An analytical study on the boundary size effect of calibration chamber studies

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ABSTRACT

Calibration chambers can provide a laboratory-controlled environment and have long been used to study, such as cone penetration tests and pressuremeter tests. However, the finite radial size of the sample that the chamber can accommodate may give rise to the boundary size effect which can influence the test results. When unsaturated soils, especially for soils where suction hardening is present, are concerned, the effect of suction on the boundary size effect is unknown. This study presents a new cavity expansion solution in unsaturated soils of finite radial extent, with which the boundary size effect can be quantified. The solution is formulated for spherical cavities subjected to two boundary conditions under the drainage condition where the contribution of suction to the effective stress is constant. It is found that samples with a higher suction value are under more influence of the boundary size effect and this is more obvious with higher mean net stress and higher void ratio.

Keywords: cavity expansion, boundary size effect, suction, finite radial extent

1 INTRODUCTION

Calibration chambers have long been used to develop correlations for interpretation and validate theoretical analysis of cone penetration tests (CPTs) and pressuremeter tests (PMTs) as the stress and strain histories, boundary condition, density and water content of the test samples can be fully controlled. However, due to the limited radial extent of tested samples, the results measured in calibration chambers for unsaturated soil samples may be remarkably different to the values measured in the field, especially for dense samples. As the experiment is analogous to a cavity expanding in finite radial extent, there have been a few attempts (e.g. Yu, 1992, 1993; Salgado *et al.*, 1997; Pournaghiazar *et al.*, 2013a) to solve this problem, though all are limited to saturated soils. Recently, Cheng *et al.* (2018) presented a cavity expansion solution for unsaturated soils for which suction hardening is present. Here the focus is given to the boundary size effect with insights gained from the new spherical cavity expansion solution for unsaturated soils.

2 SOIL WATER CHARACTERISTIC CURVE

The relationship between void ratio (e), degree of saturation (S_r) and suction (s) is defined through a soil-water characteristic curve (SWCC). A simplified model of Russell and Buzzi (2012) is used where hydraulic hysteresis is ignored. It comprises a main drying curve and an initial transition line between the

air expulsion point and the main drying curve (hereafter referred to transition line) (Figure 1):

$$S_r = \begin{cases} 1 & \text{for } s \leq s_{ex} \\ (s/s_{ex})^\beta & \text{for } C_2^{[\alpha/(\alpha-\beta)]} s_{ex} \geq s \geq s_{ex} \\ (s/s_{ae})^\alpha & \text{for } s \geq C_2^{[\alpha/(\alpha-\beta)]} s_{ex} \end{cases} \quad (1)$$

where s_{ae} and s_{ex} are the air entry value and air expulsion value. α is a negative constant. β is the slope of the transition line between s_{ex} and the main drying curve.

Figure 1. Soil-water characteristic curve and the effective stress parameter χ plotted against suction. (Extended dashed lines indicate the value where air entries.)

s_{ae} depends on void ratio e through

$$s_{ae} = C_1 e^{-\gamma} \quad (2)$$

in which C_1 is a positive constant with units of stress and γ is a constant. A fixed ratio between s_{ae} and s_{ex} is assumed,

$$s_{ae} = C_2 s_{ex} \quad (3)$$

in which C_2 is a constant. For fractal soils $\gamma = D_s$, where D_s is the fractal dimension of the particle size distribution, and $\alpha = D_p - 3$, where D_p is the fractal dimension of the pore size distribution (Russell 2014).

3 CONSTITUTIVE MODEL

The stress-strain behavior is described by a simplified version of the bounding surface plasticity model used in Yang and Russell (2015). Conventional $p' \sim q$ notation is used, where p' and q is the mean effective stress and the deviator stress, respectively. The corresponding strain variables are the volumetric strain ε_p and shear strain ε_q . Compressive stresses and strains are assumed positive and ε_p is linked to specific volume v according to $\varepsilon_p = -\ln(v/v_0)$ and $\dot{\varepsilon}_p = -\dot{v}/v$, where $v = 1 + e$ and v_0 is the specific volume at the reference configuration. A superimposed dot indicates an increment. Elastic and plastic strain increments sum to give total strain increments and superscripts e and p denote the elastic and plastic components. The stresses and strains are written in vector form, $\boldsymbol{\sigma}' = [p', q]^T$ and $\boldsymbol{\varepsilon} = [\varepsilon_p, \varepsilon_q]^T$.

