The ground stress condition under the strip load of semi-infinite length

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ABSTRACT

Uneven pressure on the strip foundations of building extensions due to the influence of their free ends often leads to the setting extensions aside from the main building, and uneven pressure on the ground of bridge abutments from adjoining embankments can cause the abutments slope in the direction of the embankments. The plane problem solution of the theory of elasticity for a loaded strip of an infinite length used in calculating is inefficient.

The presented solution of the spatial problem for a semi-infinite length strip load specifies the ground stress condition at the free ends of the strip foundation, and also in foundation grounds of the bridge abutments from the pressure of adjoining embankments. The components of even load stress and triangular load stress $\sigma_z$ are obtained by integration of stress values for the elementary concentrated force over the area of the strip. The other components of triangular load stresses, according to the known solution defining stresses, were under the rectangular foundation of the same load.

The solution: central point of coordinates moves along the $y$-axis from the center of the foundation to its end; the target stress components are of stress components under a rectangular foundation by defining the limit of these stresses, when $l \rightarrow \infty$.

The obtained solutions satisfy the conditions on infinity; the normal stresses $\sigma_z$, $\sigma_x$ and the shear stresses $\tau_{xz}$, change to the known solutions for the plane strain when $-\infty \leftarrow y \rightarrow \infty$. The stresses $\tau_{xy}$ and $\tau_{yz}$ are damped approximately on the distance of $y = 2.5b$ from the strip end, and the stress $\sigma_y$ becomes permanent and is not taken into account when calculating.

Subtracting from the solution corresponding to the strip of a semi-infinite length with the central point of coordinates of $x = 0$, $y = 0$, the same solution with the central point of coordinates of $x = 0$, $y = l$ for even stress and respectively $x = -b/2$, $y = 0$ and $x = -b/2$, $y = l$ for the triangular load we get an easier solution compared to the known one which determine the stress components at the rectangular foundation ground.

So that, knowing the solution for any kind of strip load it is possible to find the solutions for rectangular foundation and vice versa. The obtained formulas allow solving plasto-elastic tasks. Here, $b$ and $l$ are the width and the length of the strip.

Keywords: stress components, strip load, shear stresses, normal stresses, foundation end

1 INTRODUCTION

Uneven pressure on the strip foundations of building extensions due to the influence of their free ends often leads to the setting extensions aside from the main building, and uneven pressure on the ground of bridge abutments from adjoining embankments can cause the abutments slope in the direction of the embankments. This took place in Dneprodzerzhinsk (Ukraine) on railway and highway bridges at the height of embankments up to 10 m, and abutments slope nearly caused the breaking-down of the bridge superstructures. The plane problem solution for a loaded strip of an infinite length used in calculating is inefficient because it doesn’t take the influence of foundation ends into account.

2 STRESSES AT THE STRIP GROUND

In solution of the spatial problem for a semi-infinite length strip from Kushner’s works (2008) there founded at its ground all the components of even load stresses and triangular load stresses $\sigma_{zA,STR}$ (Fig. 1,a,b).
This solution is obtained by integrating of stress values from elementary concentrated force over the area of the strip by Boussinesq formulas. It clarifies the stress condition at the free ends of the strip foundations and bridge abutment grounds from the pressure of adjoining embankments.

Here, we shall give only values of triangular load stresses \( \sigma_{z_{\Delta ast}} \)

\[
\sigma_{z_{\Delta ast}} = p_0 \frac{x}{2 \pi b} \left\{ \arctg \frac{B_y x}{z R_i} + \arctg \frac{B_y y}{z} + \frac{z}{2} \left[ \frac{B_y y}{z^2 + z^2} \right]^2 \right\}
\]

where

\[ R_i = \sqrt{(x-b)^2 + y^2 + z^2}. \]

Other components of triangular load stress are founded later on the basis of Korotkin’s solution (1938) defining all the stress components at the rectangular foundation ground.

Fig. 1. Schemes of loading of tape (strip) and rectangular foundations: a and b – the strip of a semi-infinite length; c – the rectangular foundation.

