A fully coupled flow-deformation model for time-dependent analysis of unsaturated soils

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ABSTRACT

A fully coupled flow-deformation model for describing time-dependent behaviour of unsaturated soil is presented. The proposed hydro-mechanical model is formulated based on the theory of multiphase mixtures using the effective stress approach and the bounding surface viscoplastic constitutive model. The governing equations for the flow model are derived using the conservation equations of mass and momentum. The deformation of the solid skeleton is described using the bounding surface viscoplasticity framework to capture the time-dependent stress-strain behaviour of geomaterials. The constitutive model is based on the viscoplastic consistency theory and the bounding surface plasticity model. The proposed viscoplasticity model allows a smooth transition from rate-independent plasticity to rate-dependent viscoplasticity. The hardening parameter representing the size of the bounding surface is defined as a function of viscoplastic strain, viscoplastic strain rate, and suction. For unsaturated soils, the suction hardening effect is described using the coupled influence approach where suction has a multiplicative effect to the viscoplastic volumetric hardening. The numerical results are then presented demonstrating the capability of the proposed model in describing time-dependent behaviour of unsaturated soils.

Keywords: hydro-mechanical coupling, bounding surface, viscoplasticity, soils

1 INTRODUCTION

The hydraulic and mechanical behaviour of unsaturated soils is a subject of great interest in many geotechnical engineering problems. During the past few decades, several computational frameworks have been developed for the nonlinear analysis of unsaturated soils based on the theory of multiphase mixtures and the effective stress approach (Khalili *et al.* (2008); Sadeghi *et al.* (2015); Shahbodagh-Khan *et al.* (2015); Shahbodagh and Khalili (2016)). However, the time-dependent behaviour of the soil skeleton has mainly been ignored in these models. Numerous elastic-viscoplastic (EVP) constitutive models were developed based on the overstress Perzyna's theory (Perzyna, 1963, 1966) and the critical state framework to capture the time-dependent behaviour of geomaterials. Notable contributions include the works of Adachi and Oka (1982), Kaliakin and Dafalias (1990a, b), Kutter and Sathialingam (1992), Yin and Graham (1999) and Kimoto *et al.* (2013). More recently, several numerical models have been developed for the analysis of viscoplastic behaviour of unsaturated soils using the Perzyna's theory (Shahbodagh (2011); Oka *et al.* (2018)).

Presented in this paper is a fully coupled hydro mechanical model for time-dependent analysis of fully

and partially saturated soils using the bounding surface viscoplasticity constitutive model. The proposed viscoplasticity model allows a smooth transition from rate-independent plasticity to rate-dependent viscoplasticity. The hardening parameter representing the size of the bounding surface is defined as a function of viscoplastic strain, viscoplastic strain rate, and suction. For unsaturated soils, the suction hardening effect is described using the coupled influence approach where suction has a multiplicative effect to the viscoplastic volumetric hardening. The coupling between the fluid flow and the deformation model is derived using the effective stress parameters. Numerical examples are then presented, demonstrating the application of the proposed model.

2 EFFECTIVE STRESS

The effective stress concept was first introduced by Terzaghi and then widely applied for saturated soils to solve many geotechnical engineering problems. For unsaturated soils, the effective stress can be expressed by Bishop (1959) as

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + p_a \boldsymbol{\delta} - \chi(p_a - p_w) \boldsymbol{\delta} \quad (1)$$

where $\boldsymbol{\sigma}'$ is the effective stress, p_w is the pore water pressure and $\boldsymbol{\delta}$ is the identity vector, p_a is the pore

air pressure, χ is the effective stress parameter, attaining a value of one for saturated soils and zero for dry soils. Equation (1) can be rewritten as