The effective stress is:

$$p' = p_{net} + \chi s \quad \dot{p}' = \dot{p}_{net} + \psi \dot{s} \quad (4)$$

where $p_{net} = p - u_a$ is the net stress, $s = u_a - u_w$ (u_a and u_w are pore air and water pressures), χ is the effective stress parameter (having a value of 1 for saturated soils and 0 for dry soils) and $\psi = d(\chi s)/ds$. The equation for χ proposed by Khalili and Khabbaz (1998) is adopted here as:

$$\chi = \begin{cases} 1 & \text{for } s \leq s_{ex} \\ (s/s_{ex})^{-\zeta} & \text{for } C_2^{[\Omega/(\Omega-\zeta)]} s_{ex} \geq s \geq s_{ex} \\ (s/s_{ae})^{-\Omega} & \text{for } s \geq C_2^{[\Omega/(\Omega-\zeta)]} s_{ex} \end{cases} \quad (5)$$

where Ω is a parameter with a best fit value of 0.55, ζ is the slope of the transition line between the s_{ex} and main drying curve in a $\ln \chi \sim \ln s$ plane. Figure 1 plots the effective stress parameter against suction in the $\ln \chi \sim \ln s$ plane.

A simple isotropic elastic rule is adopted. K and G are defined as:

$$K = \frac{(1+e)p'}{e\kappa} \quad G = \frac{3(1-2\mu)}{2(1+\mu)} K \quad (6)$$

where κ is a material constant and is the slope of the elastic unload-reload line in the $\ln e \sim \ln p'$ plane and μ is Poisson's ratio. The incremental elastic deformation is given by $\dot{\varepsilon}_p^e = \dot{p}'/K$ and $\dot{\varepsilon}_q^e = \dot{q}/3G$.

The yield surface is defined as

$$f = q - M_{cs} p' \left[\frac{\ln(p'_c/p')}{\ln P} \right]^{1/Q} = 0 \quad (7)$$

where Q is a material constant controlling the curvature, P is another material constant controlling the ratio between p'_c (pre-consolidation pressure) and the value of p' at the intercept of f with the critical state line in the $p' \sim q$ plane.

Linear critical state lines (CSLs) and limiting isotropic compression lines (LICLs) are assumed in $\ln e \sim \ln p'$ plane. The LICLs depend on suction:

$$\ln e_{LICL} = \ln N(s) - \lambda(s) \ln p' \quad (8)$$

where, for a given s , $N(s)$ is the value of e at $\ln p' = 0$ and $\lambda(s)$ is the slope.

The CSLs are deduced implicitly by an elastic shift from the LICLs along the κ line of magnitude $\kappa \ln R$. They are also suction dependent:

$$\ln e_{CSL} = \ln N(s) - \lambda(s) \ln(Rp') + \kappa \ln R \quad (9)$$

A non-associated flow rule is assumed:

$$d = \frac{\dot{\varepsilon}_p^p}{\dot{\varepsilon}_q^p} = A \left(M_{cs} - \frac{q}{p'} \right) \quad (10)$$

where d is dilatancy and A is a material constant.

The unit vector of plastic flow ($\mathbf{m}$) at $\boldsymbol{\sigma}'$ is then:

$$\mathbf{m} = [m_p, m_q]^T = \left[\frac{d}{\sqrt{1+d^2}}, \frac{1}{\sqrt{1+d^2}} \right]^T \quad (11)$$

Imposing the consistency condition at the bounding surface under isotropic hardening leads to:

$$h = - \frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial \varepsilon_p^p} \frac{m_p}{\sqrt{\left(\frac{\partial f}{\partial p'_c} \right)^2 + \left(\frac{\partial f}{\partial q} \right)^2 + \left(\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial s} \right)^2}} \quad (12)$$

The relationship between p'_c , s and e is:

$$\frac{\dot{e}}{e} = \frac{\dot{N}(s)}{N(s)} - \dot{\lambda}(s) \ln(\bar{p}'_c) - \lambda(s) \frac{\dot{\bar{p}}'_c}{\bar{p}'_c} \quad (13)$$