The solution (Fig. 1, c): central point of coordinates moves from the center of the rectangular foundation to its end; in case of even load – along the central axis y; in case of triangular load – to the point \( C_1 \) by means of substitutions: \( y = y_1 - l/2, \quad x = x_1 + b/2 \), where \( y, x \) and \( y_1, x_1 \) are respectively old and new coordinates; the stress components are expressed by formulas that correspond a new location of central point of coordinates; for each stress component there is the limit at \( l \to \infty \). In the formula (1), defining the stress \( \sigma_{z_{\Delta ast}} \), zero ordinates of a triangular load are located on the left (Fig. 1, b), and according to Korotkin’s solution – on the right (Fig. 1, c). The final solution is founded by turning the load on the 180º, i.e. transferring the central point of coordinates from the point \( C_i \) to point \( C \) by changing the signs at coordinates x into reversed ones. In final solutions for a strip, the indexes «1» and «2» relating to new axes \( x_1, \quad y_1 \) and \( y_2 \) are omitted. The values of components of shear stresses \( \tau_{z_{\Delta ast}}, \quad \tau_{x_{\Delta ast}}, \quad \tau_{y_{\Delta ast}} \) as well as of normal stresses \( \sigma_{x_{\Delta ast}}, \quad \sigma_{y_{\Delta ast}}, \quad \sigma_{z_{\Delta ast}} \) are obtained in this way.

2.1 Shear stresses

These stresses are presented by the following formulas:

\[
\tau_{z_{\Delta ast}} = \frac{p_0}{2 \pi b} \left\{ \arctg \frac{B_y x}{z R_i} + \arctg \frac{B_y y}{z} + \frac{z}{2} \left[ \arctg \frac{x-b}{z} \right]^2 \right\}
\]

\[
R_i = \sqrt{(x-b)^2 + y^2 + z^2}, \quad R_2 = \sqrt{x^2 + y^2 + z^2}, \quad m_1 = x - b, \quad m_2 = x.
\]

\[
\tau_{z_{\Delta ast}} = \frac{p_0}{2 \pi b} \left[ \arctg \frac{x-b}{z} + \arctg \frac{y}{z R_i} \right] - \arctg \frac{z}{b} (\arctg \frac{x-b}{z} - \arctg \frac{x}{z R_i})
\]

The formula (3) represents a strip with the center point of coordinates in the point \( C \) – Fig. 1, b, extending from \( y = 0 \) to \( y = \infty \). By adding to it the same expression, but with opposite signs at \( y \), corresponding to the strip, with the coordinates from \( y = 0 \) to \( y = -\infty \), we have got Gersevanov’s solution (1933) for the strip of an infinite length in the form

\[
\tau_{z_{\Delta ast}} = \frac{p_0}{\pi} \left[ \arctg \frac{x-b}{z} + \arctg \frac{x}{z R_i} \right] + \arctg \frac{z}{b} \left[ \arctg \frac{x-b}{z} - \arctg \frac{x}{z R_i} \right]
\]

Here \( \tau_{z_{\Delta ast}} = \text{const} \) throughout the strip. We have got the following

\[
\tau_{z_{\Delta ast}} = -\frac{p_0}{2 \pi b} \left[ \arctg \frac{B_y y}{z R_i} + z \ln \frac{R_i - m_2}{R_i - m_1} - (1 - 2 \nu) \left[ z \ln \frac{R_i - m_2}{R_i - m_1} - m_2 \ln \frac{R_i + z}{R_i + z} - b + y \left( \arctg \frac{m_1}{y} + \arctg \frac{m_2}{y R_i} \right) \right] \right]
\]

\[
F_i = F_i - F_2
\]

where \( \nu \) - coefficient of transverse strain.

For the disclosure of expressions with the index \( i \) herein after the formula (6) is used.

The diagrams of distribution of shear stresses components based upon numerical examples for the
non-dimensional values of strip width and coordinates on the verticals traced through the points on the geometric axis of the strip are shown on Fig. 2, a and b.

In numerical examples it is accepted that: \( b = 2 \), \( v = 0.3 \), \( x = z = 1 \), \( 0 \leq y \leq 5 \) and \( 0 \leq y \leq \infty \).

Fig. 2. Diagrams of the stress distribution under the semi-infinite length strip: \( a \) — stresses \( \sigma_{yA,\infty} \) and \( \tau_{xy,\infty} \); \( b \) — stresses \( \tau_{xy,\Delta,xy} \); \( c \) — stresses \( \sigma_{A,\Delta,xy} \) and \( \sigma_{yA,\Delta,xy} \).

