ORIGINAL

Saturation Correction for an Ionization Chamber in a Pulsed X-Ray Field for Medical Diagnosis

Keiji ODA*,*1, Takayoshi YAMAMOTO**,*2, and Masaharu KAWANISHI***,*1

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The collection efficiency, f -value, of a parallel-plate ionization chamber exposed to diagnostic X-rays has been numerically obtained. The results of calculations show that the f -value deviates from the well-known Boag's formula when the pulse duration becomes of the same order of the ion transit time between the electrodes. Furthermore, the effects of the pulse shape on the f -value and the output currents have been clarified.

The output currents and the saturation curves have been measured in fields generated by an X-ray tube for medical diagnosis. There found a good agreement between experimental and calculated results. The true f -value has been determined by the saturation curve method previously reported. It was concluded from the experimental confirmation of the inverse square law of the exposure that the method was useful for diagnostic X-rays.

KEY WORDS: saturation correction, ionization chamber, pulsed X-rays, medical diagnosis, collection efficiency, pulse duration, pulse shape, saturation curve, output current

I INTRODUCTION

From the stand-point of radiation protection, the absorbed dose in human body must be accurately evaluated, especially for X-ray diagnosis or radiotherapy with particle accelerators. For the measurement of the dose are often used thermoluminescence dosimeters, nuclear emulsions, scintillation detectors, and so on. It is necessary for the detectors to be calibrated beforehand by standard dosimeters. An ionization chamber is in common use as a reference dosimeter, because both the absorbed dose and the exposure are

proportional to the number of ion pairs liberated in irradiated materials.¹⁾ However, there are many correction factors for the absolute measurement.^{2,3)} Among them, the factor for ion collection is most important in an X-ray field of high exposure rate.

The positive and negative ions liberated by pulsed X-rays in an ionization chamber will drift in the opposite direction to each other in the electric field. Some of them disappear owing to the recombination during the ion transit. The collection efficiency, f -value, is defined as a ratio of the number of ions collected at the electrodes to that of ions initially produced by X-rays.^{4,5)} It is well known that the f -value decreases with increasing the exposure rate. For example, the f -value of a popular chamber of Radcon III (Victreen, 550-5) becomes 0.95 for 70 mR per pulse, 0.90 for 150 mR per pulse, 0.70 for 400 mR per pulse, and so on. In this situation, the absorbed dose will be underestimated unless the f -value is accurately evaluated.

It is fundamental and important to know the dependence of the f -value on various parameters, *i.e.* the exposure rate, the pulse duration, and the

* 小田啓二, **山本幸佳, ***川西政治

*1 The Institute of Scientific and Industrial Research, Osaka University; Mihogaoka, Ibaraki, Osaka 567.
大阪大学産業科学研究所; 大阪府茨木市美穂ヶ丘 (〒567)

*2 CERN (European Organization for Nuclear Research); CH-1211, Geneva 23, Switzerland (On leave of absence from the Institute of Scientific and Industrial Research, Osaka University).

ヨーロッパ合同原子核研究機関; ジュネーブ 23, CH-1211 (大阪大学産業科学研究所より出張)

chamber constants. Some formulations of the f -value have been performed by several authors.⁴⁻⁷⁾ Among them, Boag has introduced a most useful formula for pulsed X-rays.^{4,5)} The formula gives the same f -value so long as the total exposure per pulse is kept constant. It is considered, however, that the true f -value is generally a function of the respective values of the exposure rate and pulse duration. Thus, we have obtained the true f -value by numerical calculations of the rate equations of the ion density in a parallel-plate ionization chamber.^{8,9)} The results show that the true f -value deviates from the Boag's formula when the pulse duration becomes comparable to the ion transit time of the chamber or the exposure rate becomes extremely high. The former X-ray field is often found around the X-ray tube for medical diagnosis and the latter at the vicinity of the target bombarded by high energy particles for radiotherapy. In order to ascertain the fact, some experiments have already been carried out and is in progress.^{10,11)}

In this report, the effects of the pulse duration and the pulse shape on the collection efficiency of the ionization chamber are discussed. The experiments are also carried out in a pulsed X-ray field generated by an X-ray tube for medical diagnosis. The experimental results are compared with Boag's formula and our calculations, and thereby the collection efficiency is evaluated. It is the purpose to clarify some problems in the dose-evaluation in a field of diagnostic X-rays.