$$\sigma' = \sigma_{net} - \chi s \delta \quad (2)$$

where $\sigma_{net} = \sigma + p_a \delta$ is the net stress and $s = p_a - p_w$ is the matric suction. Constitutive relations for soils are highly nonlinear, and hence they are generally expressed in the incremental format (Mac *et al.* (2014); Mac (2015); Shahbodagh Khan *et al.* (2014); Shahbodagh *et al.* (2017); Mac *et al.* (2017); Esgandani *et al.* (2017); Moghaddasi *et al.* (2017)). The rate form of the effective stress equation is obtained through a simple differentiation of Equation (2) as

$$\dot{\sigma}' = \dot{\sigma}_{net} - \psi \dot{s} \delta \quad (3)$$

where a superimposed dot indicates the rate of change, $\dot{\sigma}_{net} = \dot{\sigma} + \dot{p}_a \delta$ is the incremental net stress, $\dot{s} = \dot{p}_a - \dot{p}_w$ is the incremental suction, $\psi = d(\chi s)/ds$ is the incremental effective stress parameter.

Following Khalili and Khabbaz (1998) and Khalili *et al.* (2004), the effective stress parameter is defined as

$$\chi = \begin{cases} 1 & \text{for } \frac{s}{s_e} \leq 1 \\ \left(\frac{s}{s_e}\right)^{-\Omega} & \text{for } \frac{s}{s_e} \geq 1 \end{cases} \quad (4)$$

where Ω is a material parameter with the best fit value of 0.55, and s_e is the suction value marking the transition between saturated and unsaturated states. For the wetting process, $s_e = s_{ex}$, and for the drying process, $s_e = s_{ae}$, in which s_{ex} is the air expulsion value and s_{ae} is the air entry value.

3 SOIL WATER CHARACTERISTIC CURVE

The SWCC model proposed by Brooks and Corey (1964) is adopted as

$$S_{eff} = \begin{cases} 1 & \text{for } \frac{s}{s_e} \leq 1 \\ \left(\frac{s_e}{s}\right)^{\lambda_p} & \text{for } \frac{s}{s_e} > 1 \end{cases} \quad (5)$$

where λ_p is the pore size distribution index, $S_{eff} = (S_r - S_{res})/(1 - S_{res})$ is the effective degree of saturation, and S_{res} is the residual degree of saturation.

4 BOUNDING SURFACE VISCOPLASTICITY MODEL

The viscoplastic model is the extension of the bounding surface plasticity model proposed by Khalili *et al.* (2008) using the consistency theory of Wang *et al.* (1997) and Carosio *et al.* (2000). The core elements of the model are: elastic properties, loading surface, bounding surface, viscoplasticity flow rule, and hardening rule. The total strain increment can be decomposed into elastic (e) and viscoplastic (vp)

parts as,

$$\delta \epsilon = \delta \epsilon^e + \delta \epsilon^{vp} \quad (6)$$

where $\delta \epsilon^e$ and $\delta \epsilon^{vp}$ are the elastic and viscoplastic strain increments, respectively. The elastic response can be described using an incremental stress-strain relationship as

$$\delta \sigma' = \mathbf{D}^e \delta \epsilon^e \quad (7)$$

where $\delta \sigma'$ is the stress increment and $\mathbf{D}^e$ is the elastic stiffness matrix.

4.1 Bounding and loading surfaces

The bounding surface adopted in the present formulation is described by Khalili *et al.* (2005)

$$F(\bar{p}', \bar{q}, \bar{p}'_c) = \left(\frac{\bar{q}}{M_{cs} \bar{p}'}\right)^N - \frac{\ln(\bar{p}'_c/\bar{p}')}{\ln R} = 0 \quad (8)$$

where the superimposed bar denotes stress conditions on the bounding surface, parameter $\bar{p}'_c$ controls the size of F and is a function of viscoplastic volumetric strain ϵ_p^{vp} , viscoplastic volumetric strain rate $\dot{\epsilon}_p^{vp}$, and suction s , the material constant R represents the ratio between $\bar{p}'_c$ and the value of $\bar{p}'$ at the intercept of F with the CSL in the $(q \sim p')$ plane, and the material constant N controls the curvature of the surface.

