{ud}} N_r + \frac{\gamma B}{s_{ud}} \frac{D}{H} N_q$$

(12)

3.2 Bearing capacity of strip footing on reinforced granular bed over soft non-homogeneous ground with granular trench

The ultimate bearing capacity, $q_{nctb}$, of a strip footing in a two-layered system of reinforced granular fill over soft non-homogeneous ground stabilized with granular trench (Fig. 2), is obtained by adding the contribution of the axial resistance of the geosynthetic reinforcement to pull-out to Eq. 11. The axial tension developed in the reinforcement layer of length, $L$, is due to interface shear resistance mobilized over the top and bottom surfaces of the reinforcement. Figs. 5a & b depict the stresses developed in the reinforced granular column and the geosynthetic reinforcement respectively, due to punching of the footing through the
reinforced granular bed into the underlying soft non-homogeneous ground. The length of the reinforcement beyond the edge of the footing, \((L_r - B)/2\), is considered to be effective in contributing to the resistance to axial pullout and bearing capacity improvement. The axial tension, \(T_r\), developed in the reinforcement on either side of the footing, due to shear stresses developed over the surface of the reinforcement at the granular fill-soft non-homogeneous ground interface is

\[
T_r = \gamma H \tan \varphi_r \frac{(L_r - B)}{2} \tag{13}
\]

The ultimate bearing capacity, \(q_{ncbr}\), of a strip footing in a reinforced two-layered system of granular fill over soft ground stabilized with granular trench, thus becomes

\[
q_{ncbr} = F \left[ s_{c0} N_c + \frac{1}{4} \rho B \right] + 0.5 \gamma B s_{c0} N_g + \frac{\gamma H^2 - D_f^2}{B} \left( K_2 \tan \varphi \right) + \gamma D_f N_q + \frac{H}{B} \tan \varphi \left( L_r - B \right) \tag{14}
\]

Normalizing Eq. 14 with the undrained shear strength of soft ground, \(s_{c0}\), the normalized ultimate bearing capacity, \(N_{ncbr}\), of a strip footing in a reinforced two-layered system of granular fill over soft non-homogeneous ground stabilized with granular trench is

\[
N_{ncbr} = F \left[ N_c + \frac{1}{4} \left( \frac{\rho B}{s_{c0}} \right) \right] + 0.5 \left( \frac{s_{c0}}{s_{c0}} \right) N_g + \left( \frac{H^2}{B} \right) \left( \frac{D_f}{B} \right) N_q + \left( \frac{H}{B} \right) \tan \varphi \left( \frac{L_r}{B} - 1 \right) \tag{15}
\]

Bearing capacities ratios, \(BCR\), are defined to quantify the degrees of improvement as:

\[
(BCR)_{ncbr} = \frac{N_{ncbr}}{N_{nc}} \tag{16}
\]

\((BCR)_{ncbr} = N_{ncbr}/N_{nc}\) is the ratio of the normalized ultimate bearing capacity of a strip footing in an unreinforced two-layered system of granular fill over soft non-homogeneous ground stabilized with granular trench to that in granular trench-reinforced ground alone. The ratio \((BCR)_{ncbr}\) quantifies the contribution of the granular fill.

4 RESULTS AND DISCUSSION

The ultimate bearing capacity of a strip footing in a two-layered system of granular fill over soft non-homogeneous ground stabilized with granular trench, depends on the normalized foundation depth, \(D_f/B\), angle of shearing resistance of the granular material, \(\varphi\), normalized fill thickness, \(H/B\), \(\rho B/s_{c0}\), related to the non-homogeneity of the soft ground and \(\gamma B/s_{c0}\), related to the unit weight of the granular fill, width of the footing and undrained cohesion of soft non-homogeneous ground. If the granular fill is reinforced with a layer of geosynthetic, parameters \(L_r/B\) and \(\varphi_c/\varphi\) also influence the bearing capacity of the two-layered system. The values of the bearing capacity factors as given by Madhav and Vitkar (1978) are adopted for a normalized trench width, \(B_r/B\) of 0.5 and \(c_r/c_2\) equal to 0. The granular fill, trench and soft ground are considered to have comparable unit weights while the trench and fill materials possess comparable angles of shearing resistance. A parametric study quantifies the effect of the parameters \(\gamma B/s_{c0}\) and \(\rho B/s_{c0}\) on the normalized ultimate bearing capacity and bearing capacity ratio of the footing.

