A constitutive model related to the fabric for sand

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ABSTRACT

In order to better simulated the anisotropy of granular materials, the anisotropic state variable was introduced into the micropolar hypoplastic constitutive model, and the simulation of biaxial compression tests were carried out. The influences of fabric on the width and inclination of the shear band were studied by the FEM and DEM. The following results were found. With the increase of anisotropic state variables, at the same vertical strain, the width and inclination of the shear band remained basically unchanged and the element rotation angle at the same position was increased.

Keywords: micropolar, hypoplastic, fabric, anisotropy, shear band

1 INTRODUCTION

The geotechnical materials composed by the aggregation of various grains. The materials have obvious anisotropy in physics and have important failure precursors, such as strain localization (He *et al*, 2017; Shi and Guo, 2018; Vairaktaris *et al*, 2018). In the hypoplastic constitutive model, micropole theory is introduced to describe the microscopic effects of granular materials. The rotation of particles can be described by micropole theory, which has the advantages of simple and definite parameters (Mindlin, 1964). The fabric tensors can effectively describe the anisotropy of granular materials. Therefore, it had a great theoretical and practical significance that the hypoplastic constitutive model considering fabric was extend to the micropolar form.

Zhiwei Gao and Jidong Zhao (2013) established a plane strain model with fabric by using finite element method (FEM). Sun *et al* (2013), Fu and Dafalias (2011), Hasan and Alshibli (2012) used the discrete element method (DEM) to analyze the movement of grains in the shear bands. Lin and Wu (2015) respectively used micropolar hypoplastic constitutive model and DEM to simulate simple shear test and biaxial compression test.

In order to applied the micropolar hypoplastic constitutive model with fabric to granular materials, this paper firstly introduced the constitutive correlation function into the hypoplastic constitutive model, and

then, extended the hypoplastic constitutive model with fabric to the micropolar.

2 HYPOPLASTIC CONSTITUTIVE MODEL WITH FABRIC BASED ON MICROPOLAR THEORY

In order to understand the effect of granular materials fabric on strain localization, an anisotropic hypoplastic constitutive model of micropolar term considering fabric was established. In this model, the relevant parameters of the hypoplastic constitutive and the fabric were obtained by DEM, and then the simulations were calculated by FEM.

2.1 Micropolar hypoplastic constitutive model with fabric

In this paper, the anisotropic state variables (**A**) and Wu-Bauer hypoplastic model (Eq. (1)) were combined to establish the anisotropic hypoplastic constitutive model considering the fabric, and then the model was extended to the form of micropole (Wu *et al*, 2017). **A** are described by the fabric tensors (**F**) and **σ**. The transformation from isotropy to anisotropy was realized with the **A**. The process was as follows:

First, the components of specimen **F** (F_{ij}) and **A** (A_i) were calculated (Oda and Nakayama, 1989). The functional relationship between A_i and F_{ij} was shown in Eq. (2) (Li and Yuan, 2015).

$$\dot{\sigma} = c_1 \text{tr}(\sigma) \dot{\epsilon} + c_2 \text{tr}(\dot{\epsilon}) \sigma + c_3 \frac{\text{tr}(\sigma \dot{\epsilon})}{\text{tr}(\sigma)} \sigma + c_4 (\sigma + \sigma^*) \|\dot{\epsilon}\| \quad (1)$$

where $\dot{\sigma}$ is Jaumann stress rate tensor, c_i ($i=1, 2, 3, 4$) is a dimensionless parameter, σ is the stress tensor, $\dot{\epsilon}$ is strain rate tensor, σ^* is the deviatoric stress tensor, $\sigma^* = \sigma - \text{tr}(\sigma)I/3$.

$$A = \frac{\sigma_{ij}}{\sigma_m} F_{ij} - A_0 \quad (2)$$

where σ_{ij} is the component of the σ , σ_m is the

component of the σ^* , A_0 is the datum point (the initial anisotropy state variable), the stress state at this point is consistent with the stress state when the specimen parameters were determined.

