Comparison of the bearing capacity of an unsaturated soil obtained from the experiments, a semi-empirical model, and numerical simulations

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ABSTRACT

Bearing capacity of unsaturated soils can be interpreted extending either the modified effective stress approach (MESA) or the modified total stress approach (MTSA) depending on the type of soils and drainage conditions of pore-air and pore-water. However, the bearing capacity of unsaturated soils are typically estimated extending the MESA, rather than MTSA. In this study, an attempt was made to validate the MTSA by comparing the measured bearing capacity values from a series of model footing tests in an unsaturated cohesive soil with those estimated using a semi-empirical model and finite element analysis extending the MTSA. The commercial finite element software, SIGMA/W was used to numerically determine the bearing capacity values based on the vertical stress versus surface settlement behaviors of a model footing.

Keywords: unsaturated soil, bearing capacity, suction, modified total stress approach, finite element analysis

1 INTRODUCTION

There are several procedures or techniques available for the interpretation of the bearing capacity of saturated soils. These procedures or techniques are also conventionally used by the practicing engineers towards the estimation of bearing capacity of soils that are typically in a state of unsaturated conditions. This is mainly due to the lack of a valid framework to interpret the bearing capacity of unsaturated soils.

Various researches have been undertaken to investigate the bearing capacity of unsaturated soils considering the influence of suction (Broms 1964, Steensen-Bach et al. 1987, Fredlund and Rahardjo 1993, Oloo et al. 1997, Costa et al. 2003, Rojas et al. 2007, Balzano et al. 2012). However, most previous researches to estimate the bearing capacity of unsaturated soils were carried out extending the Modified Effective Stress Approach (MESA) using the effective shear strength parameters (i.e. c' , ϕ' , ϕ^b) rather than the Modified Total Stress Approach (MTSA). In case of unsaturated cohesionless soils, it is reasonable to use the MESA since both the pore-air and the pore-water are under drained condition while soils are experiencing shearing and/or deformation. However, there are uncertainties in terms of the pore-air and the pore-water in unsaturated cohesive soils during loading stages. This indicates that criterion for determining appropriate approach between the MESA and the MTSA should be based on the soil type and drainage

condition of the pore-air and the pore-water during the loading stages. For example, Vanapalli and Mohamed (2007) and Schanz et al. (2011) showed that the bearing capacity of unsaturated sandy soils can be reliably estimated extending the MESA taking account of the influence of suction. On the other hand, Rassam and Williams (1999) investigated the effect of suction on the bearing capacity of an unsaturated soil extending the MTSA assuming a circular slip failure following the guidelines originally proposed by Button (1953).

In this study, the model footing test results in a cohesive unsaturated soil (Oh and Vanapalli 2013) are revisited to validate the use of MTSA in estimating the bearing capacity of unsaturated cohesive soils. For this, the experimentally determined bearing capacity values were compared with those estimated using the semi-empirical model and the finite element analysis extending the MTSA. The commercial finite element software, SIGMA/W was used to numerically determine the bearing capacity values based on the vertical stress versus surface settlement behaviors of a model footing.

2 MODIFIED TOTAL STRESS APPROACH

Terzaghi (1943) bearing capacity equation is commonly recommended for soils that exhibit dilatancy, which leads to a well-defined failure surface (i.e. general shear failure conditions) (Yamamoto et al. 2008). However, this behavior is not typically observed for the in-situ plate load tests in unsaturated cohesive

soils (Larson 1997, Schnaid et al. 1995, Costa et al. 2003, Rojas et al. 2007). Consoli et al. (1998) conducted in-situ load test (circular plates ranging from 0.3 to 0.6 m diameter and square concrete footing ranging in size from 0.4 to 1.0 m) in cohesive soil (CL). The ground water table was 4.0 m deep from the soil surface; but, the influence of suction on the bearing capacity was not taken into account. The bearing capacity values were overestimated when calculated using the conventional Terzaghi (1943) bearing equation with full-strength parameters; however, good agreement was observed when calculated using 2/3 of the full-strength parameters. This complies with Terzaghi's (1943) recommendations for punching shear failure. The in-situ load test results for a reinforced concrete footing ($B \times L = 0.5 \text{ m} \times 0.5 \text{ m}$) in an unsaturated cohesive soil by Larsson (1997) also showed no signs of heave or settlements outside the footing and plate. These results manifest that bearing capacity of unsaturated cohesive soils are governed by punching shear failure mechanism (Larson 2001).

$$q_{ult(unsat)} = (s \times \xi \times N_c)_{unsat} \quad (1)$$

In this case, suction measurement is not required since the influence of suction on the compressive strength is included in s_{unsat} (Eq. (2)).

