Verification of an Efficient Monte Carlo Model that Interchanges the Geometric Shapes of Plane Source and Detector in Radiation Transport Calculation

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An efficient Monte Carlo computational model has been used to calculate the radiation transport when radioactive materials are deposited on an infinite plane surface, in which the geometrical shapes of the plane source and the point detector are interchanged. This transformation was verified by comparing the calculation results with the theoretical values only for the primary (unscattered) photons. It is desirable to confirm the validity of the transformation for both unscattered and scattered photons. The extension of the transformation to a finite plane source has not been verified yet. In this study, we performed calculations with and without transformation via the Monte Carlo method, and the results were compared with each other, then confirming the validity of the transformed model.

KEY WORDS: calculation, radionuclide, gamma ray, primary (unscattered) photons, scattered photons, photon fluence rate, Monte Carlo method.

I INTRODUCTION

The uniform deposition of radioactive materials on an infinite plane is a simple and idealized model for evaluating the environmental influence of radioactivity,1, 2 and it has been applied widely in in situ gamma ray spectrometry.3 The relationships between the gamma ray energy spectrum 1 m above the ground and dose rates such as air kerma and ambient dose equivalent have been calculated for monoenergetic sources ranging from 10 keV to 5 MeV or main radionuclides using this model and summarized in literature as conversion coefficients.4–9

In a realistic calculation model using the Monte Carlo code such as MCNP10 and PHITS,11 a sphere of diameter approximately 80 cm to 140 cm is placed as a detector at the evaluation position,1, 2 and the number of particles arriving at the detector is counted for each energy bin. As the area of the contaminated ground surface increases, the ratio of the number of particles (including scattered particles) reaching the detector rapidly decreases when compared with the number of particles emitted from the contaminated surface, and hence, the calculation efficiency becomes extremely low.

KASTLANDER and BARGHOLTZ13 indicated that any position of the detector at height \( h \) above an infinite plane source has the same photon fluence rate; similarly, the total flux of photons at height \( h \) is the same from any part of the plane source. Subsequently, they feasibly transformed the geometry for a Monte Carlo calculation and performed the calculation of the photon fluence rate and the distribution of energy from a point source reaching an infinite horizontal surface (using a circular surface with a diameter of 18 km) at the height of 1 m.

LEMERCIER et al.8 described in detail an efficient Monte Carlo method for calculating the energy spectrum of the photon fluence rate for an infinite plane with uniform contamination in accordance with KASTKANDER and BARGHOLTZ13: a realistic system consisting of an infinite plane source and a point detector is equivalent to a transformed system consisting of a point source and an infinite plane detector. NAMITO et al.14 claimed that a realistic model and its transformed model were mathematically equivalent. Although they confirmed that the Monte Carlo calculation results obtained using the transformed model were consistent with the theoretical solutions for unscattered photons, they did not provide the calculation results for the contribution of both unscattered and scattered photons, which corresponded to the actual dose. SAITO and JACOB,5, 6 LEMERCIER et al.,9 SAITO and PETOUSSI-HENSS,9 HIRAYAMA et al.,15 and MALINS et al.16 used the transformed model to calculate the environmental radiation dose.

In order to complement and solidify the efficient Monte Carlo model, it is desirable to present and compare the results calculated by considering both unscattered and scattered photons using the transformed model and the realistic model. It is also desirable to verify whether the transformed model is valid for a finite plane source. The behavior of photons that have traveled through media many times the mean free path is much more complicated than that of unscattered photons:
scattering allows a photon to reduce its energy and change its direction; a greater probability of photons being subjected to scattering results in a broadband of the energy spectrum of the photon fluence rate.

In this study, we verify the transformed model by comparing its results with those obtained using the realistic model for infinite and finite cases.

