Viscoelastic analytical solution for shallow tunnel considering time-dependent displacement boundary

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ABSTRACT

A time-dependent unified displacement function is proposed to capture main deformation responses of the tunnel cross-section. When the expansion order n equals 2, this complete displacement mode is used as displacement boundary condition (DBC) of the tunnel cross-section. Complex variable method is adopted in order to obtain an elastic analytical solution. Considering the time effect of ground mechanical parameters and tunnel cross-section deformation, a viscoelastic analytical solution of ground displacement is obtained. By parametric analyses, time effects of settlement trough curve, ground displacement and the settlement at the center point of ground surface are analyzed.

Keywords: unified displacement function, complex variable method, time effect of ground mechanical parameters, viscoelastic analytical solution of ground displacement, ground deformation analysis

1 INTRODUCTION

Most types of geomaterials exhibit time-dependent behaviors, which induce gradual deformation over time (Malan 2002; Wang et al., 2015). The deformation caused by time effect may account for 70% of the total deformation (Sulem et al., 1987). Therefore, time effects should be considered in the tunnel engineering.

For analytical solution of viscoelasticity, ground time-dependent behaviors can be accounted for by the linear viscoelastic models. Correspondence principle is a common method to obtain viscoelastic analytical solutions. The method is to develop the obtained elastic solutions into viscoelastic analytical solutions by use of Laplace transformation and inverse transformation. Most viscoelastic work focuses on deep tunnels so far (Sulem et al., 1987; Dai et al., 2004; Nomikos et al., 2011; Fahimifar et al., 2010; Wang et al., 2017) and the viscoelastic analytical solutions of shallow tunnels have been rarely covered. Wang et al., (2018) obtained analytical solution of elastic and viscoelastic when the ground surface of shallow tunnel and the tunnel cross-section subject to the stress loads. However, a viscoelastic analytical solution of a shallow tunnel is rarely considered when the tunnel cross-section boundary is the displacement.

The complex variable method is a common method

to solve elastic problems. Since Verruijt (1997; 1998) creatively obtained the elastic analytical solution of the shallow tunnel by the boundary conditions and the convergence of the series, the complex variable method (Wang et al., 2009; Fu et al., 2015a, 2015b; Fang et al., 2015; Lu et al., 2016; Zhang et al., 2018) is in a quite wide use solving the elastic analytical solution of shallow tunnel in recent years. A reasonable boundary condition is the key to obtaining an elastic solution. Compared with stress, the displacement around the tunnel is monitored in an easier way in the practical project. Hence more reasonable DBC of the tunnel cross-section are constantly proposed. In early studies of the analytical solution of tunnels, the proposed deformation modes only reflect asymmetrical deformation about horizontal center line and fail to reflect asymmetrical deformation about vertical center line (Verruijt, 1997; Loganathan and Poulos, 1998; Bobet, 2001; Park, 2004; Zhang and Huang, 2012; Fu et al., 2015a; Zhang et al., 2017). In our previous studies, DBC of the tunnel cross-section is expressed by the Fourier series (Kong et al., 2019a, 2019b, 2020). When the Fourier series expansion order takes 2, this unified displacement function reflects asymmetrical deformations about both horizontal and vertical center line of the tunnel cross-section simultaneously.

A time-dependent unified displacement function is proposed in this work. The viscoelastic analytical solution is obtained, which can dynamically describe the deformation behaviors of the tunnel cross-section with time. Based on the obtained elastic solution and the time-dependent unified displacement function, the viscoelastic analytical solution is obtained by the correspondence principle. When the ground rheology conforms to the Generalized Kelvin model, the field of time-dependent ground displacement is analyzed.

2 DISPLACEMENT FUNCTION OF THE TUNNEL CROSS-SECTION

With the economic development, tunnel cross-sections with various shapes are needed and constructed, such as circular tunnels, rectangular tunnels, horseshoe tunnels and so on. The contour line functions of tunnels with various shapes at different time, are 2π periodic functions. □ □

The deformation function of the tunnel cross-section at different time is as follows,

$$u(\theta', t) = -u_0(t) + \sum_{n=1}^{\infty} \left[a'_n(t) \sin(n\theta') + b'_n(t) \cos(n\theta') \right] \quad (1)$$

Shield tunnels are typically circular and shallowly buried. This work focuses on the analysis of initially circular cross-section, as shown in Fig. 1. Point o' is the center of the circle.

Fig. 1. Initial contour line and contour lines at different time.

When n equals 2, Eq. (1) can be simplified as,

$$u_2(\theta', t) = -u_0(t) + a'_1(t) \sin \theta' + b'_1(t) \cos \theta' + a'_2(t) \sin 2\theta' + b'_2(t) \cos 2\theta' \quad (2)$$

At any moment, five basic deformation modes can be used to reflect main deformation behaviors of circular tunnel cross-section (Kong et al, 2019a, 2019b). Each of the five items changes independently with time.