Then, by combining physical quantities such as porosity, material parameters of the critical state line equation ($f(e)$) and A with Eq. (1). The anisotropic hypoplastic constitutive model (Eq. (3)) considering the fabric was established. On the basis of Bauer critical state line (Bauer, 1995), Eq. (4) introduced the fabric related terms (Li and Yuan, 2015).

$$\dot{\sigma} = c_1 \text{tr}(\sigma) \dot{\epsilon} + c_2 \text{tr}(\dot{\epsilon}) \sigma + c_3 \frac{\text{tr}(\sigma \dot{\epsilon})}{\text{tr}(\sigma)} \sigma + c_4 f(e) (\sigma + \sigma^*) \|\dot{\epsilon}\| \quad (3)$$

$$f(e) = \left(\frac{e - e_{\min}}{e_{\Gamma} - \lambda_c (P/P_a)^\zeta + A - e_{\min}} \right)^a \quad (4)$$

where e is the porosity, $e_{\min}$ is the minimum porosity, e_{Γ} , λ_c and ζ are the material parameters that determine the critical state line, a can be measured by experiments (Wu and Kolymbas, 2000), e_{Γ} , λ_c , ζ and

a were respectively 0.934, 0.019, 0.7 and 0.1.

Finally, the Eq. 4 is extended into the form of micropole (Eq. (5) and (6)).

$$\dot{\sigma} = c_1 \text{tr}(\sigma) \dot{\epsilon} + c_2 \text{tr}(\dot{\epsilon}) \sigma + c_3 \frac{\text{tr}(\sigma \dot{\epsilon}) - \text{tr}(\mathbf{MK})}{\text{tr}(\sigma)} \sigma + c_4 f(e) \sqrt{\|\dot{\epsilon}\|^2 + l^2 \|\mathbf{K}\|^2} (\sigma + \sigma^*) \quad (5)$$

$$\dot{\mathbf{M}} = c_1 / \text{tr}(\sigma) \mathbf{K} + c_2 \frac{\text{tr}(\dot{\epsilon})}{l} \mathbf{M} + c_3 \frac{\text{tr}(\sigma \dot{\epsilon}) - \text{tr}(\mathbf{MK})}{l \text{tr}(\sigma)} \mathbf{M} + 2c_4 f(e) \sqrt{\|\dot{\epsilon}\|^2 + l^2 \|\mathbf{K}\|^2} \frac{\mathbf{M}}{l} \quad (6)$$

2.2 FEM stiffness matrix

The FEM stiffness matrix was obtained by partial differentiation for Eq. (5) and (6). The FEM stiffness matrix was 7×7 matrix, that was, $\mathbf{S} = \mathbf{H}(\sigma, \dot{\epsilon}, \mathbf{M}, \mathbf{K})$ could be expressed as Eq. (7).

By the time step Δt , the next stress state could be expressed as Eq. (8).

$$\mathbf{S} = \begin{bmatrix} \frac{\partial \dot{\sigma}_{ij}}{\partial \dot{\epsilon}_{mn}} (5 \times 5) & \frac{\partial \dot{\sigma}_{ij}}{\partial \mathbf{K}_{mn}} (5 \times 2) \\ \frac{\partial \dot{\mathbf{M}}_{ij}}{\partial \dot{\epsilon}_{mn}} (2 \times 5) & \frac{\partial \dot{\mathbf{M}}_{ij}}{\partial \mathbf{K}_{mn}} (2 \times 2) \end{bmatrix} \quad (7)$$

$$\sigma_{t_{n+1}} = \sigma_{t_n} + \mathbf{H}(\sigma, \dot{\epsilon}, \mathbf{M}, \mathbf{K}) \Delta t \quad (8)$$

where $\sigma_{t_{n+1}}$ is the stress tensor of the next time step, σ_{t_n} is the stress tensor of the current time step.