II CALCULATION PROCEDURE

1. Efficient calculation method by interchanging the geometric shapes of plane source and detector

(1) Primary photon fluence rate

LEMERCIER et al.\(^8\) and NAMITO et al.\(^14\) already mathematically described the similarity of the realistic model and the transformed model. We concisely present relevant formulas for the convenience of readers.

Firstly, consider a situation in which radionuclides are uniformly deposited on a plane circular ground surface with a radius \(x\) (m) as depicted in Fig. 1 (a). The fluence rate \(\phi\) (m\(^{-2}\) s\(^{-1}\)) of the primary photons reaching the detector located at height \(h\) (m) from the center of the ground surface is expressed by:

\[
\phi = \frac{\eta A_0}{2} \int_{\theta_0}^{\pi/2} \tan \theta e^{-\mu_x h / \sin \theta} d\theta = -\frac{\eta A_0}{2} (Ei(-\mu_x h) - Ei(-\mu_x h b)) (1)
\]

\[
\Theta = \tan^{-1}(x/h)
\]

\[
h = \sec \Theta
\]

where \(A_0\) (Bq m\(^{-2}\)) is the radioactivity per unit area of the radionuclide of interest, \(\eta\) is the yield of decay, \(\mu_x\) (m\(^{-1}\)) is the linear attenuation coefficient of air, and \(\theta\) indicates the incident angle of a photon on the point detector about the vertical axis. \(Ei(-\mu_x h)\) is the exponential integral.\(^17\)

Secondary, consider the system depicted in Fig. 1 (b), where the plane source is replaced with a point source, and the point detector is replaced with a plane detector having the same area as the plane source. The total number of photons per unit time, \(\pi x^2 \eta A_0\), corresponds to those emitted from the plane source; they are concentrated on the point source from which photons are isotropically emitted. The number of photons \(dN\) released per unit time within the solid angle consisting of zenith angle \(\theta \sim \theta + d\theta\), and azimuth angle \(0 \sim 2\pi\) is represented by:

\[
dN = \pi x^2 \eta A_0 \left( \frac{1}{2} \sin \theta d\theta \right)
\]

(4)

Taking account of the incident angle of a photon onto the plane detector, \(\theta\), and the attenuation rate while a photon travels from the point source to the plane detector, the number of photons incident per unit area and unit time on the plane detector, i.e., the photon fluence rate, is expressed by:

\[
\phi = \frac{\eta A_0}{2} \int_{\theta_0}^{\pi/2} \frac{\sin \theta}{\cos \theta} e^{-\mu_x h / \sin \theta} d\theta
\]

(5)

As Equation (5) is the same as Equation (1), in the case of primary photons, the shapes of the source and the detector can be interchanged.

(2) Fluence rates of primary and scattered photons

NAMITO et al.\(^14\) reported that the exchange of the shapes of the source and the detector holds even in the case where scattering exists as: “the structure of the shield is not changed in the transformation; therefore, the buildup factor of the realistic case equals that of the transformed case.” Although they confirmed the effectiveness of the transformation by comparing the Monte Carlo calculation results with the theoretical values for unscattered photons, they did not provide the calculation results for both unscattered and scattered photons. LEMERCIER et al.\(^8\) also confirmed the effectiveness only for unscattered photons.

In this study, we calculate the energy spectrum of the photon fluence rate for a case with both unscattered and scattered photons using the transformed and realistic models via the Monte Carlo particle and heavy ion transport calculation code PHITS.\(^1\) Then, we compare the calculation results, and thereby, confirm the validity of the transformed model.

2. Calculation domain and calculation conditions for an infinite plane source

The cylindrical calculation domain consisted of an air layer of thickness 600 m and a soil layer of thickness 1 m with a radius of 600 m. The radius of the contaminated ground surface was 600 m (the area is sufficiently large to assume the contaminated ground to be infinite), and a spherical detector with a diameter of 40 cm was placed 1 m above the ground at the center of the ground surface. It was assumed that \(^137\)Cs was uniformly deposited on the ground surface with 1 kBq m\(^{-2}\).