The loading surface adopted is of the same shape and is homologous to the bounding surface about the origin in the $(q \sim p')$ plane. The function for the loading surface takes the form

$$f(p', q, p'_c) = \left(\frac{q}{M_{cs} p'}\right)^N - \frac{\ln(p'_c/p')}{\ln R} = 0 \quad (9)$$

where p'_c is the hardening parameter controlling the size of the loading surface. The image point is defined using a mapping rule such that a straight line passing through the center of homology and σ' intersects the bounding surface at $\bar{\sigma}'$ having the same unit normal vector as σ' on the loading surface (Fig. 1). The unit normal vector at the image point defining the direction of loading is given by

Fig. 1. Bounding surface, loading surface and image point

$$\mathbf{n} = \frac{\partial f / \partial \boldsymbol{\sigma}'}{\|\partial f / \partial \boldsymbol{\sigma}'\|} = \frac{\partial F / \partial \bar{\boldsymbol{\sigma}}'}{\|\partial F / \partial \bar{\boldsymbol{\sigma}}'\|} \quad (10)$$

4.2 Viscoplastic potential

The viscoplastic potential defines the direction of viscoplastic strain increments. A plastic potential is generally expressed using a plastic flow rule relating the plastic dilatancy d to the stress ratio $\eta = q/p'$. In the current formulation, the dilatancy is given by

$$d = \frac{\delta \boldsymbol{\varepsilon}_p^{vp}}{\delta \boldsymbol{\varepsilon}_q^{vp}} = \bar{t} A \left(M_{cs} - \frac{q}{p'} \right) \quad (11)$$

where A is a material constant dependent on the mechanism and amount of energy dissipation. The viscoplastic potential g is obtained by integrating Equation (11) with respect to p' and q ,

$$g(p', q, p_0) = \bar{t} \left[q + M_{cs} p' \ln \left(\frac{p'}{p_0} \right) \right] \text{ for } A = 1 \quad (12)$$

$$g(p', q, p_0) = \bar{t} \left[q + \frac{A M_{cs} p'}{A - 1} \left(\left(\frac{p'}{p_0} \right)^{A-1} - 1 \right) \right] \quad (13)$$

for $A \neq 1$

in which p_0 is the variable controlling the size of the viscoplastic potential. The direction of viscoplastic flow is defined as

$$\mathbf{m} = \frac{\partial g / \partial \boldsymbol{\sigma}'}{\|\partial g / \partial \boldsymbol{\sigma}'\|} \quad (14)$$

4.3 Hardening rule

The hardening parameter is defined as a function of the viscoplastic strain, and the viscoplastic strain rate and the matrix suction, i.e. $\chi(\boldsymbol{\varepsilon}^{vp}, \dot{\boldsymbol{\varepsilon}}^{vp}, s)$. Following the conventional approach in the bounding surface plasticity, the strain hardening modulus h can be divided into two components

$$h = h_b + h_f \quad (15)$$

where h_b is the strain modulus and the strain rate modulus at stress point $\bar{\boldsymbol{\sigma}}'$ on the bounding surface, respectively; and h_f is the arbitrary moduli at $\boldsymbol{\sigma}'$, defined as functions of the distance between $\bar{\boldsymbol{\sigma}}'$ and $\boldsymbol{\sigma}'$.

Applying the consistency bounding surface yields

$$\delta F = \left(\frac{\partial F}{\partial \bar{\boldsymbol{\sigma}}'} \right)^T \delta \bar{\boldsymbol{\sigma}}' + \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} \delta \boldsymbol{\varepsilon}_p^{vp} + \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} \delta \boldsymbol{\varepsilon}_p^{vp} + \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial s} \delta s = 0 \quad (16)$$