$$s_{(unsat)} = [s_{(sat)} + f(\psi)] = \left(\frac{q_{u(unsat)}}{2} \right) \quad (2)$$

where $s_{(sat)}$, $s_{(unsat)}$ = shear strength of saturated and unsaturated soil, $f(\psi)$ = increment of strength due to suction, $q_{u(unsat)}$ = unconfined compressive strength of an unsaturated soil

If the intrinsic bearing capacity and shape [$\xi = 1 + 0.2(B/L)$]; Meyerhof 1963, Vesić 1973) factors for saturated soils are assumed to be valid for unsaturated soils, Eq. (1) can be rewritten as Eq. (3).

$$q_{ult(unsat)} = \left[\frac{q_{u(unsat)}}{2} \right] \left[1 + 0.2 \left(\frac{B}{L} \right) \right] N_c \quad (3)$$

where B , L = width and length of a foundation

The total stress approach such as Eq. (3) that were modified for estimating the mechanical behaviors of unsaturated soils are termed as ‘Modified Total Stress Approach’ (MTSA) to distinguish it from the conventional total stress approach (TSA) for saturated soils.

3 MODEL FOOTING TESTS IN AN UNSATURATED COHESIVE SOIL

3.1 Soil Properties

In the present study, the model footing test results in Indian Head till ($w_L = 32.5\%$, $w_P = 17.0\%$, $PI = 15.5\%$, $G_s = 2.72$) obtained from Indian Head, Saskatchewan, Canada) were used (Oh and Vanapalli 2013). Fig. 3 and Fig. 4 show the grain size distribution curve and the compaction curve of Indian Head till, respectively. The Soil-Water Characteristic Curve (SWCC) of the soil is shown in Fig. 5.

3.2 Sample preparation

Based on the compaction curve, using a specially designed compactor and high strength plastic tank (HSPT) (Fig. 6), the soil-water mixtures were statically compacted at a predetermined water content of 13.2 % (OMC) in five equal layers (compaction stress = 350 kPa) (Fig. 7(a)). The surface of each layer was scarified before compacting the following layer to ensure continuity in all the five compacted layers. The average height of the compacted samples was 200 mm.

After the compaction, the compacted soil in the HSPT was saturated using a downward flow of water simulating natural flow behavior in field conditions. At least 7 days were required for saturating the compacted soils. During saturation procedure, the compactor plate was securely placed on the surface of compacted soil and the end of the aluminum bar was fixed to the cross bar of the loading machine to prevent any possible volume change of specimens due to swelling (Fig. 7(b)).

Fig. 3. Grain size distribution curve of Indian Head till.

Fig. 4. Compaction curve of Indian Head till.

Fig. 5. SWCC of Indian Head till.

When the compacted soil is saturated four Tensiometers were installed at the depths of 10, 40, 80, and 120 mm from the surface (Fig. 7(c)). The saturated sample was then subjected to natural air-drying for several days to achieve desired suction distribution with depth in the compacted soil. The HSPT was then securely wrapped and kept in a humidity controlled chamber for at least 10 days to minimize the difference in the suction values between the surface and other depths (Fig. 7(d)). The suction values with depth were confirmed using i) Tensiometer readings, ii) suction measurement for a sample collected from a certain depth extending the axis-translation technique and iii) back-calculated suction values based on the water contents with depth using the SWCC.

Fig. 6. Equipment used by Oh and Vanapalli (2013) for conducting compaction and model footing tests.

Fig. 7. Testing technique followed.

When the desired suction distribution with depth was obtained, the HSPT was taken out of the humidity controlled chamber. Since the suction distributions were not uniform with depth, Oh and Vanapalli (2013) used the representative suction values in their study, which is defined as the suction value corresponding to the centroid of suction distribution with depth within the significant depth (i.e. depth region of 0 to $2B$, where B is width of the footing).

3.3 Model footing and unconfined compression tests

Model footing ($B \times L = 50 \text{ mm} \times 50 \text{ mm}$) tests were conducted at a loading rate of 1.14 mm/min (Fig. 7(e)) for five different representative suction values (i.e. 0 (SAT), 55 (UNSAT1), 100 (UNSAT2), 160 (UNSAT3), and 205 (ASCOMPAC) kPa). After the model footing tests, three specimens were collected from the compacted soils at the locations outside the stress bulb

using stainless steel thin-wall tubes (50 mm in diameter and 120 mm long; Fig. 7(f)). The extracted specimens were then used to conduct unconfined compression tests at a shear rate of 1.14 mm/min which is the same rate used for loading the model footings.

The summary of testing procedures is shown in Fig. 8 as a flow chart.

Fig. 8. Flow chart of the testing program.