The rate form of the elastic volumetric strain is:

$$\dot{\varepsilon}_p^e = \frac{e}{1+e} \kappa \frac{\dot{p}'}{p'} \quad (14)$$

The rate form of p'_c is:

$$p'_c = \frac{p'_c v}{\lambda(s) - \kappa} \frac{1}{v-1} \dot{\varepsilon}_p^p + \frac{p'_c}{\lambda(s) - \kappa} \left(\frac{1}{N(s)} \frac{\partial N(s)}{\partial s} - \ln(p'_c) \frac{\partial \lambda(s)}{\partial s} \right) \dot{s} \quad (15)$$

$\mathbf{n}$ defines the direction of loading and is:

$$\mathbf{n} = [n_r, n_s, n_\theta]^T = \begin{bmatrix} \frac{\frac{\partial f}{\partial p'}}{\sqrt{\left(\frac{\partial f}{\partial p'}\right)^2 + \left(\frac{\partial f}{\partial q}\right)^2 + \left(\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial s}\right)^2}}, \frac{\frac{\partial f}{\partial q}}{\sqrt{\left(\frac{\partial f}{\partial p'}\right)^2 + \left(\frac{\partial f}{\partial q}\right)^2 + \left(\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial s}\right)^2}}, \frac{\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial s}}{\sqrt{\left(\frac{\partial f}{\partial p'}\right)^2 + \left(\frac{\partial f}{\partial q}\right)^2 + \left(\frac{\partial f}{\partial p'_c} \frac{\partial p'_c}{\partial s}\right)^2}} \end{bmatrix}^T \quad (16)$$

The moisture content v_w can be written in an incremental form $\dot{v}_w = \dot{S}_r e + S_r \dot{e}$. It follows that:

$$\frac{\partial S_r}{\partial s} (v-1) \dot{s} + S_r (\alpha\gamma + 1) \dot{v} = \dot{v}_w \quad (17)$$

where $\partial S_r / \partial s = \alpha S_r / s$ when the state is on main wetting or drying curves and $\partial S_r / \partial s = \beta S_r / s$ when the state is on a scanning curve.

The complete stress-strain relationship is:

$$\begin{bmatrix} \dot{\varepsilon}_p \\ \dot{\varepsilon}_q \\ \dot{v}_w \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \begin{bmatrix} \dot{p}' \\ \dot{q} \\ \dot{s} \end{bmatrix} \quad (18)$$

where

$$\begin{aligned} D_{11} &= \frac{1}{K} + \frac{n_p m_p}{h} & D_{12} &= \frac{n_q m_p}{h} & D_{13} &= \frac{n_s m_p}{h} \\ D_{21} &= \frac{n_p m_q}{h} & D_{22} &= \frac{1}{3G} + \frac{n_q m_q}{h} & D_{23} &= \frac{n_s m_q}{h} \\ D_{31} &= -v S_r (\alpha\gamma + 1) \left(\frac{1}{K} + \frac{n_p m_p}{h} \right) & D_{32} &= -v S_r (\alpha\gamma + 1) \frac{n_q m_p}{h} \\ D_{33} &= -v S_r (\alpha\gamma + 1) \left[\frac{n_s m_p}{h} + \frac{\partial S_r}{\partial s} (v-1) \right] \end{aligned}$$

4 CAVITY EXPANSION ANALYSIS

A homogenous soil mass with radius B is initially under a mean effective stress p'_0 . A concentric cavity is then expanded from its initial radius a_0 to a , and a region consisting of elastic and plastic strains is formed around the cavity with the radius of the elastic-plastic boundary R . The elastic-plastic region is surrounded by soil which deforms purely elastically and has boundary B . The configuration is illustrated in Figure 2, where the radius of a particular soil element is denoted by r .

Figure 2. Configuration of the cavity expansion problem

The radial stress (σ'_r) represents the major principal stress and the tangential stress (σ'_θ) represents the minor principal stress. p' and q are defined in terms of σ'_r and σ'_θ . ε_p and ε_q are related to radial strain ε_r and tangential strain ε_θ .

$$p' = \frac{\sigma'_r + 2\sigma'_\theta}{3}, q = \sigma'_r - \sigma'_\theta, \varepsilon_p = \varepsilon_r + 2\varepsilon_\theta, \varepsilon_q = \frac{2(\varepsilon_r - \varepsilon_\theta)}{3} \quad (19)$$

ε_r and ε_θ at r are defined in terms of radial displacement u of the soil element at r by $\varepsilon_r = -\partial u / \partial r$ and $\varepsilon_\theta = -u / r$.