The viscoplastic strain rate is obtained from

$$\delta \boldsymbol{\varepsilon}^{vp} = \delta \lambda \frac{\partial g}{\partial \boldsymbol{\sigma}'} \quad (17)$$

where $\delta \lambda$ is the viscoplastic multiplier determined by forcing the stress point to remain on the rate-dependent

yield surface. Neglecting rate effects on the direction of the viscoplastic strains, the time derivative of Equation (17) gives

$$\delta \boldsymbol{\varepsilon}^{vp} = \delta \dot{\lambda} \frac{\partial g}{\partial \boldsymbol{\sigma}'} \quad (18)$$

Substituting Equations (17) and (18), Equation (16) can be written as

$$\delta F = \mathbf{n}^T \delta \bar{\boldsymbol{\sigma}}' - h_b \delta \lambda - \xi \delta \dot{\lambda} = 0 \quad (19)$$

where

$$h_b = - \frac{\partial F}{\partial \bar{p}'_c} \left(\frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} + \frac{\partial \bar{p}'_c}{\partial s} \frac{\partial s}{\partial \boldsymbol{\varepsilon}_p^{vp}} \right) \frac{m_p}{\|\partial F / \partial \bar{\boldsymbol{\sigma}}'\|} \quad (20)$$

and ξ is the viscoplastic strain rate hardening modulus,

$$\xi = - \frac{\partial F}{\partial \bar{p}'_c} \frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} \frac{m_p}{\|\partial F / \partial \bar{\boldsymbol{\sigma}}'\|} \quad (21)$$

in which $m_p = \partial g / \partial p' / \|\partial g / \partial \boldsymbol{\sigma}'\|$.

The strain hardening modulus h_f is defined such that it is zero on the bounding surface and infinity at the point of stress reversal. Following Russell and Khalili (2004) and Khalili *et al.* (2005), h_f assumed to be of the form

$$h_f = \bar{t} \left(\frac{\partial \bar{p}'_c}{\partial \boldsymbol{\varepsilon}_p^{vp}} + \frac{\partial \bar{p}'_c}{\partial s} \frac{\partial s}{\partial \boldsymbol{\varepsilon}_p^{vp}} \right) \frac{p'}{\bar{p}'_c} \left[\frac{\bar{p}'_c}{p'_c} - 1 \right] k_m (\eta_p - \eta) \quad (22)$$

where η_p is the slope of the peak strength line in $(q \sim p')$ plane, and k_m is a material parameter controlling the steepness of the response in the $(q \sim \varepsilon_q)$ plane.

The relationship between the strain rate hardening modulus ξ and the strain hardening modulus h_b is defined as

$$\xi = \frac{c_\beta (\lambda - \kappa)}{v (\dot{\boldsymbol{\varepsilon}}_p^{vp} + \dot{\boldsymbol{\varepsilon}}_{p,thr}^{vp})} |h_b| \quad (23)$$

where $c_\beta = C_\alpha / (\lambda - \kappa) \ln(10)$, C_α is the secondary compression index, $\dot{\boldsymbol{\varepsilon}}_{p,thr}^{vp}$ is a threshold viscoplastic volumetric strain rate below which the rate effect is negligible.

Hence, the equivalent consistency condition at the current stress state $\boldsymbol{\sigma}'$ can be written as

$$\mathbf{n}^T \delta \boldsymbol{\sigma}' - h \delta \lambda - \xi \delta \dot{\lambda} = 0 \quad (24)$$

Solving Equation (24), the incremental elasto-viscoplastic stress-strain relation becomes

$$\delta \boldsymbol{\sigma}' = \mathbf{D}^{evp} \delta \boldsymbol{\varepsilon} - \boldsymbol{\sigma}'^{vp} \quad (25)$$

in which

$$\mathbf{D}^{evp} = \mathbf{D}^e - \frac{\mathbf{D}^e \mathbf{m} \mathbf{n}^T \mathbf{D}^e}{\mathbf{n}^T \mathbf{D}^e \mathbf{m} + h + \frac{\xi}{\Delta t}} \quad (26)$$

$$\boldsymbol{\sigma}'^{vp} = \frac{\mathbf{D}^e \mathbf{m} \frac{\xi}{\Delta t} \delta \lambda^t}{\mathbf{n}^T \mathbf{D}^e \mathbf{m} + h + \frac{\xi}{\Delta t}} \quad (27)$$