Table 1 presents the values used for the composition and
The linear attenuation coefficients of soil and air were 7.68 m\(^{-1}\) and 0.00925 m\(^{-1}\), respectively, calculated from the composition and density using the radiation property calculation code. In the transformed model, the plane source was replaced with a point source at the center of the ground surface, and the spherical detector was replaced with a circular detector plane with a radius of 600 m.

3. Influence of the diameter of the spherical detector

In the realistic model, it is conceivable that the value of the photon fluence rate reaching the point detector varies depending on the size of the spherical detector installed at the evaluation position. We investigated the effect of the sphere size on the photon fluence rate and the air kerma rate by varying the diameter of the spherical detector from 20 cm to 120 cm.

Figure 2 shows the variation of the primary photon fluence rate with the diameter of the spherical detector. Its theoretical value for the point detector is 1,751 m\(^2\) s\(^{-1}\), calculated using Equation (1) with the yield of decay of 0.851. As the spherical detector becomes large, the primary photon fluence rate increases. The air kerma rate shows a similar tendency, as shown in Fig. 3. The air kerma rate (2.54 nGy h\(^{-1}\)) when the diameter of the spherical detector is 40 cm is slightly larger (0.1\%) than the air kerma rate when extrapolated to a diameter of 0 cm. In this study, we ignored the difference owing to the spherical detector size and adopted the results in the case of using the spherical detector with a diameter of 40 cm.

III RESULTS AND DISCUSSION

1. Infinite plane source

Figure 4 shows the energy spectrum of the photon fluence rate at the evaluation position when \(^{137}\)Cs is uniformly deposited on the ground surface. A small peak appeared at the photon energy of 0.184 MeV is attributed to the backscattering caused by primary photons with 0.662 MeV (\(E_\gamma = E_\gamma / [1+((E_\gamma /0.511)(1-\cos\theta))=0.662/(1+(0.662/0.511)(1-\cos180°))]\), \(\theta\) : scattering angle). The calculation result of the transformed model is consistent with that of the realistic model.

Table 2 presents the calculation results of the primary photon fluence rate and the air kerma rate when the source depth is varied by covering the contaminated ground surface with clean earth. Values in parentheses denote fractional standard deviations. The theoretical values of the primary photon fluence rate were calculated using Equation (1) by

![Table 1](image)

**Table 1** Components of the soil and air employed for the photon transport calculation in the environment.

<table>
<thead>
<tr>
<th>Element</th>
<th>Soil Weight ratio [-]</th>
<th>Air Weight ratio [-]</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>0.006</td>
<td>N</td>
</tr>
<tr>
<td>O</td>
<td>0.492</td>
<td>O</td>
</tr>
<tr>
<td>Al</td>
<td>0.130</td>
<td>Ar</td>
</tr>
<tr>
<td>Si</td>
<td>0.236</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td>0.135</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>1,000 kg/m(^3)</td>
<td>Density</td>
</tr>
</tbody>
</table>

![Fig. 3](image)

**Fig. 3** Air kerma rate as a function of the diameter of the spherical detector.

![Fig. 2](image)

**Fig. 2** Primary photon fluence rate as a function of the diameter of the spherical detector.

![Fig. 4](image)

**Fig. 4** Energy spectrum of the photon fluence rate 1 m above the ground on which radionuclide \(^{137}\)Cs is uniformly deposited on a surface with 1 kBq m\(^{-2}\).
adding a soil layer. The results of the primary photon fluence rate calculated using the transformed and realistic models are consistent with each other, and the differences between the calculation results and the theoretical values are less than 0.5%. The differences in the air kerma rate between the transformed and realistic models are 0.5% or less.