The five basic deformation modes are as follows: u_0 is the value of uniform convergence due to ground loss and it is related to two factors: the radius and the

volume loss of the cross-section of the tunnel. $a'_1 \sin \theta'$ in Eq. (2) reflects the overall vertical movement mode of the tunnel cross-section. $b'_1 \cos \theta'$ reflects the horizontal movement mode of the tunnel cross-section. $a'_2 \sin 2\theta'$ reflects oblique ovalization deformation mode. $b'_2 \cos 2\theta'$ reflects the tunnel horizontal and vertical ovalization deformation.

The asymmetrical deformation of the tunnel cross-section about the horizontal axis can be reflected by $a'_1 \sin \theta'$ and $a'_2 \sin 2\theta'$, while the asymmetrical deformation of it about the vertical axis can be reflected by $b'_1 \cos \theta'$ and $a'_2 \sin 2\theta'$. Therefore, the main deformation behaviors of the tunnel cross-section are able to be reflected by virtue of the five basic deformation modes at any moment.

3 VISCOELASTIC ANALYTICAL SOLUTION

The time-dependent ground displacement behaviors can be reflected by time-dependent ground mechanical parameters and time-dependent DBC of the tunnel cross-section. Eq. (2) can be used as DBC to reflect time-dependent deformation of the tunnel cross-section. By viscoelastic constitutive equations, time-dependent ground mechanical parameters can be obtained.

3.1 Elastic analytical solution for shallow tunnel

The mechanical model of the shallow circular tunnel is simplified as shown in Fig. 2. Assumed that (1) the surrounding rock is the homogeneous isotropic elastic material; (2) it is considered as the plane strain condition. For some moment t , γ is the weight of the material per unit volume; E and ν are the Young elastic modulus and Poisson ratio, respectively. h is the depth of the center of the tunnel.

Fig. 2. Mechanical model of shallow tunnel excavation.

In the z plane, the upper boundary ($z = \bar{z}$) is assumed to 0 stress.

$$z = \bar{z}: \quad \varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)} = 0 \quad (3)$$

The boundary condition of the tunnel cross-section ($|z + ih| = r_0$) is expressed by the following relationship, where i is the imaginary unit and κ equals $3 - 4\nu$.

$$|z + ih| = r_0: \quad 2G(u_x + i u_y) = \kappa \varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)} \quad (4)$$

When “Buoyancy effect” is considered in analytical

solutions, $\varphi(z)$ and $\psi(z)$ are written respectively,

$$\varphi(z) = -\frac{F_x + iF_y}{2\pi(1+\kappa)}[\kappa \log(z - \bar{z}_c) + \log(z - z_c)] + \varphi_0(z) \quad (5)$$

$$\psi(z) = \frac{F_x - iF_y}{2\pi(1+\kappa)}[\log(z - \bar{z}_c) + \kappa \log(z - z_c)] + \psi_0(z) \quad (6)$$

The resultant force on the circle is expressed as $F_x + iF_y$. w_t describes the ratio of the average unit weight of the tunnel to the average soil weight. F_y equals $(1-w_t)\gamma\pi r_0^2$ and F_x equals 0. z_c is the point making ζ_c equal 0 at point o of ζ plane. The value of it is $-ai$.

As shown in Fig. 2, the region R is mapped onto the circle ring by the following conformal transformation function (Verruijt, 1997), from the z plane to the ζ plane.

$$z = w(\zeta) = -ia \frac{1+\zeta}{1-\zeta} \quad (7)$$

where $a = h \frac{1-\alpha^2}{1+\alpha^2}$ and $\frac{2\alpha}{1+\alpha^2} = \frac{r_0}{h}$.

$\varphi_0(\zeta)$ and $\psi_0(\zeta)$ are represented by the Laurent series expansions,

$$\varphi_0(\zeta) = a_0 + \sum_{k=1}^{\infty} a_k \zeta^k + \sum_{k=1}^{\infty} b_k \zeta^{-k} \quad (8)$$

$$\psi_0(\zeta) = c_0 + \sum_{k=1}^{\infty} c_k \zeta^k + \sum_{k=1}^{\infty} d_k \zeta^{-k} \quad (9)$$

$a_0, b_0, a_k, b_k, c_k, d_k$ are obtained by Lu et al. (2020)

3.2 Viscoelastic analytical solution of ground displacement

Based on the obtained elastic analytical solution, the correspondence principle is usually adopted to obtain the viscoelasticity analytical solution. If material parameters are directly transformed by Laplace transformation and inverse transformation, the shear relaxation modulus of the Generalized Kelvin model can be obtained for Eq. (10),

$$G(t) = \frac{G_M G_K}{G_M + G_K} + \frac{G_M^2}{G_M + G_K} e^{-\frac{G_M + G_K}{\eta_K} t} \quad (10)$$

Here, G_M is the shear modulus of the spring of the elastic component. G_K is the shear modulus of the spring of viscous component and η_K is the viscosity coefficient. In particular, when t equals 0, $G_0 = G_M$. In the Generalized Kelvin model, G_M denotes instantaneous shear modulus.