4.1 Finite elastic region

The solution for stress distribution and displacement in an elastic region for $R \leq r \leq B$ is given as follows. For constant stress boundary condition, the stress and displacement distributions are:

$$\sigma'_r = p'_0 + (\sigma'_R - p'_0) \left[\frac{\left(\frac{R}{r}\right)^3 - \left(\frac{R}{B}\right)^3}{1 - \left(\frac{R}{B}\right)^3} \right] \quad (20)$$

$$\sigma'_\theta = p'_0 - (\sigma'_R - p'_0) \left[\frac{\frac{1}{2} \left(\frac{R}{r}\right)^3 + \left(\frac{R}{B}\right)^3}{1 - \left(\frac{R}{B}\right)^3} \right] \quad (21)$$

$$u = \frac{\sigma'_R - p'_0}{\left(\frac{1}{R}\right)^3 - \left(\frac{1}{B}\right)^3} \frac{1}{2G(1+\mu)} \quad (22)$$

$$\left(\frac{1+\mu}{kr^2} + \frac{1-2\mu}{B^3} r \right)$$

For the zero displacement boundary condition, the solutions for stress and displacement in an elastic region for $R \leq r \leq B$ are:

$$\sigma'_r = p'_0 +$$

$$(\sigma'_R - p'_0) \left[\frac{\left(\frac{R}{r}\right)^3 + \frac{1+\mu}{2(1-2\mu)} \left(\frac{R}{B}\right)^3}{\frac{1+\mu}{2(1-2\mu)} \left(\frac{R}{B}\right)^3 + 1} \right] \quad (23)$$

$$\sigma'_\theta = p'_0 -$$

$$(\sigma'_R - p'_0) \left[\frac{\frac{1}{2} \left(\frac{R}{r}\right)^3 - \frac{1+\mu}{2(1-2\mu)} \left(\frac{R}{B}\right)^3}{\frac{1+\mu}{2(1-2\mu)} \left(\frac{R}{B}\right)^3 + 1} \right] \quad (24)$$

$$u = (\sigma'_R - p'_0) r \frac{1}{4G(1+\mu)}$$

$$\left\{ \frac{(1+\mu) \left(\frac{R}{r}\right)^3 - (1+\mu) \left(\frac{R}{B}\right)^3}{\frac{1+\mu}{2(1-2\mu)} \left(\frac{R}{B}\right)^3 + 1} \right\} \quad (25)$$

where G is the elastic shear modulus and defines as $E/2(1+\mu)$.

4.2 Plastic region

Seven differential equations govern soil behavior in the plastic region. The seven equations involve seven variables and can be presented in differential form and solved as an initial value problem.

The stress equilibrium equation is:

$$\frac{\partial p'}{\partial r} + \frac{2}{3} \frac{\partial q}{\partial r} + \frac{2q}{r} = 0 \quad (26)$$

The incremental forms of plastic strains come directly from the bounding surface model:

$$D\varepsilon_p^p = D\varepsilon_p - D\varepsilon_p^e = \frac{1}{h} (n_p Dp' + n_q Dq + n_s Ds) m_p \quad (27)$$

$$D\varepsilon_q^p = D\varepsilon_q - D\varepsilon_q^e = \frac{1}{h} (n_p Dp' + n_q Dq + n_s Ds) m_q \quad (28)$$

The incremental relationship between moisture content, suction and specific volume is:

$$\frac{\partial S_r}{\partial s} (v-1) Ds + S_r (\alpha\gamma + 1) Dv = Dv_w \quad (29)$$

The rate forms for s_{ae} is

$$Ds_{ae} + C_1 \gamma (v-1)^{-\gamma-1} Dv = 0 \quad (30)$$

Suction has a multiplicative effect on hardening:

$$Dp'_c = - \frac{p'_c}{\lambda(s) - \kappa} \frac{1}{v-1} Dv - \frac{v}{K} \frac{p'_c}{\lambda(s) - \kappa} \frac{1}{v-1} Dp' + \frac{p'_c}{\lambda(s) - \kappa} \left(\frac{\partial N(s)}{\partial s} - \ln(p'_c) \frac{\partial \lambda(s)}{\partial s} \right) Ds \quad (31)$$