Figures 6 and 7 present the variations of the normalized bearing capacities, \(N_{ncbr}\) and \(N_{ncbr}\), of a strip footing in a two-layered system of unreinforced and reinforced granular fill over soft non-homogeneous ground with granular trench, respectively, with \(\gamma B/s_{c0}\), for \(\varphi\) of 35\(^{\circ}\), \(D_f/B\) of 0.5, \(H/B\) of 1.0, \(\varphi_c/\varphi\) of 0.75 (reinforced case), \(L_r/B\) of 3.0 (reinforced case) and \(B_r/B\) equal to 0.5, for \(\rho B/s_{c0}\) equal to 0, 4, 8, 12, 16, 20 and 24. \(N_{ncbr}\) and \(N_{ncbr}\) increase linearly with \(\gamma B/s_{c0}\). Relatively softer clays and wider footings display improved normalized bearing capacities upon provision of reinforced granular fill and trench. Table 1 presents values of \(N_{ncbr}\) and \(N_{ncbr}\) for varying \(\rho B/s_{c0}\) and \(\gamma B/s_{c0}\). \(N_{ncbr}\) and \(N_{ncbr}\) increase with \(\rho B/s_{c0}\) due to enhanced contribution from inherent non-homogeneity of soft ground, towards the bearing capacity of the footing.
The variation of the bearing capacity ratios \((BCR)_{nctb}\), \((BCR)_{nctbr}\) and \((BCR)_{nctbr}^*\) with \(\gamma B/s_{u0}\) for \(\varphi = 35^\circ\), \(D/B\) of 0.5, \(H/B\) of 1.0, \(\varphi_r/\varphi\) of 0.75 (reinforced case), \(L_r/B\) of 3.0 (reinforced case) and \(B_r/B\) equal to 0.5, for \(\rho B/s_{u0}\) equal to 0, 4, 8, 12, 16, 20 and 24, are shown in Figs. 8, 9 and 10 respectively. \((BCR)_{nctb}\), \((BCR)_{nctbr}\) and \((BCR)_{nctbr}^*\) values increase non-linearly with \(\gamma B/s_{u0}\) but decrease with \(\rho B/s_{u0}\). Table 2 presents the values of \((BCR)_{nctb}\), \((BCR)_{nctbr}\) and \((BCR)_{nctbr}^*\) for varying \(\rho B/s_{u0}\) and \(\gamma B/s_{u0}\). \((BCR)_{nctb}\) values are greater than those of \((BCR)_{nctbr}\) due to additional contribution from the axial resistance to pullout mobilized by the geosynthetic reinforcement towards the ultimate bearing capacity of the footing.

Table 1. \(N_{nctb}\) and \(N_{nctbr}\) values for varying \(\rho B/s_{u0}\) and \(\gamma B/s_{u0}\)

<table>
<thead>
<tr>
<th>(\rho B/s_{u0})</th>
<th>(N_{nctb})</th>
<th>(N_{nctbr})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>23.4</td>
<td>33.3</td>
</tr>
<tr>
<td>8</td>
<td>43.7</td>
<td>28.3</td>
</tr>
<tr>
<td>24</td>
<td>48.7</td>
<td>38.2</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>(\gamma B/s_{u0})</th>
<th>15</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8</td>
<td>51.2</td>
<td>79.0</td>
</tr>
<tr>
<td>8</td>
<td>61.0</td>
<td>88.9</td>
</tr>
<tr>
<td>24</td>
<td>71.1</td>
<td>99.0</td>
</tr>
<tr>
<td></td>
<td>66.0</td>
<td>103.7</td>
</tr>
<tr>
<td></td>
<td>75.8</td>
<td>113.5</td>
</tr>
<tr>
<td></td>
<td>85.9</td>
<td>123.7</td>
</tr>
</tbody>
</table>