II COLLECTION EFFICIENCY FOR PULSED X-RAYS

Boag has introduced an excellent formula of the f -value for pulsed X-rays.^{4,5)} It is based on the two simplified assumptions: ion pairs liberated per pulse distribute uniformly in the chamber at the instant when an X-ray pulse ceases, and the density of positive ions is equal to that of negative ones in the overlap region where the recombination takes place. As was pointed out in the previous paper,⁸⁾ the assumptions become invalid when the pulse duration becomes comparable to the ion transit time of the ionization chamber. In order to clarify the fact, we introduce the following partial differential equations:

$$\frac{\partial v_{\pm}}{\partial s} = \Lambda(s) - Mv_{+}v_{-} \mp \frac{\partial v_{\pm}}{\partial \xi}. \quad (1)$$

The term of the left hand side represents the rate of increase in the ion density, and those of the right one represent the production, the recombination and the drift by the collecting field, respectively. All variables are normalized by appropriate constants to be dimensionless. Namely, $v_{\pm} = n_{\pm}/q_0$, $\Lambda(s) = q(t)/q_0$, $M = \alpha q_0 \tau^2$, $s = t/\tau$ and $\xi = x/d$, where $n_{\pm}$ are the ion densities, q_0 the ion production rate averaged over the pulse duration, $q(t)$ the time-variant production rate, α the recombination coefficient, d the gap distance between the electrodes and τ the ion transit time between the electrodes. Owing to the normalization, major parameters by which the f -value is determined are reduced to only two: one is the M -value, and the other is the normalized pulse duration, s_0 , which is defined here as a ratio of the pulse duration to the ion transit time. An example of the numerical calculations is shown in Fig. 1, where the pulse duration, τ_0 , is assumed to be equal to the ion transit time, i.e. $s_0 = \tau_0/\tau = 1.0$. The temporal and spatial variations of positive ions in the build-up stage ($s \leq s_0$) are shown by solid lines, and those in the decaying stage ($s > s_0$) by broken lines. The distributions of negative ions is symmetric with respect to those of positive ones, since the mobilities, k_{+} and k_{-} , are assumed to be equal. As shown in the figure, at the instant when the pulse ceases ($s = s_0$) the density distribution is not a rectangle but nearly a tangent. Moreover, in the overlap region the distribution are not rectangular, that is, the density of positive ions in the region is differ-

Fig. 1 Temporal variation of the spatial distribution of the normalized ion density for pulsed X-rays of which duration is equal to the ion transit time. The distributions in the build-up stage are shown by solid lines, and those in the decaying stage by broken lines.

Fig. 2 Dependence of the f -value on the normalized pulse duration. The parameter, Ms_0 , corresponds to the total exposure per pulse. Boag's formula is also shown by broken lines. For X-ray tube, the s_0 -value lies between about 0.5 and 5.0

ent from that of negative ones. The results mean that Boag's assumption cannot be accepted in this case.

In **Fig. 2** the f -value calculated with eq. (1) is compared with Boag's formula. The abscissa is the normalized pulse duration, which lies between about 0.5 and 5.0 for usual X-ray tube for medical diagnosis. The parameter, Ms_0 , is $(\alpha\tau) \cdot (q_0\tau_0)$, which corresponds to the total exposure per pulse. Let α be $1.0 \times 10^{-6} \text{ cm}^3 \text{ s}^{-1}$ and τ $1.0 \times 10^{-3} \text{ s}$, the Ms_0 -value of 1.0 corresponds nearly to the total exposure of 0.5 R per pulse. As shown in the figure by horizontal broken lines, Boag's formula gives the same f -value if the total exposure is constant. The true f -value, however, deviates distinctively from the Boag's formula as the normalized pulse duration increases. This situation will be often produced at diagnosis with an X-ray tube.