The bulk modulus K is generally considered not to change over time (Feng et al., 2006; Wang et al., 2018). $\kappa(t)$ can be expressed by,

$$\kappa(t) = 3 - 4\nu(t) = \frac{3K + 7G(t)}{3K + G(t)} \quad (11)$$

When the soil is under undrained conditions, bulk modulus is infinitely large. Namely, $\nu(t)$ equals 0.5 and $\kappa(t)$ equals 1.

The elastic analytical solution of displacement can be obtained in section 3.1. The steps of the viscoelastic analytical solution should be: Laplace transform is carried out for time-dependent material parameters and time-dependent boundary conditions; then combining the Eq. (3) ~ Eq. (9) and Eq. (12), Laplace inverse transformation is conducted to obtain the viscoelastic analytical solution of displacement.

a_k and b_k in elastic analytical solution are as follows.

$$\left. \begin{aligned} (1+\kappa\alpha^{2k+2})a_{k+1} + (1-\alpha^2)(k+1)\bar{b}_{k+1} &= \alpha^2(1+\kappa\alpha^{2k})a_k \\ &+ (1-\alpha^2)k\bar{b}_k + A_{k+1}\alpha^{k+1} + B_{k+1} - B_k\alpha^2, \quad (k=1,2,\dots) \\ (1-\alpha^2)\alpha^{2k}(k+1)a_{k+1} - (\kappa+\alpha^{2k+2})\bar{b}_{k+1} &= \\ (1-\alpha^2)\alpha^{2k}ka_k - (\kappa+\alpha^{2k})\bar{b}_k + \bar{A}_k\alpha^k &+ \bar{B}_k\alpha^{2k} - \bar{B}_{-(k+1)}\alpha^{2k+2}, \quad (k=1,2,\dots) \\ (1-\alpha^2)\bar{a}_1 - (\kappa+\alpha^2)b_1 &= \bar{A}_0 - (\kappa+1)a_0 + B_0 - \alpha^2B_{-1} \\ (1+\kappa\alpha^2)a_1 + (1-\alpha^2)\bar{b}_1 &= A_1\alpha + (\kappa+1)\alpha^2a_0 + B_1 - \alpha^2B_0 \end{aligned} \right\} \quad (12)$$

where, A_k and B_k are the coefficients of the series expansion of the boundary equations.

c_0, c_k and d_k are expressed by equations containing a_k and b_k by comparing the coefficients of like powers of σ (Kong et al, 2020). From the above analysis, the coefficients of $\varphi(\zeta)$ and $\psi(\zeta)$ can be determined.

In this work, the bulk modulus is assumed to be infinity so that ν equals 0.5 when the soil or surrounding rock is under undrain condition.

a_0 is expressed as follows,

$$a_0 = \frac{A_0 + B_0 - \alpha^2B_{-1} - (1-\alpha^2)\bar{a}_1 + (\kappa+\alpha^2)b_1}{(\kappa+1)} \quad (13)$$

When the value of k is determined and "Buoyancy effect" is not considered, B_k equals zero. A_0 and A_k are proportional to G . The time-dependent parameter G can be extracted from the coefficients of $\varphi(z)$ and $\psi(z)$ in the process of calculation.

The displacement is represented as follows,

$$u_x + iu_y = \frac{\kappa\varphi(z) - z\varphi'(z) - \bar{\psi}(z)}{2G} \quad (14)$$

From above equations, the time-dependent material parameter G does not need to be taken into account in Eq. (14) without considering "Buoyancy effect".

When the "Buoyancy effect" is taken into account,

the part containing “Buoyancy effect” is written as the item $f_{x,y}$ for simplicity of writing, which do not change with time. f_0 is an item that has nothing to do with G .

$$u_x(\zeta, t) + i u_y(\zeta, t) = \int_0^t H(t - \tau) f_{x,y} d\tau + \int_0^t f_0(\zeta, \tau) d\tau \quad (15)$$

$$H(t) = L^{-1} \left[\frac{1}{s \tilde{G}(s)} \right] = \frac{1}{G_M} + \frac{1}{G_K} - \frac{1}{G_K} \exp\left(-\frac{G_K}{\eta_K} t\right) \quad (16)$$

$L^{-1}[\cdot]$ represent carrying out Laplace transform and inverse transform for $[\cdot]$. s is a positive real number.

Combining time-dependent material parameters and the obtained elastic analytical solution, the viscoelastic analytical solution of ground displacement is obtained.