From the obtained formulas and the diagrams it is seen that: at the ground of the infinite length strip (plane problem) shear stresses \( \tau_{xy,\infty} \) and \( \tau_{xy,\Delta,xy} \) are absent; at the ground of a semi-infinite length strip (spatial problem) these stresses appear due to the influence of the free end, what is more, at the end \( (y = 0) \) they are maximum and almost damped at the distance of \( y \geq 2.5b \) from it; stresses \( \tau_{xy,\Delta,xy} \) arising at the ground of the infinite length strip, under the semi-infinite strip at the distance of \( y \geq 2.5b \) increase twice from the half-value at the end to the value corresponding to the formula (4) and retain this value until infinity.

2.2 Normal stresses \( \sigma_{\Delta,\infty} \) and \( \sigma_{\Delta,\Delta,xy} \)

They are presented by the following formulas:

\[
\sigma_{yA,\infty} = \frac{p}{2\pi b} \left[ \frac{2z}{R_z} \ln \frac{R_1 + y}{R_1 + z} + \frac{m}{y} \left( \frac{1 + y}{R_1} \right) \right] - (1 - 2v) y \ln \frac{R_1 + z}{R_1 + y} - m_i \left( \arctg \frac{m_i z}{z} + \arctg \frac{m_i y}{y} \right) R_i - \left( 1 - 2v \right) m_i \left( \arctg \frac{m_i z}{z} - \arctg \frac{m_i y}{y} \right) R_i
\]

(7)

For the transition from formula (7) to the formula expressing the stresses \( \sigma_{\Delta,\infty,G} \) at the ground of the infinite length strip, we do the same as in getting the formula (4). In this case, we find

\[
\sigma_{yA,\infty,G} = \frac{p}{2\pi b} \left[ \frac{z}{b} \ln \left( \frac{(x-b)^2 + z^2}{x^2 + z^2} \right) \right] - \frac{x}{b} \left[ \arctg \frac{x-b}{z} + \arctg \frac{x}{z} \right] + \frac{(x-b)^2}{(x-b)^2 + z^2}.
\]

(8)

that it fully corresponds to Gersevanov’s solution.

\[
\sigma_{yA,\infty} = \frac{p}{2\pi b} \left[ \frac{2z}{R_z} \ln \frac{R_1 + y}{R_1 + z} - (1 - 2v) y \ln \frac{R_1 + z}{R_1 + y} + m_i \left( \arctg \frac{m_i z}{z} - \arctg \frac{m_i y}{y} \right) R_i + \left( 1 - 2v \right) m_i \left( \arctg \frac{m_i z}{z} + \arctg \frac{m_i y}{y} \right) R_i \right]
\]

(9)

For the infinite length strip we get

\[
\sigma_{yA,\infty,K} = \frac{p}{2\pi b} \left[ \frac{2z}{R_z} \ln \left( \frac{(x-b)^2 + z^2}{x^2 + z^2} \right) + 2\left( \arctg \frac{x-b}{z} + \arctg \frac{x}{z} \right) \right]
\]

(10)

The expression (10) can also be obtained from the generalized Hooke’s law. When \( \varepsilon_y = 0 \) we get the following

\[
\sigma_y = \nu(\sigma_x + \sigma_y)
\]

(11)

Substituting in (11) the values of plane strain stresses: \( \sigma_{\Delta,\infty,G} \) from (13) on the page 4 and \( \sigma_z \) from Gerevanov’s solution (1933), we have the result which exactly coincides with (10).

Diagrams (Fig. 2, c) of the distribution of the components of normal stresses \( \sigma_{yA,\infty} \) and \( \sigma_{yA,\infty,G} \) are plotted for the same parameters which were adopted on Fig. 2, a and b.

As we can see, the values of the components of normal stresses at the end of the semi-infinite strip are equal to half value of the same stresses under the infinite strip. Then, they increase quickly ( \( \sigma_{yA,\infty} \)) or slowly ( \( \sigma_{yA,\infty,G} \)) asymptotically approaching to the stresses under the infinite strip. Computational error of the values \( \sigma_{yA,\Delta,xy} \) was from 1.6 to 2.3%.

3 STRESSES AT THE RECTANGLE GROUND

Korotkin (1938) defined all the components of the stresses and displacements under the rectangular foundation basing upon the stress function which has been proposed by the Academician Galerkin, and the final formulas are presented in a very complicated way. The difficulty of calculating the stresses at the ground of the even loaded rectangular foundation according to these formulas has determined the need to use the method of the corner points for these purposes.
Universal solution for the spatial problem for a semi-infinite length strip mentioned in Kushner’s work allows easily determining the stress condition at the ground of the rectangular foundation with the help of the method of superposition.