5 GOVERNING EQUATIONS

The governing equations for fully coupled analysis of flow and deformation in partially saturated soils are formulated within the context of theory of mixtures using a systematic macroscopic approach satisfying the equations of conservation of mass and momentum. Three phases (solid, water and air) are identified. Each phase is viewed as an independent continuum, endowed with its own kinematics, mass and momentum (Habte *et al.* (2006); Khalili *et al.* (2008)).

5.1 Flow model

The differential governing equations for fluid flow model involves two phases, water phase and air phase, which are connected through the soil water characteristic curve, can be written as

$$\text{div} \left(\frac{k_{rw} \mathbf{k}}{\mu_w} (\nabla p_w + \rho_w \mathbf{g}) \right) = n_w c_w \frac{d_w p_w}{dt} + \frac{1}{V} \frac{d_s V_w}{dt} \quad (28)$$

$$\text{div} \left(\frac{k_{ra} \mathbf{k}}{\mu_a} (\nabla p_a + \rho_a \mathbf{g}) \right) = n_a c_a \frac{d_a p_a}{dt} + \frac{1}{V} \frac{d_s V_a}{dt} \quad (29)$$

in which $\mathbf{k}$ is the intrinsic permeability of the soil, k_{rw} is the relative permeability with the respect to the water phase, μ_w is the dynamic viscosity of water, $\mathbf{g}$ is the vector of gravitational acceleration, n_w is the volumetric water content, c_w is the coefficient of water compressibility; k_{ra} is the relative permeability of the air phase, μ_a is the dynamic viscosity of air, n_a is the volumetric air content, $c_a = 1/P_a$ is the compressibility of air, $P_a = p_a + p_{atm}$ is the absolute air pressure, and p_{atm} is the atmospheric pressure.

5.2 Deformation model

The deformation model of the solid phase is expressed using the condition of equilibrium on a representative volume of the soil element. Neglecting inertial effects, the linear momentum balance equation for an elemental volume is given by

$$\text{div} \boldsymbol{\sigma} + \mathbf{F} = 0 \quad (30)$$

where $\boldsymbol{\sigma}$ is the total external stress and $\mathbf{F}$ is the body force per unit volume. Rewriting the incremental form of the effective stress equation

$$\boldsymbol{\sigma}' = \boldsymbol{\sigma} + \psi \dot{p}_w \boldsymbol{\delta} + (1 - \psi) \dot{p}_a \boldsymbol{\delta} \quad (31)$$

and expressing stress-strain relationship

$$\boldsymbol{\sigma}' = \mathbf{D}^{evp} \dot{\boldsymbol{\epsilon}} - \boldsymbol{\sigma}'^{vp} \quad (32)$$

where $\mathbf{D}^{evp}$ is the drained stiffness matrix of the soil and $\dot{\boldsymbol{\epsilon}}$ is the soil skeleton strain.

The governing equation for the deformation model can be written as

$$\text{div}[\mathbf{D}^{vp}(\nabla \dot{\mathbf{u}}) - \boldsymbol{\sigma}'^{vp} - \psi \dot{p}_w \boldsymbol{\delta} - (1 - \psi) \dot{p}_a \boldsymbol{\delta}] + \dot{\mathbf{F}} = 0 \quad (33)$$