4 MODEL FOOTING AND UNCONFINED COMPRESSION TEST RESULTS

Fig. 9 summarizes model footing test results carried out on both the saturated and the unsaturated soil samples. The results suggest that footing failure conditions do not reflect well defined general shear failure conditions. The indentation from the model footing tests also demonstrated punching shear failure mode. These observations are consistent with the discussions provided by Schnaid et al. (1995) and the assumption made in developing the modified total stress approach (i.e. Eq. (3)) by Oh and Vanapalli (2013). In the case where well-defined failure is not observed, the ultimate bearing capacity can be estimated as the stress corresponding to either i) 10% of the width of the footing (Cerato and Lutenege 2007) or ii) the intersection of elastic and plastic lines (Steensen-Bach et al. 1987, Consoli et al. 1998, Costa et al. 2003, Xu 2004). Hence, Oh and Vanapalli (2013) estimated the ultimate bearing capacity using the intersection of elastic and plastic lines for the

settlement less than 10% of the width of the footing.

The UC tests were carried out on two identical specimens for each suction distribution profile. The behavior of post-peak softening is more pronounced for specimens with higher suction values as shown in Fig. 10.

Fig. 9. Model footing test results.

Fig. 10. Unconfined compression test results.

Table 1 summarizes the experiment results and estimated bearing capacity values obtained using Eq. (3) with $N_c = 5.14$.

Table 1. Summary of experiment results and estimated bearing capacities

Case	ψ (kPa)	$q_{u(unsat)}/2$ (kPa)	Measured B.C (kPa)	Estimated B.C (kPa)
SAT	0	13.1	80	80.8
UNSAT1	55	33.3	153	205.4
UNSAT2	100	52.7	233	325.1
UNSAT3	160	56.5	257	348.5
ASCOMPAC	205	63.7	384	392.9

5 NUMERICAL MODELING OF STRESS VERSUS SETTLEMENT BEHAVIOURS OF MODEL FOOTING TEST

An attempt was made to predict the bearing capacity values based on stress versus surface settlement behaviors of the model footing. For this, finite element analyses (FEA) were conducted extending the MTSA. It was assumed that $\phi_u = 0$ approach (Skempton 1948) that was originally proposed for saturated soils under undrained condition can also be applicable to the unsaturated soils based on the discussion in Section 2 with Fig. 2.

5.1 Meshes and boundary conditions

Meshes and boundary conditions used in the FEA are shown in Fig. 11. Total 714 elements were generated using Quads & Triangles mesh pattern for the soil. Element size is 0.005 m for the soil immediately below the model footing (from the surface of soil to the depth of 0.15 m (i.e. 3B)) and 0.01 m for the remainders. Boundary conditions were assumed to be restrained in horizontal direction along the center line and restrained in both horizontal and vertical directions at the bottom and the interface between soil and soil tank. Analyses were conducted as axisymmetric condition with equivalent footing size since the model footing test results used in the present study were obtained for a square footing.

5.2 Soil properties used in the numerical analyses

The undrained shear strength with respect to suction were estimated using Eq. (4) (Oh and Vanapalli 2018). The variation of elastic modulus with respect to suction was estimated using the model proposed by Oh et al. (2009) (Eq. (5)).

$$c_{u(unsat)} = c_{u(sat)} \left[1 + \frac{\psi}{(P_a / 101.3)} (S^\eta) / \mu \right] \quad (4)$$

where $c_{u(sat)}$, $c_{u(unsat)}$ = saturated and unsaturated undrained shear strength based on unconfined compressive strength, ψ = suction, S = degree of saturation, P_a = atmospheric pressure, and μ ($= 10$), η ($= 2$) = fitting parameters

Fig. 11. Meshes and boundary conditions used in the FEA (axisymmetric condition).

$$E_{unsat} = E_{sat} \left[1 + \alpha \frac{\psi}{(P_a / 101.3)} S^\beta \right] \quad (5)$$

where E_{sat} , E_{unsat} = elastic modulus of saturated and unsaturated soil, respectively, and α ($= 1/10$), β ($= 2$) = fitting parameters

The undrained shear strength and elastic modulus values were then manual assigned to the soil as shown in Fig. 12(a) and (b), respectively.