The formulas which determine the stress condition at the ground of the strip I of a semi-infinite length are used in the method of superposition for determining the stress condition at the ground of a flexible rectangular foundation loaded with even (p) or triangular (maximum ordinate \( p_i \)) load. For this purpose, the expressions, represented by the identical formulas for a given type of load corresponding to the stress condition under the strip II loaded in the same way, should be taken off the formulas responding to the strip I (Fig. 3).

The central point of coordinates is of respectively

strips II and I and the rectangular foundation:

at an even load \([ x = 0, \ y = 0, \ z = 0 ]\) and \([ x = 0, \ y = l, \ z = 0 ]\);

at a triangular load \([ x = -b/2, \ y = 0, \ z = 0 ]\) and \([ x = -b/2, \ y = l, \ z = 0 ]\) (it is not shown on Fig. 3).

In both cases \( y_1 = y - l, \ l - \) length of the foundation.

Since the \( y \) \((y_1)\) axes are transferred to a corner point (combined with the side of the strip on which the external load is zero), the new coordinates of the central point of coordinates will be \([ x = 0, \ y = 0, \ z = 0 ]\) and \([ x = 0, \ y = l, \ z = 0 ]\) respectively.

After simple transformations we have simpler formulas for determining the stresses \( \sigma_z \), as well as other components of the stresses on an arbitrary vertical traced through the plane points which are located both within and outside of the loaded rectangular foundation.

These formulas with the even load are presented as:

\[
\sigma_z = \frac{p}{2\pi} \left\{ \arctg \frac{m_z y}{z R_i} - \arctg \frac{m_z (y-l)}{z R_{i1}} + \frac{m_y z}{R_i} \left[ \frac{1}{m_i^2 + z^2} + \frac{1}{m_i^2 + (y-l)^2 + z^2} \right] \right\} \tag{12}
\]

\[
\sigma_y = \frac{p}{2\pi} \left\{ \arctg \frac{m_y y}{z R_i} - \arctg \frac{m_y (y-l)}{z R_{i1}} + \frac{m_z z}{m_i^2 + z^2} - \frac{y + y - l}{y R_i} + 2(1 - 2\nu) \times \frac{\arctg \frac{R_i - m_z + z}{y} - \arctg \frac{R_{i1} - m_z + z}{y - l}}{m_i} \right\} \tag{13}
\]

\[
\tau_{xy} = \frac{p}{2\pi} \left\{ \frac{z}{R_i} - \frac{z}{R_{i1}} + (1 - 2\nu) \ln \frac{R_i + z}{R_{i1} + z} \right\} \tag{14}
\]

\[
\tau_{yz} = \frac{p}{2\pi} \left\{ \frac{z^2}{m_i^2 + z^2} - \frac{1}{R_i} + \frac{1}{R_{i1}} \right\} \tag{15}
\]

\[
\tau_{xz} = \frac{p}{2\pi} \left\{ \frac{z^2}{m_i^2 + z^2} \left[ \frac{y + y - l}{R_i} + \frac{1}{R_{i1}} \right] \right\} \tag{16}
\]

\[
\tau_{z} = \frac{p}{2\pi} \left\{ \frac{z^2}{m_i^2 + z^2} \left[ \frac{y + y - l}{R_i} + \frac{1}{R_{i1}} \right] \right\} \tag{17}
\]

The values of the stresses corresponding to the formulas (12) - (17) and the next formulas are also calculated basing upon the formula (6).

On the basis of the formula (12) analysis in Table 1 there shown the values of the contact pressures \( p \) directly under the bottom of the foundation at \( z = 0 \).

As we can see if under the rigid foundation the boundary ordinates of the profile of contact pressures are theoretically of an infinite scope (actually in the result of plastic deformations the «peaks» are cut off), then under the flexible foundation the boundary ordinates are below average pressure \( p \) under the profile.

The analysis shows that:

the formulas obtained are satisfied with the boundary conditions and the conditions at infinity;

Fig. 3. For the defining of the stress condition at the ground of the rectangular foundation by superposition method: \( a \) - transferring of the coordinate axes in the rectangular foundation; \( b \) - definition of stresses under the rectangular foundation.
Table 1. The contact pressures under the flexible rectangular foundation.