Figure 5 shows the energy spectrum of the photon fluence rate for the source depth of 100 kg m$^{-2}$; the photon fluence rate decreases over the range of all photon energies in comparison with Fig. 4. During photons move through the soil with the weight depth of 100 kg m$^{-2}$, the chances of scattering increase remarkably, so the fraction of photons scattered among the photons reaching the evaluation point is 5 to 12 times, depending on the photon energy, compared to the case where $^{137}$Cs is deposited only on the ground surface. As a result, the number of photons scattered overall increases, and the presence of the small peak owing to the backscattering becomes unnoticeable. The calculation results of both models are in good agreement.

Thus, we can confirm that the transformed calculation model that interchanges the geometrical shape of the plane source and the point detector is applicable to the actual case with both unscattered and scattered photons.

2. Finite plane source

We investigate whether the transformation can also be applied to a finite plane source. The paths of the primary photons emitted from the plane source with a finite radius toward the point detector depicted in Fig. 6 (a) can be regarded as being analogous to the paths of the primary photons emitted from the center of the circular ground surface toward the plane detector depicted in Fig. 6 (b). Photons, except for primary photons, are scattered once or more in the soil and air before reaching the detector. We investigate whether the behavior of scattered photons reaching the plane detector in the

![Figure 5](image1.png)

**Figure 5** Energy spectrum of the photon fluence rate 1 m above the ground when the source ($^{137}$Cs with 1 kBq m$^{-2}$) is covered with clean earth of 100 kg m$^{-2}$.

![Figure 6](image2.png)

**Figure 6** Primary photon paths from the source to the detector.
Table 3: Comparisons of the primary photon fluence rate and air kerma rate between the transformed model and the realistic model with plane source radius varied.

<table>
<thead>
<tr>
<th>Source depth [kg m⁻²]</th>
<th>Radius of plane [m]</th>
<th>Primary photon fluence rate [m⁻² s⁻¹]</th>
<th>Air kerma rate [nGy h⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Transformed</td>
<td>Realistic</td>
</tr>
<tr>
<td>1</td>
<td>60</td>
<td>146 (0.08%)</td>
<td>147 (0.06%)</td>
</tr>
<tr>
<td>6</td>
<td>749 (0.05%)</td>
<td>749 (0.10%)</td>
<td>747</td>
</tr>
<tr>
<td>0</td>
<td>60</td>
<td>1,540 (0.08%)</td>
<td>1,541 (0.30%)</td>
</tr>
<tr>
<td>120</td>
<td>1,674 (0.03%)</td>
<td>1,673 (0.37%)</td>
<td>1,673</td>
</tr>
<tr>
<td>600</td>
<td>1,752 (0.11%)</td>
<td>1,753 (0.72%)</td>
<td>1,751</td>
</tr>
<tr>
<td>100</td>
<td>60</td>
<td>139 (0.01%)</td>
<td>137 (0.01%)</td>
</tr>
<tr>
<td>500</td>
<td>60</td>
<td>2 (1.3%)</td>
<td>2 (1.87%)</td>
</tr>
</tbody>
</table>

(): Fractional standard deviation.

The transformed model is consistent with that of photons reaching the spherical detector in the realistic model.

Table 3 presents the calculation results of the primary photon fluence rate and air kerma rate when the radius of the plane source surface is varied. Cases of sources being buried at the depths of 100 kg m⁻² and 500 kg m⁻² are also presented in order to confirm the validity in a strong scattered field in the limited area (radius of 60 m).

The results of both calculation models are consistent with each other. Thus, we conclude that the transformed model can also be applied to a finite plane source.

IV CONCLUSION

In this work, the energy spectrum of the photon fluence rate was calculated through the Monte Carlo method using the transformed model for a contaminated plane. The calculation results are consistent with those obtained using the realistic model. Thus, we conclude that the transformation in which a plane source surface is varied. Cases of sources being buried within the spherical detector in the realistic model.

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The results of both calculation models are consistent with each other. Thus, we conclude that the transformed model can also be applied to a finite plane source.

CONFLICT OF INTEREST DISCLOSURE

The authors indicated no conflicts of interest.

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