Table 2. \((BCR)_{nctb}\), \((BCR)_{nctbr}\) and \((BCR)_{nctbr}^*\) values for varying \(\rho B/s_{u0}\) and \(\gamma B/s_{u0}\)

<table>
<thead>
<tr>
<th>(\rho B/s_{u0})</th>
<th>((BCR)_{nctb})</th>
<th>((BCR)_{nctbr})</th>
<th>((BCR)_{nctbr}^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.37</td>
<td>1.32</td>
<td>1.28</td>
</tr>
<tr>
<td>8</td>
<td>1.66</td>
<td>1.51</td>
<td>1.43</td>
</tr>
<tr>
<td>24</td>
<td>1.31</td>
<td>1.28</td>
<td>1.25</td>
</tr>
<tr>
<td>15</td>
<td>1.44</td>
<td>1.39</td>
<td>1.36</td>
</tr>
<tr>
<td>25</td>
<td>1.46</td>
<td>1.43</td>
<td>1.40</td>
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<tr>
<td>1.11</td>
<td>1.15</td>
<td>1.11</td>
<td>1.11</td>
</tr>
<tr>
<td>1.24</td>
<td>1.21</td>
<td>1.24</td>
<td>1.21</td>
</tr>
<tr>
<td>1.28</td>
<td>1.25</td>
<td>1.28</td>
<td>1.25</td>
</tr>
</tbody>
</table>
The improvement in bearing capacity of the footing due to provision of reinforced fill over soft non-homogeneous ground stabilized with granular trench, decreases with consideration of non-homogeneity, i.e., undrained strength increase of soft ground with depth. Therefore, bearing capacity solutions that do not account for this non-homogeneous strength behavior of natural soft deposits would tend to overestimate the relative contributions of the granular trench, fill and reinforcement towards bearing capacity enhancement of the footing.

4.1 Comparison with experimental results

Figure 11 compares the present method for estimation of bearing capacity of a strip footing embedded in a granular bed over soft non-homogeneous ground stabilized with granular trench, with the experimental results of a strip footing in weak clay stabilized with granular trench, performed by Rao et al. (1994), for $\phi$ of 45°, $D_1/B$ of 0.5, $H/B$ of 0.5, $\gamma B/s_{d0}$ of 1.6 and $B/B$ of 0.8, 0.9 and 1.0. The bearing capacity ratio plotted along the ordinate is the ratio of the normalized ultimate bearing capacity of a strip footing in soft clay stabilized with granular trench to that in soft clay alone. It is observed that the bearing capacity ratios obtained by Rao et al. (1994) lie between the predictions for $\rho B/s_{d0}$ of 0 and 4.

$\rho B/s_{d0}$ value of 0 corresponds to homogeneous soft clay with nearly constant variation of undrained shear strength with depth. The presence of inherent non-homogeneity in soft ground and its effect on the bearing capacity ratio of the strip footing is visible through Fig. 11. An increase in the value of $\rho B/s_{d0}$ is indicative of an increase in the undrained shear strength of soft ground with depth. $BCR$ values increase with the normalized width of the granular trench due to larger volume of soft clay replaced by compacted granular material with relatively higher shear resistance.

5 CONCLUSIONS

A method for estimating the bearing capacity of a strip footing embedded in a geosynthetic reinforced granular bed over soft non-homogeneous ground stabilized with granular trench is presented. Consideration of in-situ non-homogeneity of soft ground yields higher bearing capacity of the footing over a homogeneous case. Relatively wider footings on dense granular fills over soft deposits display improved bearing capacity response. $BCR$ of the footing in a two-layered system of reinforced granular fill over soft non-homogeneous ground stabilized with granular trench is enhanced due to the contribution from the axial resistance of the reinforcement to pullout, compared to an unreinforced one.

REFERENCES


Fig. 11. Comparison of present theory with experimental results of Rao et al. (1994)