<table>
<thead>
<tr>
<th>Contact pressures at the points of the foundation</th>
<th>Central</th>
<th>Arbitrary</th>
<th>On the least side</th>
<th>On the larger side</th>
<th>Angular</th>
</tr>
</thead>
<tbody>
<tr>
<td>Central pressures</td>
<td>p</td>
<td>p</td>
<td>p/2</td>
<td>p/2</td>
<td>p/4</td>
</tr>
<tr>
<td>x=0, y=l/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x=b/2, y=0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x=b/2, y=1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x=0, y=b/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x=0, y=l/2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>x=0, y=0, y=l</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The contact pressures at the points of the foundation do not depend on the elastic constants, and corresponds to the first fundamental boundary problem of the theory of elasticity.

After transition of the center point of coordinates to the center of the foundation by substituting \( l/2 + y_1 = y \) and comparing with Korotkin’s solution it has been shown that:

formulas (12), (16) and (17) as well as the components of the formulas (13), (14) and (15) which do not depend on the coefficient of transverse strain fully meet this solution; the components of formulas (13), (14) and (15) which depend on this coefficient, in our solution, in Korotkin’s solution as well as in solution for the semi-infinite length strip are represented by different expressions;

numerical calculations confirm that both of these solutions are identical for the semi-infinite strip and for the rectangular foundation.

Thus, the right and left sides of the example of the formula (18) (our solution is on the left, the solution by Korotkin is on the right) are identically equal, i.e. for \( \sigma_y \) we have:

\[
\begin{aligned}
\frac{p}{2\pi} & \left\{ \cdots + 2(1 - 2\nu) \times \arctg \frac{\sqrt{(x + b/2)^2 + (y_1 + l/2)^2 + z^2} - y_1 - l/2 + z}{x + b/2} - \arctg \frac{\sqrt{(x + b/2)^2 + (y_1 + l/2)^2 + z^2} - y_1 - l/2 + z}{x - b/2} - \arctg \frac{\sqrt{(x + b/2)^2 + (y_1 - l/2)^2 + z^2} - y_1 + l/2 + z}{x + b/2} + \arctg \frac{\sqrt{(x + b/2)^2 + (y_1 - l/2)^2 + z^2} - y_1 + l/2 + z}{x - b/2} \right\} \\
& = \frac{p}{2\pi} \left\{ \cdots + 2(1 - 2\nu) \times \arctg \frac{x - b/2}{y_1 - l/2} + \arctg \frac{x + b/2}{y_1 + l/2} - \arctg \frac{x - b/2}{y_1 - l/2} - \arctg \frac{x + b/2}{y_1 + l/2} \right\} \\
& + \arctg \frac{(x - b/2)\sqrt{(x - b/2)^2 + (y_1 + l/2)^2 + z^2}}{(y_1 + l/2)z} + \\
& + \arctg \frac{(x + b/2)\sqrt{(x + b/2)^2 + (y_1 - l/2)^2 + z^2}}{(y_1 - l/2)z} - \\
& - \arctg \frac{(x - b/2)\sqrt{(x - b/2)^2 + (y_1 - l/2)^2 + z^2}}{(y_1 - l/2)z} \right\} \quad (18)
\end{aligned}
\]

The similar expression can be written for \( \sigma_x \).

Example 1. The stresses \( \sigma_x \) founded by the formula (12) at the point with dimensionless coordinates \( x=b/4=l, \ y=3l/4=6, \ z=2 \) at the ground of the rectangular foundation loaded with even load with the intensity \( p \) and dimensions \( b=4 \) and \( l=8 \) are equal to \( \sigma_x = 0,669p \).

The stresses at the same point obtained on the basis of the method of angular points are \( \sigma_x = 0,678p \). The difference of 1.3% is determined by a less accuracy of calculations with the help of tables according to the method of angular points.

Example 2. In table 2 in dimensionless coordinates \( x=0 \) and \( y \) from \( 0 \) to \( l \) there shown the distribution of stresses \( \sigma_y \) in fractions from even load with the intensity \( p \) at the depth \( z=1 \) at the ground of rectangular foundation with dimensions \( b=2, \ l=12, \ \eta = l/b = 6 \).

Table 2. Distribution of stresses \( \sigma_y \) along the longitudinal axis of the foundation.