5.3 Fully coupled equations

The fully coupled equations are obtained by combining the governing equations for the flow and deformation models

$$\text{div} \left(\frac{k_{rw} \mathbf{k}}{\mu_w} (\nabla p_w + \rho_w \mathbf{g}) \right) = \psi \text{div} \dot{\mathbf{u}} + \bar{a}_{11} \dot{p}_w - a_{12} \dot{p}_a \quad (34)$$

$$\begin{aligned} \text{div} \left(\frac{k_{ra} \mathbf{k}}{\mu_a} (\nabla p_a + \rho_a \mathbf{g}) \right) \\ = (1 - \psi) \text{div} \dot{\mathbf{u}} - \bar{a}_{21} \dot{p}_w + a_{22} \dot{p}_a \end{aligned} \quad (35)$$

with $\bar{a}_{11} = n_w c_w + a_{12}$; $\bar{a}_{22} = n_a c_a + a_{21}$; and $a_{12} = a_{21} = -n \frac{\partial s_r}{\partial s}$

6 FINITE ELEMENT IMPLEMENTATION

Applying the Galerkin's method and the finite difference approach, the spatially discretized form of the governing partial differential equations can be written as

$$[K]\{\Delta u\} - \psi[C]\{\Delta p_w\} - (1 - \psi)[C]\{\Delta p_a\} = \{\Delta R\} \quad (36)$$

$$\begin{aligned} & -\psi[C]^T\{\Delta u\} - \bar{a}_{11}[M]\{\Delta p_w\} - \\ & [H_w](\beta\{p_w\} + \{p_w\}_t)\Delta t + a_{12}[M]\{\Delta p_a\} = \\ & (\{Q_w\}_t + \beta\{\Delta Q_w\})\Delta t \end{aligned} \quad (37)$$

$$\begin{aligned} & -(1 - \psi)[C]^T\{\Delta u\} + \bar{a}_{21}[M]\{\Delta p_w\} - \\ & [H_a](\beta\{p_a\} + \{p_a\}_t)\Delta t - a_{22}[M]\{\Delta p_a\} = \\ & (\{Q_a\}_t + \beta\{\Delta Q_a\})\Delta t \end{aligned} \quad (38)$$

in which $[K]$ is the element stiffness matrix, $[C]$ is the coupling matrix, $[M]$ is the mass matrix, $[H_w]$ and $[H_a]$ are flow matrices corresponding to the permeability of the water and air phases, respectively, $\{u\}$ is the vector of nodal displacements, $\{p_w\}$ is the vector of nodal pore water pressures, $\{p_a\}$ is the vector of nodal pore air pressures, $\{P\}$ is the vector of nodal forces, $\{Q_w\}$ and $\{Q_a\}$ are vectors of nodal fluxes of the water and air flows, respectively.

7 NUMERICAL EXAMPLES

In this section, several numerical examples are presented to demonstrate the capability of the proposed fully coupled model to capture the time-dependent behaviour of geomaterials.

Effect of strain rate

Henkel (1956) presented a series of conventional drained triaxial tests on heavily overconsolidated samples of Weald Clay. The physical properties and material constants are presented in Table 1. The bounding surface viscoplasticity model constants used in the simulation were $N = 4.5$, $R = 2.714$, $k = 2.0$ and $A = 0.72$, $k_m = 0.7$. The viscosity parameters were: $c_\beta = 0.07$, $\dot{\epsilon}_{p,ref}^{vp} = 2.0 \times 10^{-5}/\text{min}$ and $\dot{\epsilon}_{p,thr}^{vp} = 1.0 \times 10^{-8}/\text{min}$.

Table 1. Physical properties and material constants for Weald Clay

Swelling index	κ	0.025	-
Poisson's ratio	ν	0.3	-
The slope of the critical state line	M_{cs}	0.96	-
Compression index	λ	0.093	-
Specific volume on the critical state line at a unit confining pressure	Γ	2.06	-
Initial void ratio	e_0	0.617	-
Initial mean effective stress	p'_0	34.5	kPa

Fig. 2. Effect of strain rate on stress-strain response of heavily overconsolidated Weald Clay: a) Axial strain vs deviatoric stress; b) Axial strain vs volumetric strain.

The effect of strain rate was simulated based on the data of the triaxial test in drained compression condition with cell pressure of 34.5kPa. The

stress-strain response and the change in volumetric strain are shown in Fig. 2 for different strain rates. It is clear that the stiffness and strength of the soil increase with the increase of strain rate, implying the proposed model is capable in simulating the rate-dependent behaviour of clayey soils.