In unsaturated soils, the coefficient of earth pressure at rest in terms of total (K_0) and effective (K'_0) stress can be estimated using Eq. (6) and Eq. (7), respectively. In this study, $K_0 = 1$ was used in the analyses since it was assumed that $\phi_u = 0$.

$$K_0 = \frac{(\sigma_h - u_a)}{(\sigma_v - u_a)} = \frac{\nu}{1 - \nu} - K_m \frac{\psi}{(\sigma_v - u_a)} \quad (6)$$

$$K_0 = K'_0 - (1 - K'_0) \frac{\psi}{(\sigma_v - u_a)} \sigma^s \quad (7)$$

where $K_m = E/[(1 - \nu)H_s]$, E = elastic modulus, ν = Poisson's ratio, $(\sigma_h - u_a)$ = net normal stress, ψ = suction, H_s = elastic modulus with respect to a change in ψ , σ^s = suction stress, u_a = pore-air pressure, and u_w = pore-water pressure

Fig. 12. Variation of (a) undrained shear strength and (b) elastic modulus with respect to depth.

Oh and Vanapalli (2010) revisited the measured maximum elastic (E) and shear (G) moduli for both saturated and unsaturated conditions (Mendoza et al. 2005) to investigate the variation of Poisson's ratio with respect to degree of saturation. The results showed that Poisson's ratio is relatively high for saturated condition and then gradually decreases as soils desaturate. This suggests that Poisson's ratio can be expressed as a function of degree of saturation. In this study, the FEA was carried out with five different Poisson's ratios (i.e. 0.1, 0.2, 0.3, 0.4, and 0.495).

6 ANALYSIS OF FEA RESULTS

Fig. 13 shows the comparisons between the measured stress versus settlement behaviors and those predicted from the FEA for five different Poisson's ratio values (i.e. $\nu = 0.1, 0.2, 0.3, 0.4$, and 0.495). Based on the results in Fig. 13, the measured and estimated bearing capacities are summarized in Table 2

Fig. 13. Comparisons between measured stress versus settlement behaviors and those estimated using the FEA for different Poisson's ratios.

Table 2. Comparisons between measured and estimated bearing capacity values.

Case	ν (kPa)	B.C (kPa)	B.C _{0.1} (kPa)	B.C _{0.2} (kPa)	B.C _{0.3} (kPa)	B.C _{0.4} (kPa)	B.C _{0.495} (kPa)
SAT	0	80	63	64	68	73	73
UNSAT1	55	153	180	188	195	197	205
UNSAT2	100	233	235	249	262	269	269
UNSAT3	160	257	296	296	300	312	323
ASCOMPAC	205	384	333	350	353	361	360

B.C: measured bearing capacity from model footing test

Subscript: Poisson's ratio used in the FEA

The bearing capacity values increase with increasing Poisson's ratio. This is because the higher Poisson's ratio contributes to higher lateral displacement, which leads to an increase in the resistance to the penetration of model footing into soil under confinement. Poisson's ratio close to 0.5 provided best agreement when compared with measured bearing capacity for saturated condition. For unsaturated conditions (except as compacted soil, ASCOMPAC), better comparisons were achieved for the Poisson's ratios less than 0.3. For the ASCOMPAC, the estimated bearing capacity values were slightly lower than the measured values regardless of Poisson's ratio. Fig. 14 shows a comparison between the measured bearing capacity values and those estimated from different approaches such as the MTSA (Eq. (3)) and the FEA along with MTSA. The bearing capacity values estimated from the FEA using $\nu = 0.5$ and $\nu = 0.1$ were used as representative bearing capacity values for saturated and unsaturated conditions, respectively for the purpose of comparison. As can be seen (Fig. 14), the bearing capacity values estimated using the FEA with MTSA provided the best agreement when compared with the measured bearing capacity values.

7 CONCLUSIONS

In the present study, an attempt was made to estimate the bearing capacity of an unsaturated fine-grained soil using a semi-empirical model and finite element analysis extending the Modified Total Stress Approach (MTSA). For this, the bearing capacities values of a model footing for various representative suction values were compared with those estimated using the aforementioned two different approaches. The conclusions obtained from the research can be summarized as below.

1. Poisson's ratio, ν affects the stress versus settlement behaviors of shallow foundations in unsaturated cohesive soils in the finite element analysis. Bearing capacity values estimated based on the stress versus settlement behaviors increase with increasing the Poisson's ratio. For saturated condition, the stress versus settlement behavior predicted using $\nu = 0.5$ provided better agreement with the measured stress versus settlement behavior. On the other hand, low values of

Poisson's ratio (i.e. less than 0.3) are recommended in predicting the stress versus settlement behaviors for unsaturated cohesive soils.

2. The stress versus settlement behaviors are not influenced by K_0 when the FEA is carried out extending the Modified Total Stress Approach (MTSA) due to the use of zero internal friction angle.
3. The FEA extending the MTSA provided better comparison than the MTSA (Eq. (3)) when compared with the measured bearing capacity values from a series of model footing tests in an unsaturated cohesive soil.

Fig. 14. Comparison between the measured bearing capacity values and those estimated with different approaches.

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