Even though the pulse duration is very short ($s_0 < 1.0 \times 10^{-3}$), the f -value becomes smaller than the formula with increasing the exposure rate. This is attributed mainly to the recombination loss during pulse which Boag omitted. We will have an opportunity of discussing about the f -value for ps- and ns-pulses of X-rays generated by an electron linear accelerator.¹⁰⁾

III EFFECT OF PULSE SHAPE ON COLLECTION EFFICIENCY

In above calculations, the shape of X-ray pulses

Photo 1 Typical waveform of pulsed X-rays emitted from an X-ray tube. It was observed with a plastic scintillator. Sweep rate: 1 ms/div.

Fig. 3 Normalized production rate, $A(s)$, for three pulse shapes of rectangle, triangle and that of **Photo 1**.

was assumed to be a rectangle for simplicity. However, a typical pulse shape of X-rays emitted from an X-ray tube is far from rectangular as shown in **Photo 1**, which was observed with a plastic scintillator.

In order to obtain the f -value for an arbitrary pulse shape, we introduce the normalized production rate, $A(s)$, as a function of the normalized time under the following condition:

$$\int_0^{s_0} A(s) ds = s_0. \quad (2)$$

Figure 3 shows the normalized production rate for three pulse shapes, *i.e.* rectangle, triangle and that of **Photo 1**. For rectangular pulse, the production rate is expressed as follows:

$$q(t) = \begin{cases} q_0 & 0 \leq t \leq \tau_0 \\ 0 & \tau_0 < t \leq \tau + \tau_0 \end{cases} \quad (3)$$

The equation is rewritten with the normalized variables as follows:

$$A(s) = \begin{cases} 1 & 0 \leq s \leq s_0 \\ 0 & s_0 < s \leq 1 + s_0 \end{cases} \quad (4)$$

For triangular pulse,

$$A(s) = \begin{cases} 4s/s_0 & 0 \leq s \leq s_0/2 \\ 4 - 4s/s_0 & s_0/2 < s \leq s_0 \\ 0 & s_0 < s \leq 1 + s_0 \end{cases} \quad (5)$$

The f -values for three pulse shapes are compared in Fig. 4, which shows the deviation from the f -value for rectangular pulse. In general, the f -value including the pulse shape effect becomes smaller than that for rectangular pulse shown in Fig. 3. It is found from Fig. 4 that the deviation reaches the maximum of about 10% near an s_0 -value of 1.0. Namely, the effect becomes most significant for pulsed X-rays in the diagnostic region. Another notice should be taken that the waveform of the pulsed X-rays from the X-ray tube may be approximated by a triangle within a few percent error.

On the other hand, the effect of the pulse shape on the output currents of the ionization chamber is clarified as follows. The waveforms of the currents have been derived theoretically by a few authors,^{6,12)} but they had neglected the pulse shape. Here, we define the normalized current density, $\zeta(s)$, in the following formula:⁸⁾

$$\zeta(s) = \frac{j(t)}{eq_0 d} = \int_0^1 [v_+(s, \xi) + v_-(s, \xi)] d\xi \quad (6)$$

where $j(t)$ is the output current density, e the electric charge and $v_{\pm}(s, \xi)$ are the solution of eq. (1). For pulsed X-rays of which s_0 -value is near 1.0, the current density cannot be obtained by Boag's formula, because the drift of ions during pulse is neglected in the assumptions. Figure 5 shows the results of numerical calculations of eq. (6) in the case that $s_0 = 1.0$ and $M = 10.0$. The values correspond nearly to the pulse duration of 1.0 ms and the total exposure of 5.0 R. The normalized current density for rectangular pulse is shown by a solid line, which is similar to that obtained by LIVINGSTON.⁶⁾ A broken line in the figure shows the current density for triangular pulse. It is clear that the output currents of the

Fig. 4 Deviation from the f -value for rectangular pulse. The effect of the pulse shape is serious at an s_0 -value of about 1.0. The shape for the X-ray tube can be approximated by a triangle with a few percent deviation.