For one-dimensional analysis, a boundary value problem consisting of a multi-phase porous layer of 100m depth of heavily overconsolidated Weald Clay is considered. The finite element mesh and the boundary conditions for the simulations are shown in Fig. 3. The upper boundary is drained and subjected to a load of intensity $f(t)$, in which t is time in second. On the other hand, the remaining boundaries are impervious. All nodes are horizontally fixed and the ones at the bottom are vertically constrained. The material parameters are presented in Table 2.

Table 2. Material parameters for the numerical analysis.

Initial porosity	n_0	0.33	-
Lame's constant	λ	9355	kPa
Lame's constant	μ	6237	kPa
Initial permeability of liquid phase at $S_r=1\text{m/s}$	k_{ws0}	4.883×10^{-3}	m/s
Initial permeability of gas phase at $S_r=S_{res}$	k_{gs0}	0.05	m/s
Compressibility coefficient of liquid phase	c_w	2.2×10^{-6}	kPa^{-1}
Unit weight of liquid phase	γ_w	9.8	kN/m^3
Density of saturated mixture	γ_{sat}	1.8	t/m^3
Air entry suction value	s_{ae}	10.0	kPa
Air expulsion suction value	s_{ex}	10.0	kPa
Pore size distribution index	λ_p	0.15	-
Residual degree of saturation	S_{res}	0.2	-
Permeability constant	C_k	1.0	-

An 8-node isoparametric quadrilateral element with a reduced Gaussian integration is adopted. For the coupled analysis, the 4-node quadrilateral element is utilized for pore fluid pressure fields. The mesh pattern of 1×100 (100 mixed elements) is considered as the default mesh configuration for the analyses.

The response of the porous media subjected to the uniform step loading under different conditions (dry, fully saturated and partially saturated) is shown in Fig. 4. The step load increases from zero to a maximum value of 500kPa within 100s and remains constant afterward. It demonstrates the continuity of response at transition between saturated and unsaturated states and between unsaturated and dry states.

The distributions of suction, pore water pressure and pore gas pressure at various time steps are shown in Fig. 5. High levels of pore water and pore gas pressures are generated within the first 100 sec, and dissipated gradually afterward. Fig. 6 shows the settlement of the soil layer under the step load of 500kPa applied within different time duration $t_m = 100\text{s}$, 200s , 500s and 1000s . The figure demonstrates the effect of the loading rate on the behaviour of the unsaturated soil. As observed,

the longer the time duration to reach the maximum load, the larger settlement of the soil layer due to more dissipation of pore water and pore gas pressures and the softer behaviour of the soil skeleton under lower strain rates.

Fig. 3. Porous layer under uniform load: finite element mesh, boundary conditions and applied load.

Fig. 4. Displacement developed over time using viscoplastic model under dry, fully saturated and partially saturated conditions.

Fig. 5. Analysis of porous media in one-dimensional problem using Bounding Surface Viscoplasticity Model under unsaturated conditions with suction $s = 20\text{kPa}$

Fig. 6. Displacement of soil column under the step load of 500kPa within different time t_m of 100s, 200s, 500s and 1000s.

8 CONCLUSIONS

A fully coupled flow-deformation model is presented for the analysis of time-dependent behaviour of unsaturated geomaterials. The viscoplastic behaviour of the soil skeleton is captured using the bounding surface viscoplasticity constitutive model. The hardening parameter is defined as a function of the viscoplastic strain, the viscoplastic strain rate and the matric suction. The coupling between the flow model and the deformation model is derived using the concept of effective stress and the compatibility of the volumetric deformation of the three phases. The Garlerkin's approach and the finite difference method are applied for spatial and time discretisation of the governing equations. Numerical examples are presented to show the capability of the model to capture the time-dependent behaviour of unsaturated soils.

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