Taking advantage of the findings of Yang and Russell (2015), that constant χs can be assumed without loss of significant accuracy, when the actual drainage conditions are different, the constant χs condition is assumed here:

$$\frac{\partial(\chi s)}{\partial r} = \Psi \frac{\partial s}{\partial r} + \chi \Omega \frac{s}{s_e} \frac{\partial s_e}{\partial r} = 0 \quad (32)$$

In the elastic region the solution procedure is straightforward and involves using the above closed form expressions for stresses, strains and velocity in terms of r and R . In the plastic region the solution procedure is more complex and the governing equations are not described in the same scheme, in that Eqs. (33-38) are formulated at material time whereas Eq. (32) is at local time. The auxiliary independent variable proposed by Chen and Abousleiman (2013) is adopted here to bring these equations into the same scheme, which is defined as:

$$\xi = \frac{u}{r} = \frac{r - r_0}{r} \quad (33)$$

After some manipulations of ξ , the governing Eqs. (32-37) and drainage condition (38) can also be expressed with respect to ξ . These governing equations may be presented in matrix form as:

$$\begin{bmatrix} S_{11} & S_{12} & 0 & 0 & S_{15} & 0 & S_{17} \\ S_{21} & S_{22} & 0 & S_{24} & S_{25} & 0 & 0 \\ S_{31} & S_{32} & 0 & S_{34} & S_{35} & 0 & 0 \\ 0 & S_{42} & 0 & 0 & S_{45} & S_{46} & 0 \\ 0 & 0 & 0 & S_{54} & 0 & 0 & S_{57} \\ S_{61} & 0 & S_{63} & S_{64} & S_{65} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{75} & S_{76} & S_{77} \end{bmatrix} \begin{bmatrix} Dp'/D\xi \\ Dq/D\xi \\ Dp'_c/D\xi \\ Dv/D\xi \\ Ds/D\xi \\ Dv'_s/D\xi \\ Ds_{ae}/D\xi \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{2q}{\left[1-\xi-\frac{v_0}{v(1-\xi)^2}\right]} \\ 0 \\ 2/(1-\xi) \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

(34)

where

$$\begin{aligned} S_{11} &= 1 & S_{12} &= \frac{2}{3} & S_{15} &= -\Psi & S_{17} &= -\chi \Omega \frac{s}{s_{ae}} \\ S_{21} &= \frac{1}{K} + \frac{1}{h} n_p m_p & S_{22} &= \frac{1}{h} n_q m_p & S_{24} &= \frac{1}{v} & S_{25} &= \frac{1}{h} n_s m_p \\ S_{31} &= \frac{1}{h} n_p m_q & S_{32} &= \frac{1}{3G} + \frac{1}{h} n_q m_q & S_{34} &= \frac{2}{3} \frac{1}{v} \\ S_{35} &= \frac{1}{h} n_s m \\ S_{42} &= S_r (\alpha\gamma + 1) & S_{45} &= \frac{\partial S_r}{\partial s} (v-1) & S_{46} &= -1 \\ S_{54} &= C_1 \gamma (v-1)^{-\gamma-1} & S_{57} &= 1 \\ S_{61} &= \frac{v}{K} \frac{p'_c}{\lambda(s) - \kappa} & S_{63} &= 1 & S_{64} &= \frac{p'_c}{\lambda(s) - \kappa} \\ S_{65} &= -\frac{p'_c}{\lambda(s) - \kappa} \left(\frac{\partial N(s)}{\partial s} - \ln(p'_c) \frac{\partial \lambda(s)}{\partial s} \right) \\ S_{75} &= \Psi \text{ and } S_{77} = \frac{\chi \Omega s}{s_e} \end{aligned}$$

Now the governing differential equations in the plastic region are all ordinary differential equations with respect to ξ and can be solved as initial value problem once the initial conditions at the elastic-plastic boundary are known.