By the above method, the displacement is not equal 0 at infinity (Wang et al., 2009; Fu et al., 2015; Fang et al., 2015; Zhang et al., 2018). Verruijt (1997) concludes that the point at infinity could be considered to be the non-moving reference and suggests that a rigid body translation should be added to overcome the drawback. When the “Buoyancy effect” is considered in analytical solutions, the displacement is infinite at infinity owing to upward resultant force and the choice of a non-moving reference point is arbitrary. A point at a distance of five times the tunnel depth on the ground surface is set the vertical displacements to 0 (Strack and Verruijt, 2002; Verruijt and Strack, 2008). In this work, the horizontal translation is also carried out owing to the effects of the asymmetrical deformation terms (Kong et al., 2019). The final analytical solution of displacement consists of two parts: the first part is the displacement field caused by the assumed DBC; the second part is rigid body translation.

4 GROUND DISPLACEMENT FIELD ANALYSIS

Viscoelastic analytical solutions of ground displacement relate to time-dependent material parameters and DBC. In this work, the time effect of material parameters is adopted as expressed in Eq. (16). The time-dependent DBC of the tunnel cross-section is considered to be the following mode. Here, the tunnel cross-section is considered to be instantaneously excavated. The displacement law of the tunnel cross-section is assumed as,

$$u(\theta', t) = (-u_0 + a'_1 \sin \theta' + b'_1 \cos \theta' + a'_2 \sin 2\theta' + b'_2 \cos 2\theta') \lambda(t) \quad (17)$$

where, $\lambda(t) = 1 + t / (l_a + l_b \cdot t)$; l_a and l_b are the parameters of function. When t equals 0, $\lambda(t)$ equals 1 and it corresponds to the moment that the whole tunnel cross-section is just excavated completely.

The DBC of the tunnel cross-section is designed when $l_a=3$ and $l_b=1$,

$$u(\theta', t) = (-0.05 - 0.05 \sin \theta' - 0.05 \cos \theta' + 0.05 \sin 2\theta' + 0.05 \cos 2\theta') \lambda(t) \quad (18)$$

The time-dependent contour of the tunnel cross-section is shown in Fig. 3. The displacement of the tunnel cross-section changes dynamically with time. After 100 Day, the DBC tends to be stable.

Fig. 3. Contours of the tunnel cross-section at different time.

Geometric parameters for the tunnel are considered to be: $r_0=3\text{m}$, $h=10\text{m}$; the mechanical parameters of surrounding soft clay with undrained behavior: $\gamma=17.64\text{kN/m}$, $w_t=0.5$. The Generalized Kelvin viscoelastic model is adopted with: $G_M = 20\text{MPa}$, $G_K = 10\text{MPa}$, $\eta_K = 100 \text{ MPa Day}$, and $\nu(t)=0.5$.

Ground displacement on 0 Day, 2 Day, 10 Day, 30 Day, 100 Day and 200 Day are shown in Fig. 4. The displacement vector arrows represent displacement relative values and displacement directions.

Fig. 4. Cloud charts of ground displacement.

The ground displacement in Fig. 4, the settlement trough curve of the ground surface in Fig. 5 and the central point settlement in Fig. 6 firstly increase rapidly, then increase slowly and finally approach a stable value as t increases. The asymmetrical deformation is formed about the vertical centerline of the tunnel as shown in Eq. (18) due to coupling effects of five deformation parameters. The maximum vertical displacement value of the ground surface is located at the left side of center point, which is determined by DBC. The uplift of the ground surface tends to be stable after 10 Day. The ground surface central point settlement increases rapidly before about 50 Day. The ground displacement mainly takes place on the upper left side of

cross-section of the tunnel and gradually expands to the ground surface as the time increases.

Fig. 5. Ground surface settlement trough curve.

Fig. 6. Ground surface center point o settlement curve.

5 CONCLUSIONS

The Fourier series is adopted to represent the DBC of the tunnel cross-section at any time. When series expansion order n takes 2, liner combinations of these five deformation modes can be used to present main deformation behaviors of circular tunnel cross-section under complex geological conditions and construction methods. In particular, this complete deformation mode can reflect asymmetrical deformation not only about the vertical but also about the horizontal center line of the cross-section of the tunnel at any time.

Time-dependent ground displacement behaviors can be reflected by time-dependent ground mechanical parameters and time-dependent DBC. The complex variable method is adopted to obtain the elastic analytical solution when a complete deformation mode is employed as the DBC of the tunnel cross-section. By means of correspondence principle, the viscoelastic analytical solution is obtained. When coupling effects of time-dependent material parameters and DBC are considered, settlement trough curve, ground displacement and the settlement of the ground surface center point firstly increase rapidly, then increase slowly and finally approach a stable value with the increase of time.

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