Fig. 5 Normalized current densities as a function of the normalized time for both rectangular and triangular pulses. The difference between them is clearly shown.

ionization chamber is affected sensitively by the pulse shape of X-rays.

IV EXPERIMENTAL RESULTS AND DISCUSSION

1. Experimental apparatus

The pulsed X-rays in this experiment were generated by an X-ray tube (Circlex 1/2 P38C) installed at the Hospital of RIMD (Research Institute for Microbial Diseases) of Osaka University. The tube was operated under the conditions of the tube voltage of 60 kV and the current of the electron beam of 500 mA. The

Photo 2 Output current shapes for the X-ray tube, which were observed with an ionization chamber of parallel plate. To the chamber were applied the voltages of (a) 300 V, (b) 200 V and (c) 100 V. Sweep rate: 2 ms/div.

total filtration was 1.2 mm-aluminum equivalent. The duration of pulsed X-rays was 5.46 ms as shown in **Photo 1**.

An ionization chamber of parallel plate was used in order to compare the experimental results with the calculations of the one-dimensional

Fig. 6 Output current shapes calculated under the same conditions as in **Fig. 5**. The normalized pulse durations are (a) 2.20, (b) 1.46 and (c) 0.73. A good agreement with **Fig. 5** is recognized.

equations. The chamber was made of polyethylene except the connectors and the conducting materials. The wall was a thin polyethylene sheet of 200 μm thickness to attain the C.P.E. (charged particle equilibrium) condition. The electrodes and the guard ring were the evaporated aluminum films. The gap distance between the electrodes was 10.0 mm, and the active volume was 12.6 cm^3 . The ion mobility in air has already been estimated to be $1.34 \text{ cm}^2\text{s}^{-1}\text{V}^{-1}$ from the experiments in the X-ray field of very short pulse generated by an electron linear accelerator. Accordingly, the ion transit time at an applied voltage of V is expressed as $\tau = 0.746/V$.

2. Output currents of ionization chamber

Photo 2 shows the waveforms of the output currents of the ionization chamber, which was placed at a distance of 26.2 cm from the target of the X-ray tube. To the chamber were applied the voltages of (a) 300 V, (b) 200 V and (c) 100 V, respectively. The normalized pulse duration, therefore, are 2.20, 1.46 and 0.73, respectively. The time constant of the detecting circuit is adjusted to be less than 10 μs , so the photographs represent the exact ion current.

In order to obtain the currents theoretically, the calculations of eq. (6) are carried out by putting the waveform of **Photo 1** into eq. (1). The results are shown in **Fig. 6**, where the abscissa is the time normalized by the pulse duration, that is, $s/s_0 = t/\tau_0 = 183t$. The M -values of 1.91, 4.29 and 17.2 used in the calculations will be evaluated in the succeeding subsection. **Figure 6** and **Photo 2** show a good agreement between the calculated

and experimental results. This will mean that some assumptions made in eq. (1) are valid, and therefore our computation is applicable to pulsed X-rays in question as well as those generated by the particle accelerator.

3. Saturation curves

It is difficult to determine the f -value in an unknown pulsed X-ray field, because the f -value itself is a function of the exposure which one wants to know. Thus, we proposed a method in the previous paper,^{10,11)} and the procedure is briefly stated.

It is necessary in the procedure to measure the pulse duration and the ionic charges at several applied voltages. By plotting the collected charges against the applied voltage, a so-called saturation curve is drawn. On the other hand, the saturation curve can be obtained theoretically in the following procedure provided that both M - and s_0 -values are given for a reference applied voltage (V_r):

(1) Both M - and s_0 -values for an arbitrary voltage (V_1) are calculated easily by multiplying factors of $(V_r/V_1)^2$ and V_1/V_r , respectively.

(2) The f -values for all voltages are found from Fig. 2.

(3) The f -values are plotted against the applied voltage.

The M -value for the reference voltage can be evaluated by finding the best fit to the experimental curve among theoretical ones for various M -values. Then, the corresponding f -value can be determined uniquely.