4.3 Initial conditions and elastic-plastic boundary

By insisting that the conditions at the elastic-plastic boundary ($r=R$) must satisfy the governing equations in elastic region (20) and (21) or (23) and (24) (depending on the boundary condition), after expansion of the cavity to a certain radius, the driver starts from ξ_p which for constant stress boundary conditions is:

$$\xi_p = \left(\frac{u}{r} \right)_{r=r_p} = \frac{\sigma'_{r,p} - p'_0}{\left(\frac{1}{R} \right)^3 - \left(\frac{1}{B} \right)^3} \frac{1}{2G(1+\mu)} \left[\frac{1+\mu}{2R^3} + \frac{1-2\mu}{B^3} \right] \quad (35)$$

where $\sigma'_{r,p}$ is obtained by substituting Eqs. (20) and (21) into η_p and rearranging as:

$$\sigma'_{r,p} = p'_0 \left[\frac{1 + \frac{2}{3} \eta_p}{1 + \frac{2}{3} \left(\frac{R}{B} \right)^3 \eta_p} \right] \quad (36)$$

Similarly for zero displacement boundary conditions:

$$\xi_p = \left(\frac{u}{r} \right)_{r=r_p} = (\sigma'_{r,p} - p'_0) \frac{1}{4G(1+\mu)} \left\{ \frac{1 - \left(\frac{R}{B} \right)^3}{\frac{1}{2(1-2\mu)} \left(\frac{R}{B} \right)^3 + 1} \right\} \quad (37)$$

where $\sigma'_{r,p}$ is:

$$\sigma'_{r,p} = p'_0 \left[\frac{1 + \frac{2}{3} \eta_p}{1 - \frac{1+\mu}{3(1-2\mu)} \left(\frac{R}{B} \right)^3 \eta_p} \right] \quad (38)$$

To link the auxiliary independent variable with radial coordinate, it has:

$$\frac{dr}{r} = \frac{d\xi}{1 - \xi - v_0 / \left[v(\xi)(1-\xi)^2 \right]} \quad (39)$$

Integrating Eq.(39) from the cavity wall to the radial distant to the plastic region, the relation between r/a and ξ can be obtained as:

$$\frac{r}{a} = \exp \left(\int_{\xi_0}^{\xi} \frac{d\xi}{1 - \xi - v_0 / \left[v(\xi)(1 - \xi)^2 \right]} \right) \quad (40)$$

5 RESULTS AND DISCUSSIONS

5.1 Input parameters

Model parameters for a silty sand from Yang and Russell (2015) are adopted here. The particle size and pore size distributions are fractal with $D_s = 2.6$ and $D_p = 2.48$. Also, $C_1 = 1.5$ kPa and $C_2 = 30$. It follows that $s_{ae} = 1.5e^{-2.61}$ kPa, $s_{ae} = 30s_{ex}$, $\alpha = -0.52$ and $\gamma = 2.61$. $\beta = -0.2$ fits the transition line. As $\Omega = 0.55$ it follows that $\zeta = \beta\Omega/\alpha = 0.2115$. The elasticity parameters are $\kappa = 0.012$ and $\mu = 0.4$. The critical state strength is $M_{cs} = 1.45$ (corresponding to $\phi'_{tr} = 35.7^\circ$). The parameter $A = 1.2$ is adopted. $Q = 2.8$ and $P = 10$ define the shape of the yield surface. $\lambda(s)$ and $N(s)$ for different suction values are listed in Table 1. $\lambda(s)$ and $N(s)$ for intermediate suction values are found by linear interpolation and for suctions larger than 300 kPa by linear extrapolation from the 100 kPa to 300 kPa interval.

Table 1. $\lambda(s)$ and $N(s)$ for different suction values.

s (kPa)	$s \leq s_{ex}$	$s = 10$	100	300
$N(s)$	0.515	1.4765	1.58	1.81
$\lambda(s)$	0.08	0.1555	0.16	0.17

Cavity expansion results are then generated for a range of initial stress states involving $p_{net0} = 60$ kPa, 90 kPa, 120 kPa or 240 kPa, $s_0 = 20$ kPa, 80 kPa or 160 kPa and $e_0 = 0.43, 0.51, 0.58$ or 0.66 . The initial stress states are illustrated in the $\ln e \sim \ln p'$ plane together with corresponding CSL and LICL, as shown in Figure 3. The specific volumes selected represent loose, medium and dense state, respectively.

5.2 Boundary size effect

To explore the boundary size effect, results are plotted in the $\sigma_L/\sigma_{L,\infty} \sim B/a$ plane (where $\sigma_{L,\infty}$ denotes the limiting cavity pressure in infinite radial extent and is obtained when $R/B=0$) as shown in Figure 4 for spherical cavity expansion. The ratio of B/a is calculated as the product of R/a at the elastic-plastic boundary and input value of R/B . The results for $s_0 = 20$ kPa and 160 kPa are plotted to highlight the effect of suction.