<table>
<thead>
<tr>
<th>( y )</th>
<th>0.0</th>
<th>0.25</th>
<th>0.50</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_y )</td>
<td>0.00</td>
<td>0.042</td>
<td>0.083</td>
<td>0.167</td>
<td>0.250</td>
<td>0.333</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>0.409</td>
<td>0.650</td>
<td>0.760</td>
<td>0.810</td>
<td>0.813</td>
<td>0.818</td>
</tr>
<tr>
<td>( y )</td>
<td>4.0</td>
<td>4.5</td>
<td>5.0</td>
<td>5.5</td>
<td>6.0</td>
<td></td>
</tr>
<tr>
<td>( \sigma_y )</td>
<td>0.667</td>
<td>0.750</td>
<td>0.833</td>
<td>0.917</td>
<td>0.958</td>
<td>1.0</td>
</tr>
<tr>
<td>( \sigma_x )</td>
<td>0.818</td>
<td>0.813</td>
<td>0.810</td>
<td>0.760</td>
<td>0.650</td>
<td>0.409</td>
</tr>
</tbody>
</table>

If at the boundary verticals the stresses are equal to the half of the stresses under the center of the foundation, then at the distances which are equal to \( b \) or \( 0,167l \) from the ends of the foundation they increase practically twice up to values (0,810 ... 0,818)p and remain constant. Thus, with the relation of the sides of a considered rectangular foundation \( \eta = 1/6 \ (l = 6b) \) on the area with a length \( 4b \) (0,667l) the plane problem takes place, i. e. this area is taken into operation under plane strain.

In certain works, for example by Gorbunov-Possadov et al. (1984), it is indicated that if the length of the bearing surface of the structure exceeds its width more than threefold, the use of plane strain scheme can be considered appropriate. We consider that in such conditions the influence of the ends will be extremely significant. In our point of view in Russian grounds design standards (1995) with the calculation of the
precipitate to strip foundations (the conditions of plane strain) the strips with the sides ratio \( \eta = l/b \geq 10 \) are related on a reasonable basis.

The formula obtained in this way which defines the values \( \sigma_z \) from the triangular load on an arbitrary vertical at the ground of the rectangular foundation gives the results that fully coincide with the results arising from the solution by Korotkin.

The analysis of the formula defining stresses \( \sigma_z \) shows that:

when \( z = 0 \), \( x = b \) and \( 0 < y < l \) the contact pressure along the long side of the foundation under the maximum ordinate of the triangular load is equal to

\[
\sigma_z(z=0) = p_0 \frac{b}{2\pi b} \left( \frac{\pi}{2} + \frac{\pi}{2} \right) = \frac{p_0}{2} \quad (19)
\]

when \( z = 0 \) and abscissa \( x \) represented by periodic decimal fraction \( x = 0.9b \) and \( 0 < y < l \) the contact pressure will be

\[
\sigma_z(z=0) = p_0 \frac{0.9b}{2\pi b} \left( \frac{\pi}{2} + \frac{\pi}{2} + \frac{\pi}{2} \right) = 0.9p \quad (20)
\]

when \( z = 0 \), \( x < 0 \), \( 0 < y < l \) and moving along the \( x \)-axis to the \( y \)-axis contact pressure is respectively reduced from \( 0.9p_0 \) to 0.

This solution also coincides with the solution by Korotkin.

4 CONCLUSION

1. The coincidence of solutions obtained by various methods confirms their high accuracy.

2. The expressions that define all components of stresses at the ground of an even loaded rectangular foundation according to this solution, even in the expanded form, are easier than to the solution by Korotkin (they have a less number of components) that is connected with the placement of the center point of coordinates in one of the ends of the foundation.

3. The formula (12) can be used along with the method of angular points, and tables can be made on the basis of it.

4. These results allow:
   - in case of the solution of one of the problems for any type of load on the area it is easy to come up to the solution of the second problem, i.e. from the loaded rectangle to loaded strip of a semi-infinite length and vice versa;
   - to solve plasto-elastic tasks with known values of all the stress components;
   - to determine stress components at the grounds of the embankments with trapezoidal form;
   - taking into account the influence of stress components from adjoining embankments on the grounds of bridge abutments, approach viaducts, initial areas of dams and other constructions, and also the influence of free ends in the extensions on strip foundations;
   - in some cases to use the founded solutions in determining horizontal pressure on retaining structures on the basis of the theory of elasticity.

REFERENCES