In Fig. 7 is shown an example of the normalized saturation curves, where the reference voltage is 1,000 V. Both ordinate and abscissa are normalized by respective value for the reference voltage. Namely, the ordinate is the relative collected-charges and the abscissa the relative applied voltage. They correspond to the relative f -value and the inverse ratio of Ms_0 -value, respectively. The experimental results observed at a distance of 26.2 cm from the X-ray source are shown by dots. The three theoretical curves are also shown in the figure. We found no curve fit to all measured points among the curves predicted by Boag's formula. Two of them are shown by broken lines. The upper curve agrees well with the points at the applied voltage higher than 600 V, but deviates from the rest. The

Fig. 7 Normalized saturation curve. The experimental results are shown by dots, and the theoretical curves are shown by solid and broken lines. Only our calculations fit well to all measured points. ---, Boag's formula; —, our calculations; ●, experiments (26.2 cm from X-ray source).

Fig. 8 Dependence of the ionic charge density or the exposure on the distance from the X-ray source. The exposure corrected by our method follows the inverse square law, whereas that corrected by Boag's formula becomes much larger near the source. Δ , measured at 400 V; $\circ$, corrected by our calculations; $\bullet$, corrected by Boag's formula.

lower does not pass the points except for two at the voltages of 100 and 1,000 V. On the other hand, according to our calculations a best fit of the saturation curves to all points is found as shown in the figure by a solid line. The curve gives a true f -value of 0.971 at a voltage of 1,000 V. In general, Boag's formula brings about the underestimate of the f -value, that is, 0.81 or 0.90 in this case.

Figure 8 shows the measured and corrected values of the ionic charge density (exposure) as a function of the distance from the X-ray source. The measurement was performed by adjusting the

beam current as reproducible as possible. The radius of the beam of the X-ray tube is so small that X-rays are considered to be emitted from a point source. Furthermore, the absorption of the X-rays in air is negligible, since the mean energy is about 28 keV and the absorption at a distance of 80 cm from the filter is about 4%. Accordingly, the exposure should obey the inverse square law about the distance. As shown in the figure, the exposure corrected by our method lies near a straight line on log-log scale, whereas that corrected by Boag's formula deviates near the source. It is concluded from the result that the f -value in the X-ray field can be accurately evaluated by our method.

In view of a practical situation, we propose the following procedure to estimate the exposure of diagnostic X-rays:

- (1) One measures the pulse duration, and if possible, the pulse shape of X-rays with some detector such as a scintillator. Otherwise, the pulse shape may be approximated by a triangle.
- (2) One measures the ionic charge densities at two or more applied voltages with a parallel-plate ionization chamber.
- (3) One draws the saturation curve, and then obtains the f -value by the computation of the fitting of the theoretical curve.

V CONCLUSION

The collection efficiency of a parallel-plate ionization chamber exposed to pulsed X-rays for medical diagnosis was obtained numerically. It was found that the true f -value deviated from the well-known Boag's formula when the pulse duration became comparable with the ion transit time between the electrodes. The effect of the pulse shape of the f -value was also clarified. It was pointed out that the effect became important especially for pulsed X-rays of which duration was nearly equal to the ion transit time.

Experiments were carried out in a diagnostic X-ray field by adjusting the normalized pulse duration to about 1.0. The output currents could be analyzed by our calculations including the effect of the pulse shape. The saturation curve were also obtained. The f -value was evaluated by such a method as was previously reported. The exposure corrected by the f -value was obtained as a function of the distance from

the X-ray source. It was found from the experimental confirmation of the inverse square law that our calculations and the method for determining the f -value could be applied to pulsed X-rays for medical diagnosis.

In the present report has been discussed some problems only for parallel-plate ionization chambers. However, in most cases the chambers are thimble or cylindrical. In order to discuss about the problems quantitatively for such chambers, complicated and troublesome computation to obtain the f -value must be made by combining the equation of continuity in cylindrical co-ordinates with Poisson's equation. Further treatment is the same as has been described in the present report. It goes without saying that the normalized pulse duration is a most important parameter in both cases. It is considered that one should make the ion transit time as long as possible lest the f -value should deviate from Boag's formula.

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