As shown in Figure 4 for a certain e_0 , the reduction in the σ_L under constant stress boundary conditions and the increase in the σ_L under zero displacement boundary conditions becomes smaller as B/a increases, as expected. At a certain value of B/a , the value of $\sigma_L/\sigma_{L,\infty}$ at $s_0 = 160$ kPa deviates more from unity than that at s_0

$= 20$ kPa and this is more obvious with higher values of p_{net0} , which is probably due to the stiffer response at higher s_0 and p_{net0} . This suggests samples with a higher suction value are under more influence of the boundary size effect, especially at larger initial mean net stresses. The effect is less significant as e_0 is reduced as the lines for different suction values become more indistinguishable and also the results for different p_{net0} prone to converge so that the contribution of suction to p'_0 has less effect on $\sigma_L/\sigma_{L,\infty}$.

Figure 3. Initial stress states with CSL and LICL lines for the cavity expansion analysis.

The effect of suction on R/a can also be observed in these figures. Taking Figure 4 (b) for example, for constant stress condition, at a certain R/B a larger value of R/a resulting from higher suction is associated with a higher value of B/a . And the larger R/a is associated with smaller value of $\sigma_L/\sigma_{L,\infty}$. The opposite is for zero displacement condition, that higher suction results in a smaller R/a but a higher value of $\sigma_L/\sigma_{L,\infty}$. Again the effect is more obvious for larger e_0 and higher p_{net0} .

Figure 4. Constant stress and zero displacement boundary effects on spherical cavity expansion results for various initial mean effective stresses, (a) $e_0 = 0.66$, (b) $e_0 = 0.58$, (c) $e_0 = 0.51$ and (d) $e_0 = 0.43$. Dashed lines indicate results at $s_0 = 160$ kPa.

6 CONCLUSIONS

This paper presents a generalized solution for a spherical cavity expanding in unsaturated soils of finite radial extent subjected to two boundary conditions. The finite radial extent is represented by the value of R/B (radius ratio of plastic region to finite radial extent) which is taken as a shape factor. The effects of boundary conditions under different values of shape factor are reflected by the values at the elastic-plastic boundary and thus the initial values for the solution in the plastic region.

The role of suction on the boundary size effect is examined and is found to be significant, especially at higher initial mean net stress and at higher initial void ratio. This suggests the soil mass with a higher suction value are under more influence of the boundary size effect.

REFERENCES

- 1) Chen S.L and Abousleiman Y.N. (2013): Exact drained solution for cylindrical cavity expansion in modified Cam Clay soil. *Geotechnique* 2013, 63(6), 510-517.
- 2) Cheng, Y., Yang, HW and Sun DA. (2018): Cavity expansion in unsaturated soils of finite radial extent. *Computers and Geotechnics*, 102, 216-228.
- 3) Khalili N. and Khabbaz, M.H. (1998): Unique relationship for χ for the determination of the shear strength of unsaturated soils. *Geotechnique*, 48(5), 681-687.
- 4) Russell AR, Buzzi O. (2012): A fractal basis for soil-water characteristics curves with hydraulic hysteresis. *Geotechnique*, 62(3), 269-274.
- 5) Russell AR. (2014): How water retention in fractal soils depends on particle and pore sizes, shapes, volumes and surface areas. *Geotechnique*, 64(5), 379-90.
- 6) Salgado R, Mitchell JK, Jamiolkowski M. (1997) Cavity expansion and penetration resistance in sand. *J Geotech Geoenviron*, 123(4), 344-54.
- 7) Pournaghiazar M, Russell AR, Khalili N. (2013): Drained cavity expansions in soils of finite radial extent subjected to two boundary conditions. *International Journal for Numerical and Analytical Methods in Geomechanics*, 37(4), 331-352.
- 8) Yang, H and Russell, A.R. (2015): Cavity expansion in unsaturated soils exhibiting hydraulic hysteresis considering three drainage conditions. *International Journal of Numerical and Analytical Methods in Geomechanics* 2015, 39, 1975-2016.
- 9) Yu, H.S., Schnaid, F. and Collins, I.F. (1996): Analysis of cone pressuremeter tests in sands. *Journal of Geotechnical Engineering ASCE*, 122(8), 623-632