3 THE NUMERICAL SIMULATION

3.1 Calculation model

In this paper, two plane strain biaxial compression test models were established. The DEM models height (H) and the width (W) were 70mm and 35mm respectively (Fig. 1(a)). The FEM models H and W were 48mm and 16mm respectively (Fig. 1(b)). The relevant parameters of the DEM models and FEM models were shown in table 1 and 2.

Fig. 1. The two calculation models and corresponding relationship.

Table 1 The relevant parameters of the DEM model.

density/ ρ (kg/m ³)	modulus of elasticity/ E (MPa)	normal contact stiffness/ K_n (N/m)	granular normal to tangential stiffness ratio/ (K_n/K_s)	damping coefficient	coefficient of friction between grains/ f_g	coefficient of friction between grain and wall/ f_w
1600	20	1×10^8	1.00	0.7	0.4	0

Table 2 The relevant parameters of the FEM model.

E /(MPa)	poisson ratio / μ	friction angle / φ (°)	c_1	c_2	c_3	c_4
20	0.25	45	-53.33	-234.55	-210.29	-105.90

3.2 Calculation results

Fig. 2 shown the distribution of α in schemes 1~4 at different ε_v . Fig. 3 shown the evolution of the average stress ratio n (the average the maximum principal stress (σ_1) over the mean minimum principal stress (σ_3) at the top of the FEM model with the ε_v in schemes 1~4. Fig. 4 shown the evolution of rotation angle (α_m) of monitoring points with the ε_v in schemes 1~4. The monitoring points in schemes 1~4 were the same, and

the monitoring points position are the element with the maximum shear stress in scheme 1 when $\varepsilon_v=2.8\%$.

Fig. 2 shown that, in terms of width (w_s) and inclination of the shear band (θ), at the same ε_v , with the increase of A_i , θ and w_s were basically unchanged. For example, when $\varepsilon_v=4.4\%$, the θ is about 50° , and w_s was about 9mm in schemes 1~4 (Fig. 2(e)~(h)). In terms of the dilatation phenomenon of the FEM model, with the increase of A_i , θ and w_s were basically unchanged.

Fig. 2. The distribution of α in schemes 1~4.

Fig. 3 shown that, the curves of n - ε_v for schemes 1~4 could be divided into three stages: elastic stage, hardening stage and softening stage.

Fig. 3. The curves of n - ε_v .

In the hardening stage, with the increase of A_i , the peak values of the curves of n - ε_v increased in schemes 1~4. In the softening stage, with the increase of A_i , the evolution of the curves of n - ε_v in schemes 1~4 was basically the same, and the slopes after the peak of the curves of n - ε_v were basically the same. The reason was the fact that shear band has been formed and the FEM model was fully softened.

Fig. 4 shown that, the curves of α_m - ε_v of FEM model for schemes 1~4 could be divided into three stages: approximate 0 stage, nonlinear stage and linear stage. In the nonlinear stage, with the increase of A_i , the α_m of schemes 1~4 increased at the same ε_v . This indicates that the higher degree of anisotropy had the faster relative motion speed of the hanger and dropped side of the shear band in the process of forming shear band.

Fig. 4. The curves of α_m - ε_v .

4 CONCLUSIONS

The FEM simulation of the biaxial compression

plane strain models were carried out, and the following conclusions were obtained:

(1) In order to better simulated the anisotropy of granular materials, the correlation function of fabric was added into the micropolar hypoplastic constitutive model, and the micropolar hypoplastic constitutive model considering the fabric was established.

(2) The curves of average stress ratio of the upper end of the FEM model vertical strain can be divided into three stages: elastic stage, hardening stage and softening stage.

(3) With the increase of anisotropic state variables, at the same vertical strain, the width and inclination of the shear band remained basically unchanged and the rotation angle at the same position increased.

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